

# Computer algebra independent integration tests

3-Logarithms/3.2.3-u-log-e-f-a+b-x<sup>p</sup>-c+d-x<sup>q</sup>-r<sup>s</sup>

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## Contents

<b>1</b>	<b>Introduction</b>	<b>9</b>
1.1	Listing of CAS systems tested . . . . .	9
1.2	Results . . . . .	10
1.3	Performance . . . . .	13
1.4	list of integrals that has no closed form antiderivative . . . . .	14
1.5	list of integrals solved by CAS but has no known antiderivative . . . . .	14
1.6	list of integrals solved by CAS but failed verification . . . . .	14
1.7	Timing . . . . .	15
1.8	Verification . . . . .	15
1.9	Important notes about some of the results . . . . .	15
1.10	Design of the test system . . . . .	17
<b>2</b>	<b>detailed summary tables of results</b>	<b>19</b>
2.1	List of integrals sorted by grade for each CAS . . . . .	19
2.2	Detailed conclusion table per each integral for all CAS systems . . . . .	21
2.3	Detailed conclusion table specific for Rubi results . . . . .	43
<b>3</b>	<b>Listing of integrals</b>	<b>47</b>
3.1	$\int \left(f + \frac{g}{x}\right)^3 \left(A + B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$ . . . . .	47
3.2	$\int \left(f + \frac{g}{x}\right)^2 \left(A + B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$ . . . . .	53

3.3	$\int \left(f + \frac{g}{x}\right) \left(A + B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$	59
3.4	$\int \frac{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}} dx$	64
3.5	$\int \frac{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2} dx$	69
3.6	$\int \frac{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3} dx$	75
3.7	$\int (a+bx)^4 \log \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$	81
3.8	$\int (a+bx)^3 \log \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$	86
3.9	$\int (a+bx)^2 \log \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$	90
3.10	$\int (a+bx) \log \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$	94
3.11	$\int \frac{\log \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx} dx$	98
3.12	$\int \frac{\log \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2} dx$	102
3.13	$\int \frac{\log \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3} dx$	106
3.14	$\int \frac{\log \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4} dx$	110
3.15	$\int \frac{\log \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5} dx$	115
3.16	$\int (a+bx)^4 \log^2 \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$	120
3.17	$\int (a+bx)^3 \log^2 \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$	129
3.18	$\int (a+bx)^2 \log^2 \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$	137
3.19	$\int (a+bx) \log^2 \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$	144
3.20	$\int \frac{\log^2 \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx} dx$	151
3.21	$\int \frac{\log^2 \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2} dx$	159
3.22	$\int \frac{\log^2 \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3} dx$	165
3.23	$\int \frac{\log^2 \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4} dx$	172
3.24	$\int \frac{\log^2 \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5} dx$	180
3.25	$\int (g+hx)^4 \log \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$	187

3.26	$\int (g + hx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$	192
3.27	$\int (g + hx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$	197
3.28	$\int (g + hx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$	201
3.29	$\int \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$	205
3.30	$\int \frac{\log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{g + hx} dx$	209
3.31	$\int \frac{\log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{(g + hx)^2} dx$	213
3.32	$\int \frac{\log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{(g + hx)^3} dx$	217
3.33	$\int \frac{\log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{(g + hx)^4} dx$	221
3.34	$\int \frac{\log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{(g + hx)^5} dx$	226
3.35	$\int (g + hx)^3 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$	233
3.36	$\int (g + hx)^2 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$	242
3.37	$\int (g + hx) \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$	251
3.38	$\int \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$	258
3.39	$\int \frac{\log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{g + hx} dx$	263
3.40	$\int \frac{\log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{(g + hx)^2} dx$	272
3.41	$\int \frac{\log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{(g + hx)^3} dx$	281
3.42	$\int \frac{\log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{(g + hx)^4} dx$	290
3.43	$\int \frac{\left( a + b \log \left( \frac{\sqrt{1 - cx}}{\sqrt{1 + cx}} \right) \right)^n}{1 - c^2 x^2} dx$	300
3.44	$\int \frac{\left( a + b \log \left( \frac{\sqrt{1 - cx}}{\sqrt{1 + cx}} \right) \right)^3}{1 - c^2 x^2} dx$	304
3.45	$\int \frac{\left( a + b \log \left( \frac{\sqrt{1 - cx}}{\sqrt{1 + cx}} \right) \right)^2}{1 - c^2 x^2} dx$	308
3.46	$\int \frac{a + b \log \left( \frac{\sqrt{1 - cx}}{\sqrt{1 + cx}} \right)}{1 - c^2 x^2} dx$	312
3.47	$\int \frac{1}{(1 - c^2 x^2) \left( a + b \log \left( \frac{\sqrt{1 - cx}}{\sqrt{1 + cx}} \right) \right)} dx$	316

- 3.48  $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2} dx \dots\dots\dots 320$
- 3.49  $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3} dx \dots\dots\dots 324$
- 3.50  $\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx \dots\dots\dots 328$
- 3.51  $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)(s+t\log(i(g+hx)^n))^2}{gk+hkx} dx \dots\dots\dots 331$
- 3.52  $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)(s+t\log(i(g+hx)^n))}{gk+hkx} dx \dots\dots\dots 337$
- 3.53  $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{gk+hkx} dx \dots\dots\dots 343$
- 3.54  $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)(s+t\log(i(g+hx)^n))} dx \dots\dots\dots 347$
- 3.55  $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)(s+t\log(i(g+hx)^n))^2} dx \dots\dots\dots 350$
- 3.56  $\int \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} dx \dots\dots\dots 354$
- 3.57  $\int \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} dx \dots\dots\dots 362$
- 3.58  $\int \frac{\log\left(i(j(hx)^t)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} dx \dots\dots\dots 368$
- 3.59  $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} dx \dots\dots\dots 373$
- 3.60  $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log\left(i(j(hx)^t)^u\right)} dx \dots\dots\dots 377$
- 3.61  $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log^2\left(i(j(hx)^t)^u\right)} dx \dots\dots\dots 380$
- 3.62  $\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx \dots\dots\dots 383$
- 3.63  $\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx \dots\dots\dots 386$
- 3.64  $\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx \dots\dots\dots 389$
- 3.65  $\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)} dx \dots\dots\dots 393$
- 3.66  $\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx \dots\dots\dots 396$
- 3.67  $\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)} dx \dots\dots\dots 399$

3.68	$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)} dx \dots\dots\dots$	407
3.69	$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)} dx \dots\dots\dots$	414
3.70	$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx \dots\dots\dots$	420
3.71	$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx \dots\dots\dots$	424
3.72	$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} dx \dots\dots\dots$	428
3.73	$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} dx \dots\dots\dots$	432
3.74	$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right) dx \dots\dots\dots$	436
3.75	$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right) dx \dots\dots\dots$	440
3.76	$\int \frac{x^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx \dots\dots\dots$	444
3.77	$\int \frac{x^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx \dots\dots\dots$	450
3.78	$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx \dots\dots\dots$	456
3.79	$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx \dots\dots\dots$	461
3.80	$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f-gx^2)} dx \dots\dots\dots$	466
3.81	$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)} dx \dots\dots\dots$	472
3.82	$\int \frac{x^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} dx \dots\dots\dots$	478
3.83	$\int \frac{x^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} dx \dots\dots\dots$	486
3.84	$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} dx \dots\dots\dots$	494
3.85	$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} dx \dots\dots\dots$	500

3.86	$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f+gx+hx^2)} dx$	505
3.87	$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)} dx$	512
3.88	$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx} dx$	520
3.89	$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)} dx$	524
3.90	$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)} dx$	527
3.91	$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)} dx$	531
3.92	$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)} dx$	535
3.93	$\int \log\left(\frac{c(b+ax)}{x}\right) dx$	540
3.94	$\int \log^2\left(\frac{c(b+ax)}{x}\right) dx$	544
3.95	$\int \log^3\left(\frac{c(b+ax)}{x}\right) dx$	548
3.96	$\int \log\left(\frac{c(b+ax)^2}{x^2}\right) dx$	553
3.97	$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx$	556
3.98	$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right) dx$	561
3.99	$\int \log\left(\frac{cx^2}{(b+ax)^2}\right) dx$	565
3.100	$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right) dx$	568
3.101	$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right) dx$	573
3.102	$\int \frac{\text{PolyLog}\left(2, 1 + \frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)} dx$	577
3.103	$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)} dx$	580
3.104	$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx} dx$	584
3.105	$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)} dx$	588
3.106	$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)} dx$	592
3.107	$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx} dx$	597
3.108	$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)} dx$	604

**4 Listing of Grading functions****611**





# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 108 ]. This is test number [ 61 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 98.15 ( 106 )	% 1.85 ( 2 )
Mathematica	% 100. ( 108 )	% 0. ( 0 )
Maple	% 23.15 ( 25 )	% 76.85 ( 83 )
Maxima	% 60.19 ( 65 )	% 39.81 ( 43 )
Fricas	% 36.11 ( 39 )	% 63.89 ( 69 )
Sympy	% 13.89 ( 15 )	% 86.11 ( 93 )
Giac	% 35.19 ( 38 )	% 64.81 ( 70 )

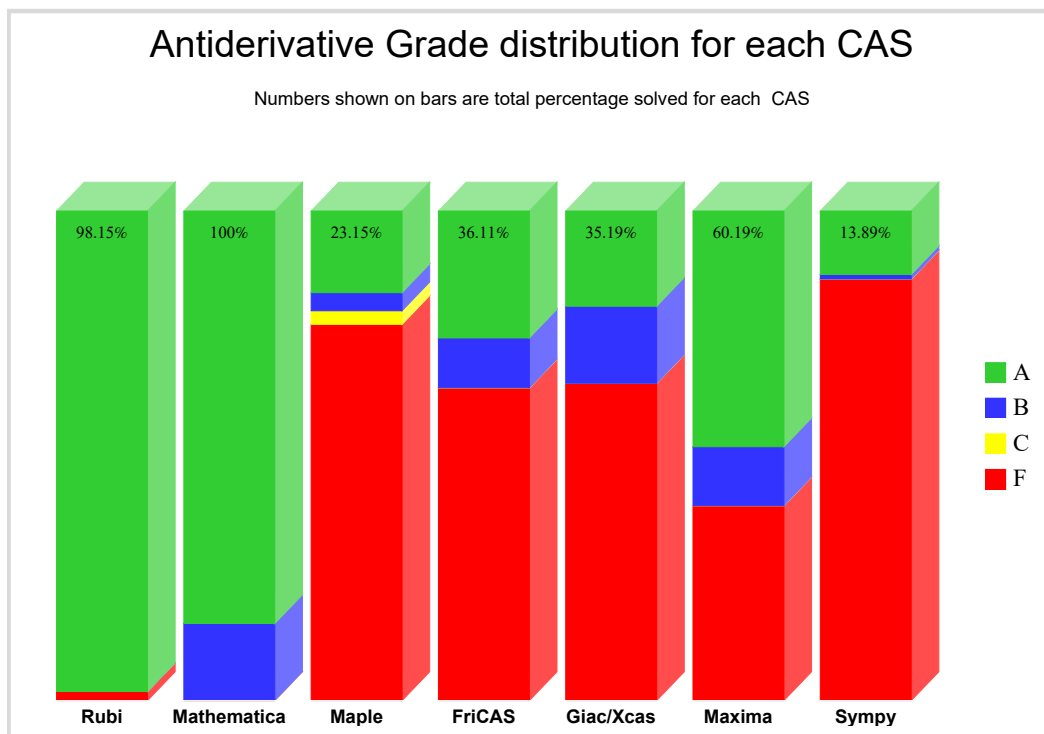
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

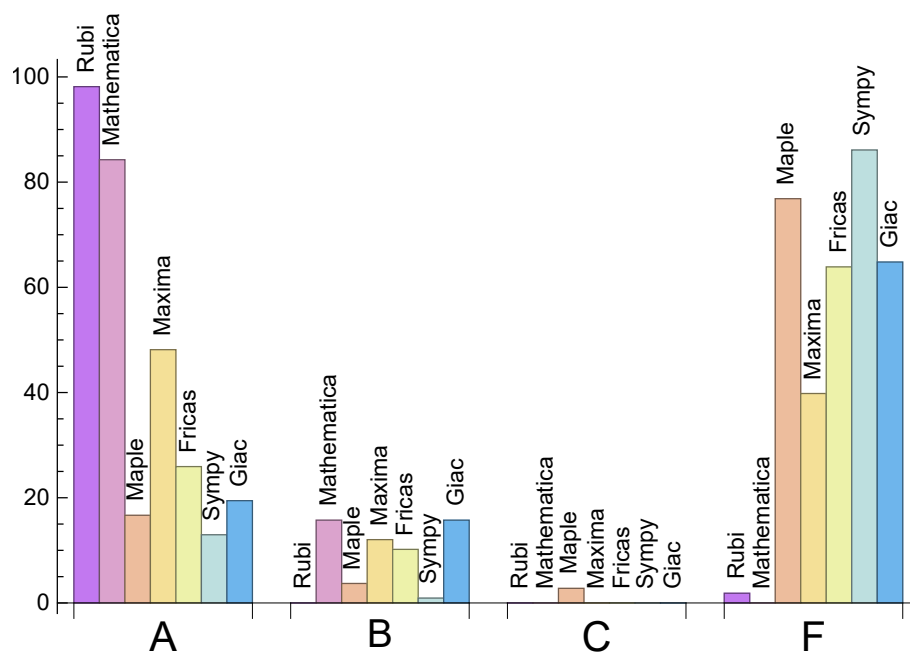
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	98.15	0.	0.	1.85
Mathematica	84.26	15.74	0.	0.
Maple	16.67	3.7	2.78	76.85
Maxima	48.15	12.04	0.	39.81
Fricas	25.93	10.19	0.	63.89
Sympy	12.96	0.93	0.	86.11
Giac	19.44	15.74	0.	64.81

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



## 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.42	350.24	0.9	182.5	1.
Mathematica	1.66	1551.08	2.54	165.	0.96
Maple	3.93	318.8	2.63	29.	1.
Maxima	1.22	558.37	2.09	193.	1.65
Fricas	1.06	353.05	2.82	103.	2.81
Sympy	19.15	32.2	0.95	26.	0.93
Giac	0.94	548.61	2.82	96.5	1.8

## 1.4 list of integrals that has no closed form antiderivative

{54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {67, 68}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used



#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

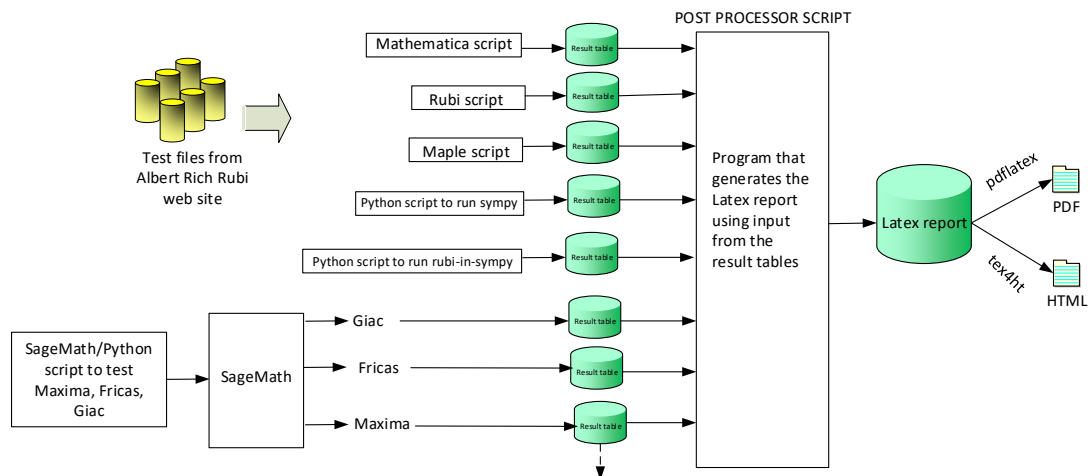
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
        leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

Nasser M. Abbasi  
June 22, 2018

# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

B grade: { }

C grade: { }

F grade: { 74, 75 }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 66, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105 }

B grade: { 16, 17, 24, 40, 41, 42, 51, 56, 57, 58, 67, 68, 69, 92, 106, 107, 108 }

C grade: { }

F grade: { }

## 2.1.3 Maple

A grade: { 29, 54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 89, 93, 96, 102, 106 }

B grade: { 88, 99, 104, 107 }

C grade: { 64, 74, 75 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 94, 95, 97, 98, 100, 101, 103, 105, 108 }

## 2.1.4 Maxima

A grade: { 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 40, 41, 47, 48, 53, 54, 55, 59, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 74, 75, 93, 94, 96, 97, 99, 100 }

B grade: { 7, 15, 24, 25, 34, 42, 44, 45, 46, 49, 50, 88, 89 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 20, 39, 43, 51, 52, 56, 57, 58, 64, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 98, 101, 102, 103, 104, 105, 106, 107, 108 }

## 2.1.5 FriCAS

A grade: { 10, 12, 28, 29, 43, 46, 47, 48, 49, 50, 54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 74, 75, 89, 93, 96, 99 }

B grade: { 7, 8, 9, 13, 14, 15, 25, 26, 27, 44, 45 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

## 2.1.6 Sympy

A grade: { 44, 45, 46, 47, 62, 63, 65, 66, 74, 75, 89, 93, 96, 99 }

B grade: { 50 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60,

61, 64, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

## 2.1.7 Giac

A grade: { 12, 13, 31, 47, 48, 54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 89, 93, 96, 99 }

B grade: { 7, 8, 9, 10, 14, 15, 25, 26, 27, 28, 29, 32, 33, 34, 46, 49, 50 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 30, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	385	336	0	0	0	0	0
normalized size	1	0.95	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.493	0.413	0.154	0.	0.	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	242	217	0	0	0	0	0
normalized size	1	0.92	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.337	0.23	0.086	0.	0.	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	135	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.225	0.096	0.089	0.	0.	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	185	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.332	0.148	0.199	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	352	295	0	0	0	0	0
normalized size	1	1.09	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.494	0.594	0.084	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	562	470	0	0	0	0	0
normalized size	1	1.06	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.797	1.653	0.084	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	185	0	533	1315	0	1002
normalized size	1	1.	0.92	0.	2.65	6.54	0.	4.99
time (sec)	N/A	0.095	0.308	0.734	1.307	0.864	0.	1.26

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	154	0	385	980	0	774
normalized size	1	1.	0.9	0.	2.24	5.7	0.	4.5
time (sec)	N/A	0.072	0.211	0.422	1.336	0.957	0.	1.406

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	127	0	262	693	0	566
normalized size	1	1.	0.89	0.	1.83	4.85	0.	3.96
time (sec)	N/A	0.061	0.135	0.386	1.288	0.88	0.	1.308

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	105	0	159	435	0	381
normalized size	1	1.	0.91	0.	1.37	3.75	0.	3.28
time (sec)	N/A	0.043	0.193	0.174	1.354	0.82	0.	1.35

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	93	0	221	0	0	0
normalized size	1	1.	0.87	0.	2.07	0.	0.	0.
time (sec)	N/A	0.083	0.109	0.436	3.137	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	89	0	134	267	0	151
normalized size	1	1.	0.94	0.	1.41	2.81	0.	1.59
time (sec)	N/A	0.037	0.058	0.421	1.265	0.902	0.	1.197

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	116	0	223	689	0	332
normalized size	1	1.	0.86	0.	1.65	5.1	0.	2.46
time (sec)	N/A	0.056	0.273	0.424	1.238	0.871	0.	1.192

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	141	0	390	1177	0	633
normalized size	1	1.	0.86	0.	2.38	7.18	0.	3.86
time (sec)	N/A	0.069	0.392	0.425	1.278	0.854	0.	1.211

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	164	0	620	1758	0	1010
normalized size	1	1.	0.85	0.	3.21	9.11	0.	5.23
time (sec)	N/A	0.085	0.407	0.424	1.279	0.849	0.	1.35

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	920	920	2508	0	1918	0	0	0
normalized size	1	1.	2.73	0.	2.08	0.	0.	0.
time (sec)	N/A	0.845	2.686	0.414	1.579	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	805	805	1853	0	1446	0	0	0
normalized size	1	1.	2.3	0.	1.8	0.	0.	0.
time (sec)	N/A	0.665	1.859	0.402	1.471	0.	0.	0.



Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	686	686	1211	0	1038	0	0	0
normalized size	1	1.	1.77	0.	1.51	0.	0.	0.
time (sec)	N/A	0.533	1.127	0.404	1.428	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	540	540	781	0	680	0	0	0
normalized size	1	1.	1.45	0.	1.26	0.	0.	0.
time (sec)	N/A	0.391	0.57	0.166	1.504	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	460	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.492	0.171	0.416	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	465	465	411	0	529	0	0	0
normalized size	1	1.	0.88	0.	1.14	0.	0.	0.
time (sec)	N/A	0.387	0.836	0.41	1.342	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	632	632	872	0	1019	0	0	0
normalized size	1	1.	1.38	0.	1.61	0.	0.	0.
time (sec)	N/A	0.49	1.605	0.412	1.481	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	764	764	1407	0	1690	0	0	0
normalized size	1	1.	1.84	0.	2.21	0.	0.	0.
time (sec)	N/A	0.606	2.731	0.414	1.761	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	884	884	14573	0	2452	0	0	0
normalized size	1	1.	16.49	0.	2.77	0.	0.	0.
time (sec)	N/A	0.738	7.025	0.418	2.175	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	275	0	842	1932	0	1715
normalized size	1	1.	0.82	0.	2.52	5.78	0.	5.13
time (sec)	N/A	0.186	0.352	0.393	1.203	1.542	0.	1.522

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	231	0	582	1377	0	1254
normalized size	1	1.	0.84	0.	2.11	4.99	0.	4.54
time (sec)	N/A	0.127	0.31	0.397	1.216	1.249	0.	1.328

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	209	0	363	917	0	837
normalized size	1	1.	0.96	0.	1.67	4.21	0.	3.84
time (sec)	N/A	0.099	0.254	0.431	1.258	1.161	0.	1.305

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	120	0	193	524	0	479
normalized size	1	1.	0.75	0.	1.21	3.28	0.	2.99
time (sec)	N/A	0.069	0.196	0.174	1.202	1.208	0.	1.31

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	57	61	101	182	0	242
normalized size	1	1.	0.93	1.	1.66	2.98	0.	3.97
time (sec)	N/A	0.015	0.063	0.071	1.157	1.089	0.	1.238

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	163	0	251	0	0	0
normalized size	1	1.	1.1	0.	1.7	0.	0.	0.
time (sec)	N/A	0.12	0.083	0.622	1.22	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	93	0	166	0	0	257
normalized size	1	1.	0.73	0.	1.3	0.	0.	2.01
time (sec)	N/A	0.052	0.233	0.493	1.253	0.	0.	1.469

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	206	0	313	0	0	1413
normalized size	1	1.	1.02	0.	1.55	0.	0.	7.
time (sec)	N/A	0.112	0.55	0.496	1.259	0.	0.	1.579

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	254	0	616	0	0	2383
normalized size	1	1.	0.98	0.	2.37	0.	0.	9.17
time (sec)	N/A	0.152	0.96	0.519	1.298	0.	0.	1.864

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	480	0	1048	0	0	6884
normalized size	1	1.	1.51	0.	3.3	0.	0.	21.65
time (sec)	N/A	0.195	1.822	0.509	1.412	0.	0.	2.777

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2240	2220	1386	0	2429	0	0	0
normalized size	1	0.99	0.62	0.	1.08	0.	0.	0.
time (sec)	N/A	2.436	3.013	0.392	1.452	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1645	1657	899	0	1516	0	0	0
normalized size	1	1.01	0.55	0.	0.92	0.	0.	0.
time (sec)	N/A	1.758	1.89	0.385	1.379	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1063	1097	480	0	841	0	0	0
normalized size	1	1.03	0.45	0.	0.79	0.	0.	0.
time (sec)	N/A	1.163	1.127	0.176	1.376	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	437	0	402	0	0	0
normalized size	1	1.	1.62	0.	1.49	0.	0.	0.
time (sec)	N/A	0.153	0.216	0.131	1.366	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1471	2096	1370	0	0	0	0	0
normalized size	1	1.42	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	1.931	0.281	0.578	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	832	878	2930	0	1006	0	0	0
normalized size	1	1.06	3.52	0.	1.21	0.	0.	0.
time (sec)	N/A	0.932	2.671	0.498	1.577	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1304	1362	15976	0	2507	0	0	0
normalized size	1	1.04	12.25	0.	1.92	0.	0.	0.
time (sec)	N/A	1.408	6.322	0.5	2.45	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1957	2013	47110	0	6388	0	0	0
normalized size	1	1.03	24.07	0.	3.26	0.	0.	0.
time (sec)	N/A	2.102	6.541	0.52	5.212	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	138	0	0
normalized size	1	1.	1.	0.	0.	3.29	0.	0.
time (sec)	N/A	0.08	0.029	0.485	0.	2.193	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	710	248	65	0
normalized size	1	1.	1.	0.	19.19	6.7	1.76	0.
time (sec)	N/A	0.061	0.009	0.411	1.352	2.012	47.02	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	362	184	65	0
normalized size	1	1.	1.	0.	9.78	4.97	1.76	0.
time (sec)	N/A	0.063	0.01	0.342	1.277	2.129	23.341	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	142	119	61	116
normalized size	1	1.	1.	0.	3.84	3.22	1.65	3.14
time (sec)	N/A	0.038	0.006	0.364	1.207	1.995	30.809	1.263

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	49	72	53	42
normalized size	1	1.	1.	0.	1.44	2.12	1.56	1.24
time (sec)	N/A	0.068	0.036	0.351	1.482	2.005	164.757	1.161

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	46	72	0	46
normalized size	1	1.	1.	0.	1.35	2.12	0.	1.35
time (sec)	N/A	0.065	0.01	0.356	1.583	1.991	0.	1.259

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	108	142	0	115
normalized size	1	1.	1.	0.	2.92	3.84	0.	3.11
time (sec)	N/A	0.065	0.011	0.349	1.609	1.847	0.	1.304

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	112	59	65	78
normalized size	1	1.	1.	0.	3.73	1.97	2.17	2.6
time (sec)	N/A	0.023	0.009	0.389	1.924	1.947	13.519	1.298

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	22595	0	0	0	0	0
normalized size	1	1.	55.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.471	7.545	0.982	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	436	0	0	0	0	0
normalized size	1	1.	1.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.288	2.552	0.75	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	166	0	275	0	0	0
normalized size	1	1.	0.97	0.	1.6	0.	0.	0.
time (sec)	N/A	0.136	0.089	0.491	1.262	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	50	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.362	0.723	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	50	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	2.743	0.739	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	1241	0	0	0	0	0
normalized size	1	1.	3.78	0.	0.	0.	0.	0.
time (sec)	N/A	1.252	1.827	2.213	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	839	0	0	0	0	0
normalized size	1	1.	3.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.91	0.913	1.8	0.	0.	0.	0.



Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	451	0	0	0	0	0
normalized size	1	1.	2.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.603	0.448	1.784	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	0	170	0	0	0
normalized size	1	1.	0.96	0.	2.1	0.	0.	0.
time (sec)	N/A	0.066	0.063	0.123	1.208	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.545	0.445	1.772	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.425	2.462	1.78	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	5.094	0.385	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	5.084	0.209	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	66	450	0	0	0	0
normalized size	1	1.	0.8	5.49	0.	0.	0.	0.
time (sec)	N/A	0.177	5.027	0.121	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	5.098	0.089	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	33.103	0.327	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	620	649	18164	0	0	0	0	0
normalized size	1	1.05	29.3	0.	0.	0.	0.	0.
time (sec)	N/A	1.133	21.544	7.167	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	496	517	9211	0	0	0	0	0
normalized size	1	1.04	18.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.819	10.071	5.039	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	384	1408	0	0	0	0	0
normalized size	1	1.04	3.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.558	6.028	4.115	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	106	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.5	1.908	38.288	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	92	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	1.287	48.165	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	112	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.519	0.632	1.07	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	112	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.484	0.575	1.008	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	A	A	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	45	0	44	662	80	104	44	0
normalized size	1	0.	0.98	14.71	1.78	2.31	0.98	0.
time (sec)	N/A	0.516	0.312	1.322	1.497	2.068	3.06	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	A	A	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	45	0	45	503	78	103	44	0
normalized size	1	0.	1.	11.18	1.73	2.29	0.98	0.
time (sec)	N/A	0.512	0.085	1.214	1.68	1.992	3.329	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	560	461	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.735	0.387	0.493	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	550	467	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.565	0.24	0.418	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	413	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.352	0.121	0.44	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	468	421	0	0	0	0	0
normalized size	1	1.61	1.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.317	0.102	0.474	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	518	518	487	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.602	0.187	0.457	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	596	596	479	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.583	0.309	0.455	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1046	1046	1240	0	0	0	0	0
normalized size	1	1.	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	1.717	1.377	1.511	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	831	831	1105	0	0	0	0	0
normalized size	1	1.	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	1.068	5.472	1.393	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	685	685	539	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.636	0.685	1.349	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	545	515	0	0	0	0	0
normalized size	1	1.36	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.518	0.308	1.362	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	800	800	625	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.98	0.94	1.414	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	995	995	721	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	1.291	0.825	1.398	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	84	97	128	0	0	0
normalized size	1	1.	1.83	2.11	2.78	0.	0.	0.
time (sec)	N/A	0.159	0.017	0.098	1.129	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	29	190	38	14	24
normalized size	1	1.	1.	1.45	9.5	1.9	0.7	1.2
time (sec)	N/A	0.058	0.096	0.059	1.195	1.982	0.391	1.154

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	76	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	0.014	0.531	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	110	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.248	0.038	1.102	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	559	0	0	0	0	0
normalized size	1	1.	3.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.25	0.456	5.443	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	28	44	34	63	20	35
normalized size	1	1.	1.12	1.76	1.36	2.52	0.8	1.4
time (sec)	N/A	0.012	0.005	0.188	1.177	1.937	0.342	1.257

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	63	0	153	0	0	0
normalized size	1	1.	0.94	0.	2.28	0.	0.	0.
time (sec)	N/A	0.074	0.014	0.655	1.179	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	91	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.025	0.549	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	40	38	93	26	39
normalized size	1	1.	1.	1.43	1.36	3.32	0.93	1.39
time (sec)	N/A	0.007	0.003	0.125	1.192	1.877	0.312	1.147

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	106	0	159	0	0	0
normalized size	1	1.	1.58	0.	2.37	0.	0.	0.
time (sec)	N/A	0.157	0.016	0.526	1.224	0.	0.	0.



Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	98	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	0.025	0.504	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	79	38	88	26	39
normalized size	1	1.	1.	2.82	1.36	3.14	0.93	1.39
time (sec)	N/A	0.007	0.003	0.224	1.241	1.962	0.364	1.263

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	106	0	159	0	0	0
normalized size	1	1.	1.58	0.	2.37	0.	0.	0.
time (sec)	N/A	0.155	0.014	0.94	1.145	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	102	98	0	0	0	0	0
normalized size	1	1.04	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.025	0.786	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	36	0	0	0	0
normalized size	1	1.	0.86	1.03	0.	0.	0.	0.
time (sec)	N/A	0.064	0.012	0.062	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	68	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	0.024	1.318	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	149	135	879	0	0	0	0
normalized size	1	1.06	0.96	6.28	0.	0.	0.	0.
time (sec)	N/A	0.18	0.042	0.065	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	96	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.164	0.027	2.268	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	206	1636	357	0	0	0	0
normalized size	1	1.01	8.02	1.75	0.	0.	0.	0.
time (sec)	N/A	0.254	0.469	0.063	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	334	1080	4733	0	0	0	0
normalized size	1	1.04	3.35	14.7	0.	0.	0.	0.
time (sec)	N/A	0.507	0.234	0.072	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	445	908	0	0	0	0	0
normalized size	1	1.03	2.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.594	0.799	2.336	0.	0.	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [42] had the largest ratio of [ 0.5484 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	20	10	0.95	32	0.312
2	A	16	10	0.92	32	0.312
3	A	12	7	1.	30	0.233
4	A	14	8	1.	32	0.25
5	A	18	11	1.09	32	0.344
6	A	22	11	1.06	32	0.344
7	A	4	3	1.	29	0.103
8	A	4	3	1.	29	0.103
9	A	4	3	1.	29	0.103
10	A	4	2	1.	27	0.074
11	A	6	6	1.	29	0.207
12	A	5	4	1.	29	0.138
13	A	4	3	1.	29	0.103
14	A	4	3	1.	29	0.103
15	A	4	3	1.	29	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	32	14	1.	31	0.452
17	A	28	14	1.	31	0.452
18	A	24	14	1.	31	0.452
19	A	20	13	1.	29	0.448
20	A	19	15	1.	31	0.484
21	A	20	12	1.	31	0.387
22	A	24	13	1.	31	0.419
23	A	28	13	1.	31	0.419
24	A	32	13	1.	31	0.419
25	A	5	2	1.	29	0.069
26	A	5	2	1.	29	0.069
27	A	5	2	1.	29	0.069
28	A	5	2	1.	27	0.074
29	A	3	3	1.	21	0.143
30	A	7	4	1.	29	0.138
31	A	7	3	1.	29	0.103
32	A	5	2	1.	29	0.069
33	A	5	2	1.	29	0.069
34	A	5	2	1.	29	0.069
35	A	49	14	0.99	31	0.452
36	A	47	15	1.01	31	0.484
37	A	39	15	1.03	29	0.517
38	A	10	9	1.	23	0.391
39	A	29	14	1.42	31	0.452
40	A	35	12	1.06	31	0.387
41	A	47	16	1.04	31	0.516
42	A	61	17	1.03	31	0.548
43	A	3	3	1.	40	0.075
44	A	3	3	1.	40	0.075
45	A	3	3	1.	40	0.075
46	A	2	2	1.	38	0.053
47	A	3	3	1.	40	0.075

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	3	3	1.	40	0.075
49	A	3	3	1.	40	0.075
50	A	1	1	1.	34	0.029
51	A	11	6	1.	48	0.125
52	A	9	5	1.	46	0.109
53	A	7	4	1.	32	0.125
54	A	0	0	0.	0	0.
55	A	0	0	0.	0	0.
56	A	13	6	1.	39	0.154
57	A	11	6	1.	39	0.154
58	A	9	5	1.	37	0.135
59	A	5	3	1.	25	0.12
60	A	0	0	0.	0	0.
61	A	0	0	0.	0	0.
62	A	0	0	0.	0	0.
63	A	0	0	0.	0	0.
64	A	5	5	1.	26	0.192
65	A	0	0	0.	0	0.
66	A	0	0	0.	0	0.
67	A	12	7	1.05	45	0.156
68	A	10	7	1.04	45	0.156
69	A	8	6	1.04	43	0.14
70	A	0	0	0.	0	0.
71	A	0	0	0.	0	0.
72	A	0	0	0.	0	0.
73	A	0	0	0.	0	0.
74	F	0	0	N/A	0	N/A
75	F	0	0	N/A	0	N/A
76	A	30	9	1.	32	0.281
77	A	27	10	1.	32	0.312
78	A	18	6	1.	30	0.2
79	A	18	6	1.61	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	29	11	1.	32	0.344
81	A	31	12	1.	32	0.375
82	A	37	14	1.	34	0.412
83	A	30	12	1.	34	0.353
84	A	21	9	1.	32	0.281
85	A	19	7	1.36	31	0.226
86	A	31	12	1.	34	0.353
87	A	40	16	1.	34	0.471
88	A	5	5	1.	19	0.263
89	A	1	1	1.	24	0.042
90	A	3	3	1.	34	0.088
91	A	3	3	1.	55	0.055
92	A	3	3	1.	58	0.052
93	A	4	4	1.	11	0.364
94	A	5	5	1.	13	0.385
95	A	7	7	1.	13	0.538
96	A	2	2	1.	13	0.154
97	A	6	6	1.	15	0.4
98	A	4	4	1.	15	0.267
99	A	2	2	1.	13	0.154
100	A	6	6	1.	15	0.4
101	A	4	4	1.04	15	0.267
102	A	1	1	1.	38	0.026
103	A	2	2	1.	50	0.04
104	A	3	3	1.06	42	0.071
105	A	2	2	1.	62	0.032
106	A	3	3	1.01	49	0.061
107	A	7	5	1.04	42	0.119
108	A	8	6	1.03	65	0.092

# Chapter 3

## Listing of integrals

**3.1** 
$$\int \left(f + \frac{g}{x}\right)^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$

Optimal. Leaf size=404

$$-3Bf^2gn\text{PolyLog}\left(2, -\frac{bx}{a}\right) + 3Bf^2gn\text{PolyLog}\left(2, -\frac{dx}{c}\right) - \frac{1}{2}Bg^3n \log(x) \left(\frac{b^2}{a^2} - \frac{d^2}{c^2}\right) + \frac{b^2Bg^3n \log(a+bx)}{2a^2} + 3f^2g \log(\dots)$$

[Out]  $-(B*(b*c - a*d)*g^3*n)/(2*a*c*x) + A*f^3*x - (B*(b^2/a^2 - d^2/c^2)*g^3*n*\text{Log}[x])/2 + (b^2*B*g^3*n*\text{Log}[a + b*x])/(2*a^2) - 3*B*f^2*g*n*\text{Log}[x]*\text{Log}[1 + (b*x)/a] + (B*f^3*(a + b*x)*\text{Log}[e*((a + b*x)/(c + d*x))^n])/b - (g^3*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]))/(2*x^2) + (3*(b*c - a*d)*f*g^2*(a + b*x)*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]))/(a*(c + d*x)*(a - (c*(a + b*x))/(c + d*x))) + 3*f^2*g*\text{Log}[x]*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f^3*n*\text{Log}[c + d*x])/(b*d) - (B*d^2*g^3*n*\text{Log}[c + d*x])/(2*c^2) + 3*B*f^2*g*n*\text{Log}[x]*\text{Log}[1 + (d*x)/c] + (3*B*(b*c - a*d)*f*g^2*n*\text{Log}[a - (c*(a + b*x))/(c + d*x)])/(a*c) - 3*B*f^2*g*n*\text{PolyLog}[2, -((b*x)/a)] + 3*B*f^2*g*n*\text{PolyLog}[2, -((d*x)/c)]$

---

**Rubi [A]** time = 0.493055, antiderivative size = 385, normalized size of antiderivative = 0.95, number of steps used = 20, number of rules used = 10, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {2528, 2486, 31, 2525, 12, 72, 2524, 2357, 2317, 2391}

$$-3Bf^2gn\text{PolyLog}\left(2, -\frac{bx}{a}\right) + 3Bf^2gn\text{PolyLog}\left(2, -\frac{dx}{c}\right) - \frac{1}{2}Bg^3n \log(x) \left(\frac{b^2}{a^2} - \frac{d^2}{c^2}\right) + \frac{b^2Bg^3n \log(a+bx)}{2a^2} + 3f^2g \log(\dots)$$

Antiderivative was successfully verified.

```
[In] Int[(f + g/x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -(B*(b*c - a*d)*g^3*n)/(2*a*c*x) + A*f^3*x + (3*B*(b*c - a*d)*f*g^2*n*Log[x
])/ (a*c) - (B*(b^2/a^2 - d^2/c^2)*g^3*n*Log[x])/2 - (3*b*B*f*g^2*n*Log[a +
b*x])/a + (b^2*B*g^3*n*Log[a + b*x])/ (2*a^2) - 3*B*f^2*g*n*Log[x]*Log[1 + (
b*x)/a] + (B*f^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])/ (2*x^2) - (3*f*g^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])/x + 3*f^2*g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
- (B*(b*c - a*d)*f^3*n*Log[c + d*x])/ (b*d) + (3*B*d*f*g^2*n*Log[c + d*x])/c
- (B*d^2*g^3*n*Log[c + d*x])/ (2*c^2) + 3*B*f^2*g*n*Log[x]*Log[1 + (d*x)/c]
- 3*B*f^2*g*n*PolyLog[2, -(b*x)/a] + 3*B*f^2*g*n*PolyLog[2, -(d*x)/c])
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^ (n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)
^q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^ (n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12



```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

### Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

### Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e, Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \left(f + \frac{g}{x}\right)^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx &= \int \left[ f^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + \frac{g^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x^3} + \frac{3fg^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x^2} \right] dx \\
&= f^3 \int \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx + (3f^2g) \int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x} dx + \frac{3fg^2}{2} \int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2} dx \\
&= Af^3x - \frac{g^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2} - \frac{3fg^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x} + 3f^2g \log(x) \\
&= Af^3x + \frac{Bf^3(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{g^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2} - \frac{3fg^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x} \\
&= Af^3x + \frac{Bf^3(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{g^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2} - \frac{3fg^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x} \\
&= -\frac{B(bc-ad)g^3n}{2acx} + Af^3x + \frac{3B(bc-ad)fg^2n \log(x)}{ac} - \frac{B(bc-ad)(bc+ad)g^3}{2a^2c^2} \\
&= -\frac{B(bc-ad)g^3n}{2acx} + Af^3x + \frac{3B(bc-ad)fg^2n \log(x)}{ac} - \frac{B(bc-ad)(bc+ad)g^3}{2a^2c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.413278, size = 336, normalized size = 0.83

$$-3Bf^2gn \left( \text{PolyLog}\left(2, -\frac{bx}{a}\right) - \text{PolyLog}\left(2, -\frac{dx}{c}\right) + \log(x) \left( \log\left(\frac{bx}{a} + 1\right) - \log\left(\frac{dx}{c} + 1\right) \right) \right) + \frac{Bg^3n \left( \log(x) (a^2d^2x - b^2c) \right)}{2a^2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g/x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] A\*f^3\*x + (B\*f^3\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/b - (g^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(2\*x^2) - (3\*f\*g^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/x + 3\*f^2\*g\*Log[x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - (B\*(b\*c - a\*d)\*f^3\*n\*Log[c + d\*x])/(b\*d) + (3\*B\*f\*g^2\*n\*((b\*c - a\*d)\*Log[x] - b\*c\*Log[a + b\*x] + a\*d\*Log[c + d\*x]))/(a\*c) + (B\*g^3\*n\*((-(b^2\*c^2\*x) + a^2\*d^2\*x)\*Log[x] + b^2\*c^2\*x\*Log[a + b\*x] + a\*(-(b\*c^2) + a\*c\*d - a\*d^2\*

$x \cdot \text{Log}[c + d \cdot x])]) / (2 \cdot a^2 \cdot c^2 \cdot x) - 3 \cdot B \cdot f^2 \cdot g \cdot n \cdot (\text{Log}[x] \cdot (\text{Log}[1 + (b \cdot x)/a] - \text{Log}[1 + (d \cdot x)/c]) + \text{PolyLog}[2, -((b \cdot x)/a)] - \text{PolyLog}[2, -((d \cdot x)/c)])$

**Maple [F]** time = 0.154, size = 0, normalized size = 0.

$$\int \left( f + \frac{g}{x} \right)^3 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f+g/x)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

[Out] `int((f+g/x)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$Bf^3n \left( \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right) - 3Bfg^2n \left( \frac{b \log(bx+a)}{a} - \frac{d \log(dx+c)}{c} - \frac{(bc-ad) \log(x)}{ac} \right) + \frac{1}{2} Bg^3n \left( \frac{b^2 \log(bx+a)}{a^2} - \frac{d^2 \log(dx+c)}{c^2} - \frac{(b^2c^2 - a^2d^2) \log(x)}{a^2c^2} \right) + Bf^3x \log(e*(b*x/(d*x+c) + a/(d*x+c))^n) + Af^3x - 3Bf^2g \int (-\log((b*x+a)^n) - \log((d*x+c)^n) + \log(e)) / x, x) + 3A*f^2*g*log(x) - 3B*f*g^2*log(e*(b*x/(d*x+c) + a/(d*x+c))^n) / x - 3A*f*g^2/x - 1/2*B*g^3*log(e*(b*x/(d*x+c) + a/(d*x+c))^n) / x^2 - 1/2*A*g^3/x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")`

[Out] `B*f^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) - 3*B*f*g^2*n*(b*log(b*x + a)/a - d*log(d*x + c)/c - (b*c - a*d)*log(x)/(a*c)) + 1/2*B*g^3*n*(b^2*log(b*x + a)/a^2 - d^2*log(d*x + c)/c^2 - (b^2*c^2 - a^2*d^2)*log(x)/(a^2*c^2)) + B*f^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f^3*x - 3*B*f^2*g*integrate(-log((b*x + a)^n) - log((d*x + c)^n) + log(e))/x, x) + 3*A*f^2*g*log(x) - 3*B*f*g^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) / x - 3*A*f*g^2/x - 1/2*B*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) / x^2 - 1/2*A*g^3/x^2`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{A f^3 x^3 + 3 A f^2 g x^2 + 3 A f g^2 x + A g^3 + (B f^3 x^3 + 3 B f^2 g x^2 + 3 B f g^2 x + B g^3) \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] integral((A*f^3*x^3 + 3*A*f^2*g*x^2 + 3*A*f*g^2*x + A*g^3 + (B*f^3*x^3 + 3*
B*f^2*g*x^2 + 3*B*f*g^2*x + B*g^3)*log(e*((b*x + a)/(d*x + c))^n))/x^3, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right) \left( f + \frac{g}{x} \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(f + g/x)^3, x)
```

$$3.2 \quad \int \left( f + \frac{g}{x} \right)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$$

**Optimal.** Leaf size=263

$$-2Bfgn \text{PolyLog} \left( 2, -\frac{bx}{a} \right) + 2Bfgn \text{PolyLog} \left( 2, -\frac{dx}{c} \right) + 2fg \log(x) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) + \frac{g^2(a+bx)(bc-ad)}{a(c+dx)}$$

```
[Out] A*f^2*x - 2*B*f*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + ((b*c - a*d)*g^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*(c + d*x)*(a - (c*(a + b*x))/(c + d*x))) + 2*f*g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f^2*n*Log[c + d*x])/(b*d) + 2*B*f*g*n*Log[x]*Log[1 + (d*x)/c] + (B*(b*c - a*d)*g^2*n*Log[a - (c*(a + b*x))/(c + d*x)])/(a*c) - 2*B*f*g*n*PolyLog[2, -((b*x)/a)] + 2*B*f*g*n*PolyLog[2, -((d*x)/c)]
```

**Rubi [A]** time = 0.337362, antiderivative size = 242, normalized size of antiderivative = 0.92, number of steps used = 16, number of rules used = 10, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {2528, 2486, 31, 2525, 12, 72, 2524, 2357, 2317, 2391}

$$-2Bfgn \text{PolyLog} \left( 2, -\frac{bx}{a} \right) + 2Bfgn \text{PolyLog} \left( 2, -\frac{dx}{c} \right) + 2fg \log(x) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) - \frac{g^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g/x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f^2*x + (B*(b*c - a*d)*g^2*n*Log[x])/(a*c) - (b*B*g^2*n*Log[a + b*x])/a - 2*B*f*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/x + 2*f*g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f^2*n*Log[c + d*x])/(b*d) + (B*d*g^2*n*Log[c + d*x])/c + 2*B*f*g*n*Log[x]*Log[1 + (d*x)/c] - 2*B*f*g*n*PolyLog[2, -((b*x)/a)] + 2*B*f*g*n*PolyLog[2, -((d*x)/c)]
```

**Rule 2528**

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*RFX^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunctionQ[RGx, x]
```

onQ[RGx, x] && IGtQ[n, 0]

### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))  
^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)  
^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c +  
d\*x)^q]^r]^(s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s  
}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x,  
x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.  
, x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1))  
, x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(  
a + b\*Log[c\*RFx^p])^n - 1)\*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d  
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||  
IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match  
Q[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 72

Int[((e\_.) + (f\_.)\*(x\_))^(p\_.)/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))),  
x\_Symbol] := Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x]  
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_S  
ymbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e  
, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n - 1)\*D[RFx, x])/RFx, x], x] /;  
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

### Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] := With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

### Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e, Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \left(f + \frac{g}{x}\right)^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx &= \int \left[ f^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + \frac{g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x^2} + \frac{2fg \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x} \right] dx \\
&= f^2 \int \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx + (2fg) \int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x} dx + \int \frac{g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x^2} dx \\
&= Af^2x - \frac{g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x} + 2fg \log(x) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + \frac{2fg \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x} \\
&= Af^2x + \frac{Bf^2(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x} + 2fg \log(x) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \\
&= Af^2x + \frac{Bf^2(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x} + 2fg \log(x) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \\
&= Af^2x + \frac{B(bc-ad)g^2n \log(x)}{ac} - \frac{bBg^2n \log(a+bx)}{a} - 2Bfgn \log(x) \log\left(1 + \frac{a+bx}{c+dx}\right) \\
&= Af^2x + \frac{B(bc-ad)g^2n \log(x)}{ac} - \frac{bBg^2n \log(a+bx)}{a} - 2Bfgn \log(x) \log\left(1 + \frac{a+bx}{c+dx}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.230476, size = 217, normalized size = 0.83

$$-2Bfgn \left( \text{PolyLog} \left( 2, -\frac{bx}{a} \right) - \text{PolyLog} \left( 2, -\frac{dx}{c} \right) + \log(x) \left( \log \left( \frac{bx}{a} + 1 \right) - \log \left( \frac{dx}{c} + 1 \right) \right) \right) + 2fg \log(x) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(f + g/x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]),x]

[Out] A\*f^2\*x + (B\*f^2\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/b - (g^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/x + 2\*f\*g\*Log[x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - (B\*(b\*c - a\*d)\*f^2\*n\*Log[c + d\*x])/(b\*d) + (B\*g^2\*n\*((b\*c - a\*d)\*Log[x] - b\*c\*Log[a + b\*x] + a\*d\*Log[c + d\*x]))/(a\*c) - 2\*B\*f\*g\*n\*(Log[x]\*(Log[1 + (b\*x)/a] - Log[1 + (d\*x)/c]) + PolyLog[2, -((b\*x)/a)] - PolyLog[2, -((d\*x)/c)])

**Maple [F]** time = 0.086, size = 0, normalized size = 0.

$$\int \left( f + \frac{g}{x} \right)^2 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f+g/x)^2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int((f+g/x)^2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$Bf^2n \left( \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right) - Bg^2n \left( \frac{b \log(bx+a)}{a} - \frac{d \log(dx+c)}{c} - \frac{(bc-ad) \log(x)}{ac} \right) + Bf^2x \log \left( e \left( \frac{bx}{dx+c} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f+g/x)^2\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")



```
[Out] B*f^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) - B*g^2*n*(b*log(b*x + a)/a -
d*log(d*x + c)/c - (b*c - a*d)*log(x)/(a*c)) + B*f^2*x*log(e*(b*x/(d*x + c)
) + a/(d*x + c))^n) + A*f^2*x - 2*B*f*g*integrate(-(log((b*x + a)^n) - log(
(d*x + c)^n) + log(e))/x, x) + 2*A*f*g*log(x) - B*g^2*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/x - A*g^2/x
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{A f^2 x^2 + 2 A f g x + A g^2 + (B f^2 x^2 + 2 B f g x + B g^2) \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right)}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] integral((A*f^2*x^2 + 2*A*f*g*x + A*g^2 + (B*f^2*x^2 + 2*B*f*g*x + B*g^2)*l
og(e*((b*x + a)/(d*x + c))^n))/x^2, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A \right) \left( f + \frac{g}{x} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(f + g/x)^2, x)
```

$$3.3 \quad \int \left( f + \frac{g}{x} \right) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$$

**Optimal.** Leaf size=143

$$-BgnPolyLog\left(2, -\frac{bx}{a}\right) + BgnPolyLog\left(2, -\frac{dx}{c}\right) + g \log(x) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) + \frac{Bf(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b}$$

```
[Out] A*f*x - B*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f*n*Log[c + d*x])/(b*d) + B*g*n*Log[x]*Log[1 + (d*x)/c] - B*g*n*PolyLog[2, -((b*x)/a)] + B*g*n*PolyLog[2, -((d*x)/c)]
```

---

**Rubi [A]** time = 0.224888, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$ , Rules used = {2528, 2486, 31, 2524, 2357, 2317, 2391}

$$-BgnPolyLog\left(2, -\frac{bx}{a}\right) + BgnPolyLog\left(2, -\frac{dx}{c}\right) + g \log(x) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) + \frac{Bf(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g/x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

```
[Out] A*f*x - B*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f*n*Log[c + d*x])/(b*d) + B*g*n*Log[x]*Log[1 + (d*x)/c] - B*g*n*PolyLog[2, -((b*x)/a)] + B*g*n*PolyLog[2, -((d*x)/c)]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]
```

### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)]^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
```

```
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

### Rule 31

```
Int[((a_) + (b_)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 2524

```
Int[((a_) + Log[(c_)*(RFx_)^(p_)]*(b_))^(n_)/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

### Rule 2357

```
Int[((a_) + Log[(c_)*(x_)^(n_)]*(b_))^(p_)*(RFx_), x_Symbol] := With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

### Rule 2317

```
Int[((a_) + Log[(c_)*(x_)^(n_)]*(b_))^(p_)/((d_) + (e_)*(x_)), x_Symb
ol] := Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \left(f + \frac{g}{x}\right) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx &= \int \left[ f \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + \frac{g \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x} \right] dx \\
&= f \int \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx + g \int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x} dx \\
&= Afx + g \log(x) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + (Bf) \int \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) dx - \\
&= Afx + \frac{Bf(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + g \log(x) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) - \\
&= Afx + \frac{Bf(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + g \log(x) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) - \\
&= Afx - Bgn \log(x) \log\left(1 + \frac{bx}{a}\right) + \frac{Bf(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + g \log(x) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \\
&= Afx - Bgn \log(x) \log\left(1 + \frac{bx}{a}\right) + \frac{Bf(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + g \log(x) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0960526, size = 135, normalized size = 0.94

$$-Bgn \left( \text{PolyLog}\left(2, -\frac{bx}{a}\right) - \text{PolyLog}\left(2, -\frac{dx}{c}\right) + \log(x) \left( \log\left(\frac{bx}{a} + 1\right) - \log\left(\frac{dx}{c} + 1\right) \right) \right) + g \log(x) \left( B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(f + g/x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]),x]

[Out] A\*f\*x + (B\*f\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/b + g\*Log[x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - (B\*(b\*c - a\*d)\*f\*n\*Log[c + d\*x])/(b\*d) - B\*g\*n\*(Log[x]\*(Log[1 + (b\*x)/a] - Log[1 + (d\*x)/c]) + PolyLog[2, -((b\*x)/a)] - PolyLog[2, -((d\*x)/c)])

**Maple [F]** time = 0.089, size = 0, normalized size = 0.

$$\int \left( f + \frac{g}{x} \right) \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f+g/x)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int((f+g/x)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$Bfn \left( \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right) + Bfx \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + Afx - Bg \int - \frac{\log((bx+a)^n) - \log((dx+c)^n)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f+g/x)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out] B\*f\*n\*(a\*log(b\*x + a)/b - c\*log(d\*x + c)/d) + B\*f\*x\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A\*f\*x - B\*g\*integrate(-(log((b\*x + a)^n) - log((d\*x + c)^n) + log(e))/x, x) + A\*g\*log(x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{Afx + Ag + (Bfx + Bg) \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f+g/x)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out] integral((A\*f\*x + A\*g + (B\*f\*x + B\*g)\*log(e\*((b\*x + a)/(d\*x + c))^n))/x, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f+g/x)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right) \left( f + \frac{g}{x} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f+g/x)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)\*(f + g/x), x)

$$3.4 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}} dx$$

**Optimal.** Leaf size=217

$$\frac{BgnPolyLog\left(2, -\frac{b(fx+g)}{af-bg}\right)}{f^2} - \frac{BgnPolyLog\left(2, -\frac{d(fx+g)}{cf-dg}\right)}{f^2} - \frac{g \log(fx+g) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{f^2} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf}$$

[Out] (A\*x)/f + (B\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(b\*f) - (B\*(b\*c - a\*d)\*n\*Log[c + d\*x])/(b\*d\*f) + (B\*g\*n\*Log[(f\*(a + b\*x))/(a\*f - b\*g)]\*Log[g + f\*x])/f^2 - (g\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[g + f\*x])/f^2 - (B\*g\*n\*Log[(f\*(c + d\*x))/(c\*f - d\*g)]\*Log[g + f\*x])/f^2 + (B\*g\*n\*PolyLog[2, -((b\*(g + f\*x))/(a\*f - b\*g))])/f^2 - (B\*g\*n\*PolyLog[2, -((d\*(g + f\*x))/(c\*f - d\*g))])/f^2

**Rubi [A]** time = 0.33225, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2528, 2486, 31, 2524, 2418, 2394, 2393, 2391}

$$\frac{BgnPolyLog\left(2, -\frac{b(fx+g)}{af-bg}\right)}{f^2} - \frac{BgnPolyLog\left(2, -\frac{d(fx+g)}{cf-dg}\right)}{f^2} - \frac{g \log(fx+g) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{f^2} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g/x), x]

[Out] (A\*x)/f + (B\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(b\*f) - (B\*(b\*c - a\*d)\*n\*Log[c + d\*x])/(b\*d\*f) + (B\*g\*n\*Log[(f\*(a + b\*x))/(a\*f - b\*g)]\*Log[g + f\*x])/f^2 - (g\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[g + f\*x])/f^2 - (B\*g\*n\*Log[(f\*(c + d\*x))/(c\*f - d\*g)]\*Log[g + f\*x])/f^2 + (B\*g\*n\*PolyLog[2, -((b\*(g + f\*x))/(a\*f - b\*g))])/f^2 - (B\*g\*n\*PolyLog[2, -((d\*(g + f\*x))/(c\*f - d\*g))])/f^2

### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] :> With[{u = ExpandIntegrand[(a + b\*Log[c\*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x]



onQ[RGx, x] && IGtQ[n, 0]

### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))  
^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)  
^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c +  
d\*x)^q]^r]^(s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s  
}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

### Rule 31

Int[((a\_.) + (b\_.)\*(x\_))^-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x,  
x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_S  
ymbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e  
, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /;  
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(RFx\_), x\_Sy  
mbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]},  
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[  
RFx, x] && IntegerQ[p]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_  
)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)  
^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x)  
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_  
Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x  
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*  
(e\*f - d\*g), 0]

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f + \frac{g}{x}} dx &= \int \left( \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f} - \frac{g\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f(g+fx)} \right) dx \\
&= \frac{\int \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx}{f} - \frac{g \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx} dx}{f} \\
&= \frac{Ax}{f} - \frac{g\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(g+fx)}{f^2} + \frac{B \int \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) dx}{f} + \frac{(Bgn) \int \frac{(c+dx)\left(-\frac{d}{c}\right)}{c+dx} dx}{f} \\
&= \frac{Ax}{f} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{g\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(g+fx)}{f^2} - \frac{(B(bc-ad)n)}{bf} \\
&= \frac{Ax}{f} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n \log(c+dx)}{bdf} - \frac{g\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(g+fx)}{f^2} \\
&= \frac{Ax}{f} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n \log(c+dx)}{bdf} + \frac{Bgn \log\left(\frac{f(a+bx)}{af-bg}\right) \log(g+fx)}{f^2} \\
&= \frac{Ax}{f} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n \log(c+dx)}{bdf} + \frac{Bgn \log\left(\frac{f(a+bx)}{af-bg}\right) \log(g+fx)}{f^2} \\
&= \frac{Ax}{f} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n \log(c+dx)}{bdf} + \frac{Bgn \log\left(\frac{f(a+bx)}{af-bg}\right) \log(g+fx)}{f^2}
\end{aligned}$$

**Mathematica [A]** time = 0.148163, size = 185, normalized size = 0.85

$$\frac{Bgn \left( \text{PolyLog}\left(2, \frac{b(fx+g)}{bg-af}\right) - \text{PolyLog}\left(2, \frac{d(fx+g)}{dg-cf}\right) + \log(fx+g) \left( \log\left(\frac{f(a+bx)}{af-bg}\right) - \log\left(\frac{f(c+dx)}{cf-dg}\right) \right) \right) - g \log(fx+g) \left( B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \right)}{f^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g/x), x]

[Out] (A\*f\*x + (B\*f\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/b - (B\*(b\*c - a\*d)\*f\*n\*Log[c + d\*x])/(b\*d) - g\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[g + f\*x] + B\*g\*n\*((Log[(f\*(a + b\*x))/(a\*f - b\*g)] - Log[(f\*(c + d\*x))/(c\*f - d\*g]))\*Log[g + f\*x] + PolyLog[2, (b\*(g + f\*x))/(-a\*f + b\*g)] - PolyLog[2, (d\*(g + f\*x))/(-c\*f + d\*g)]/f^2

**Maple [F]** time = 0.199, size = 0, normalized size = 0.

$$\int \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right) \left( f + \frac{g}{x} \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(f+g/x), x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(f+g/x), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$A \left( \frac{x}{f} - \frac{g \log(fx + g)}{f^2} \right) - B \int - \frac{x \log((bx + a)^n) - x \log((dx + c)^n) + x \log(e)}{fx + g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(f+g/x), x, algorithm="maxima")

[Out] A\*(x/f - g\*log(f\*x + g)/f^2) - B\*integrate(-(x\*log((b\*x + a)^n) - x\*log((d\*x + c)^n) + x\*log(e))/(f\*x + g), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{Bx \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + Ax}{fx + g}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(f+g/x),x, algorithm="fricas")

[Out] integral((B\*x\*log(e\*((b\*x + a)/(d\*x + c))^n) + A\*x)/(f\*x + g), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(f+g/x),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A}{f + \frac{g}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(f+g/x),x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)/(f + g/x), x)

$$3.5 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2} dx$$

**Optimal.** Leaf size=322

$$\frac{2BgnPolyLog\left(2, -\frac{b(fx+g)}{af-bg}\right)}{f^3} - \frac{2BgnPolyLog\left(2, -\frac{d(fx+g)}{cf-dg}\right)}{f^3} - \frac{g^2(a+bx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{f^2(fx+g)(af-bg)} - \frac{2g \log(fx+g)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{f^3}$$

[Out] (A\*x)/f^2 + (B\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(b\*f^2) - (g^2\*(a + b\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(f^2\*(a\*f - b\*g)\*(g + f\*x)) - (B\*(b\*c - a\*d)\*n\*Log[c + d\*x])/(b\*d\*f^2) + (2\*B\*g\*n\*Log[(f\*(a + b\*x))/(a\*f - b\*g)]\*Log[g + f\*x])/f^3 - (2\*g\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[g + f\*x])/f^3 - (2\*B\*g\*n\*Log[(f\*(c + d\*x))/(c\*f - d\*g)]\*Log[g + f\*x])/f^3 + (B\*(b\*c - a\*d)\*g^2\*n\*Log[(g + f\*x)/(c + d\*x])/(f^2\*(a\*f - b\*g)\*(c\*f - d\*g)) + (2\*B\*g\*n\*PolyLog[2, -((b\*(g + f\*x))/(a\*f - b\*g))])/f^3 - (2\*B\*g\*n\*PolyLog[2, -((d\*(g + f\*x))/(c\*f - d\*g))])/f^3

**Rubi [A]** time = 0.493759, antiderivative size = 352, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {2528, 2486, 31, 2525, 12, 72, 2524, 2418, 2394, 2393, 2391}

$$\frac{2BgnPolyLog\left(2, -\frac{b(fx+g)}{af-bg}\right)}{f^3} - \frac{2BgnPolyLog\left(2, -\frac{d(fx+g)}{cf-dg}\right)}{f^3} - \frac{g^2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{f^3(fx+g)} - \frac{2g \log(fx+g)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{f^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g/x)^2, x]

[Out] (A\*x)/f^2 - (b\*B\*g^2\*n\*Log[a + b\*x])/(f^3\*(a\*f - b\*g)) + (B\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(b\*f^2) - (g^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(f^3\*(g + f\*x)) - (B\*(b\*c - a\*d)\*n\*Log[c + d\*x])/(b\*d\*f^2) + (B\*d\*g^2\*n\*Log[c + d\*x])/(f^3\*(c\*f - d\*g)) + (B\*(b\*c - a\*d)\*g^2\*n\*Log[g + f\*x])/(f^2\*(a\*f - b\*g)\*(c\*f - d\*g)) + (2\*B\*g\*n\*Log[(f\*(a + b\*x))/(a\*f - b\*g)]\*Log[g + f\*x])/f^3 - (2\*g\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[g + f\*x])/f^3 - (2\*B\*g\*n\*Log[(f\*(c + d\*x))/(c\*f - d\*g)]\*Log[g + f\*x])/f^3 + (2\*B\*g\*n\*PolyLog[2, -((b\*(g + f\*x))/(a\*f - b\*g))])/f^3 - (2\*B\*g\*n\*PolyLog[2, -((d\*(g + f\*x))/(c\*f - d\*g))])/f^3

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*RFX^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunc
tionQ[RGx, x] && IGtQ[n, 0]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*
(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFX^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /;
```

FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(RFx\_), x\_Symbol] :> With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n]^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f + \frac{g}{x}\right)^2} dx &= \int \left( \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f^2} + \frac{g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^2(g+fx)^2} - \frac{2g \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^2(g+fx)} \right) dx \\
&= \frac{\int \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx}{f^2} - \frac{(2g) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx} dx}{f^2} + \frac{g^2 \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(g+fx)^2} dx}{f^2} \\
&= \frac{Ax}{f^2} - \frac{g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - \frac{2g \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(g+fx)}{f^3} + \frac{B \int \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) dx}{f^3} \\
&= \frac{Ax}{f^2} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - \frac{2g \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3} \\
&= \frac{Ax}{f^2} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - \frac{B(bc-ad)n \log(c+dx)}{bdf^2} \\
&= \frac{Ax}{f^2} - \frac{bBg^2n \log(a+bx)}{f^3(af-bg)} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - \frac{B(bc-ad)n \log(c+dx)}{bdf^2} \\
&= \frac{Ax}{f^2} - \frac{bBg^2n \log(a+bx)}{f^3(af-bg)} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - \frac{B(bc-ad)n \log(c+dx)}{bdf^2} \\
&= \frac{Ax}{f^2} - \frac{bBg^2n \log(a+bx)}{f^3(af-bg)} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - \frac{B(bc-ad)n \log(c+dx)}{bdf^2}
\end{aligned}$$

**Mathematica [A]** time = 0.593835, size = 295, normalized size = 0.92

$$2Bgn \left( \text{PolyLog}\left(2, \frac{b(fx+g)}{bg-af}\right) - \text{PolyLog}\left(2, \frac{d(fx+g)}{dg-cf}\right) + \log(fx+g) \left( \log\left(\frac{f(a+bx)}{af-bg}\right) - \log\left(\frac{f(c+dx)}{cf-dg}\right) \right) \right) - \frac{g^2 \left( B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A \right)}{fx+g}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g/x)^2,x]

[Out] (A\*f\*x + (B\*f\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/b - (g^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(g + f\*x) - (B\*(b\*c - a\*d)\*f\*n\*Log[c + d\*x])/



$$(b*d) - 2*g*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])*\text{Log}[g + f*x] + (B*g^2*n*(b*(-(c*f) + d*g)*\text{Log}[a + b*x] + d*(a*f - b*g)*\text{Log}[c + d*x] + (b*c - a*d)*f*\text{Log}[g + f*x]))/((a*f - b*g)*(c*f - d*g)) + 2*B*g*n*((\text{Log}[(f*(a + b*x))/(a*f - b*g)] - \text{Log}[(f*(c + d*x))/(c*f - d*g)])*\text{Log}[g + f*x] + \text{PolyLog}[2, (b*(g + f*x))/(-(a*f) + b*g)] - \text{PolyLog}[2, (d*(g + f*x))/(-(c*f) + d*g)]))/f^3$$

**Maple [F]** time = 0.084, size = 0, normalized size = 0.

$$\int \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) \left( f + \frac{g}{x} \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(f+g/x)^2,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(f+g/x)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-A \left( \frac{g^2}{f^4x + f^3g} - \frac{x}{f^2} + \frac{2g \log(fx + g)}{f^3} \right) - B \int -\frac{x^2 \log((bx+a)^n) - x^2 \log((dx+c)^n) + x^2 \log(e)}{f^2x^2 + 2fgx + g^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(f+g/x)^2,x, algorithm="maxima")

[Out] -A\*(g^2/(f^4\*x + f^3\*g) - x/f^2 + 2\*g\*log(f\*x + g)/f^3) - B\*integrate(-(x^2\*log((b\*x + a)^n) - x^2\*log((d\*x + c)^n) + x^2\*log(e))/(f^2\*x^2 + 2\*f\*g\*x + g^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{Bx^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + Ax^2}{f^2x^2 + 2fgx + g^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^2*log(e*((b*x + a)/(d*x + c))^n) + A*x^2)/(f^2*x^2 + 2*f*g*x + g^2), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{\left(f + \frac{g}{x}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(f + g/x)^2, x)
```

$$3.6 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3} dx$$

**Optimal.** Leaf size=531

$$\frac{3BgnPolyLog\left(2, -\frac{b(fx+g)}{af-bg}\right)}{f^4} - \frac{3BgnPolyLog\left(2, -\frac{d(fx+g)}{cf-dg}\right)}{f^4} + \frac{g^3\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{2f^4(fx+g)^2} - \frac{3g^2(a+bx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{f^3(fx+g)(af-bg)}$$

[Out] (A\*x)/f^3 + (B\*(b\*c - a\*d)\*g^3\*n)/(2\*f^3\*(a\*f - b\*g)\*(c\*f - d\*g)\*(g + f\*x)) - (b^2\*B\*g^3\*n\*Log[a + b\*x])/(2\*f^4\*(a\*f - b\*g)^2) + (B\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(b\*f^3) + (g^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(2\*f^4\*(g + f\*x)^2) - (3\*g^2\*(a + b\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(f^3\*(a\*f - b\*g)\*(g + f\*x)) - (B\*(b\*c - a\*d)\*n\*Log[c + d\*x])/(b\*d\*f^3) + (B\*d^2\*g^3\*n\*Log[c + d\*x])/(2\*f^4\*(c\*f - d\*g)^2) + (B\*(b\*c - a\*d)\*g^3\*(b\*c\*f + a\*d\*f - 2\*b\*d\*g)\*n\*Log[g + f\*x])/(2\*f^3\*(a\*f - b\*g)^2\*(c\*f - d\*g)^2) + (3\*B\*g\*n\*Log[(f\*(a + b\*x))/(a\*f - b\*g)]\*Log[g + f\*x])/f^4 - (3\*g\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[g + f\*x])/f^4 - (3\*B\*g\*n\*Log[(f\*(c + d\*x))/(c\*f - d\*g)]\*Log[g + f\*x])/f^4 + (3\*B\*(b\*c - a\*d)\*g^2\*n\*Log[(g + f\*x)/(c + d\*x)])/(f^3\*(a\*f - b\*g)\*(c\*f - d\*g)) + (3\*B\*g\*n\*PolyLog[2, -(b\*(g + f\*x))/(a\*f - b\*g)])/f^4 - (3\*B\*g\*n\*PolyLog[2, -(d\*(g + f\*x))/(c\*f - d\*g)])/f^4

**Rubi [A]** time = 0.796941, antiderivative size = 562, normalized size of antiderivative = 1.06, number of steps used = 22, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {2528, 2486, 31, 2525, 12, 72, 2524, 2418, 2394, 2393, 2391}

$$\frac{3BgnPolyLog\left(2, -\frac{b(fx+g)}{af-bg}\right)}{f^4} - \frac{3BgnPolyLog\left(2, -\frac{d(fx+g)}{cf-dg}\right)}{f^4} + \frac{g^3\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{2f^4(fx+g)^2} - \frac{3g^2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{f^4(fx+g)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g/x)^3, x]

[Out] (A\*x)/f^3 + (B\*(b\*c - a\*d)\*g^3\*n)/(2\*f^3\*(a\*f - b\*g)\*(c\*f - d\*g)\*(g + f\*x)) - (b^2\*B\*g^3\*n\*Log[a + b\*x])/(2\*f^4\*(a\*f - b\*g)^2) - (3\*b\*B\*g^2\*n\*Log[a + b\*x])/(f^4\*(a\*f - b\*g)) + (B\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(b\*f^3) + (g^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(2\*f^4\*(g + f\*x)^2) - (3\*g^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(f^4\*(g + f\*x)) - (B\*(b\*c - a\*d)\*g^3\*n\*Log[c + d\*x])/(b\*d\*f^3) + (B\*d^2\*g^3\*n\*Log[c + d\*x])/(2\*f^4\*(c\*f - d\*g)^2) + (B\*(b\*c - a\*d)\*g^3\*(b\*c\*f + a\*d\*f - 2\*b\*d\*g)\*n\*Log[g + f\*x])/(2\*f^3\*(a\*f - b\*g)^2\*(c\*f - d\*g)^2) + (3\*B\*g\*n\*Log[(f\*(a + b\*x))/(a\*f - b\*g)]\*Log[g + f\*x])/f^4 - (3\*g\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[g + f\*x])/f^4 - (3\*B\*g\*n\*Log[(f\*(c + d\*x))/(c\*f - d\*g)]\*Log[g + f\*x])/f^4 + (3\*B\*(b\*c - a\*d)\*g^2\*n\*Log[(g + f\*x)/(c + d\*x)])/(f^3\*(a\*f - b\*g)\*(c\*f - d\*g)) + (3\*B\*g\*n\*PolyLog[2, -(b\*(g + f\*x))/(a\*f - b\*g)])/f^4 - (3\*B\*g\*n\*PolyLog[2, -(d\*(g + f\*x))/(c\*f - d\*g)])/f^4

$$d)*n*\text{Log}[c + d*x]/(b*d*f^3) + (B*d^2*g^3*n*\text{Log}[c + d*x])/(2*f^4*(c*f - d*g)^2) + (3*B*d*g^2*n*\text{Log}[c + d*x])/(f^4*(c*f - d*g)) + (3*B*(b*c - a*d)*g^2*n*\text{Log}[g + f*x])/(f^3*(a*f - b*g)*(c*f - d*g)) + (B*(b*c - a*d)*g^3*(b*c*f + a*d*f - 2*b*d*g)*n*\text{Log}[g + f*x])/(2*f^3*(a*f - b*g)^2*(c*f - d*g)^2) + (3*B*g*n*\text{Log}[(f*(a + b*x))/(a*f - b*g)]*\text{Log}[g + f*x])/f^4 - (3*g*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])*\text{Log}[g + f*x])/f^4 - (3*B*g*n*\text{Log}[(f*(c + d*x))/(c*f - d*g)]*\text{Log}[g + f*x])/f^4 + (3*B*g*n*\text{PolyLog}[2, -((b*(g + f*x))/(a*f - b*g))])/f^4 - (3*B*g*n*\text{PolyLog}[2, -((d*(g + f*x))/(c*f - d*g))])/f^4$$

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)
^q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
 ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
 , Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
 mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
 Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
 RFx, x] && IntegerQ[p]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
 )), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
 )^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
 , x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
 Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
 ], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
 (e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
 , -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f + \frac{g}{x}\right)^3} dx &= \int \left( \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f^3} - \frac{g^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)^3} + \frac{3g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)^2} - \dots \right) dx \\
&= \frac{\int \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx}{f^3} - \frac{(3g) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx} dx}{f^3} + \frac{(3g^2) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(g+fx)^2} dx}{f^3} - \dots \\
&= \frac{Ax}{f^3} + \frac{g^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^4(g+fx)} - \frac{3g \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^4} - \dots \\
&= \frac{Ax}{f^3} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3} + \frac{g^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^4(g+fx)} - \dots \\
&= \frac{Ax}{f^3} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3} + \frac{g^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^4(g+fx)} - \dots \\
&= \frac{Ax}{f^3} + \frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)} - \frac{b^2Bg^3n \log(a+bx)}{2f^4(af-bg)^2} - \frac{3bBg^2n \log(a+bx)}{f^4(af-bg)} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f^4} - \dots \\
&= \frac{Ax}{f^3} + \frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)} - \frac{b^2Bg^3n \log(a+bx)}{2f^4(af-bg)^2} - \frac{3bBg^2n \log(a+bx)}{f^4(af-bg)} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f^4} - \dots \\
&= \frac{Ax}{f^3} + \frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)} - \frac{b^2Bg^3n \log(a+bx)}{2f^4(af-bg)^2} - \frac{3bBg^2n \log(a+bx)}{f^4(af-bg)} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f^4} - \dots
\end{aligned}$$

**Mathematica [A]** time = 1.65344, size = 470, normalized size = 0.89

$$6Bgn \left( \text{PolyLog}\left(2, \frac{b(fx+g)}{bg-af}\right) - \text{PolyLog}\left(2, \frac{d(fx+g)}{dg-cf}\right) + \log(fx+g) \left( \log\left(\frac{f(a+bx)}{af-bg}\right) - \log\left(\frac{f(c+dx)}{cf-dg}\right) \right) \right) + \frac{g^3 \left( B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A \right)}{(fx+g)^2} - \dots$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g/x)^3,x]

```
[Out] (2*A*f*x + (2*B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + (g^3*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(g + f*x)^2 - (6*g^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(g + f*x) - (2*B*(b*c - a*d)*f*n*Log[c + d*x])/(b*d) -
6*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x] + (6*B*g^2*n*(b*(-(
c*f) + d*g)*Log[a + b*x] + d*(a*f - b*g)*Log[c + d*x] + (b*c - a*d)*f*Log[g
+ f*x]))/((a*f - b*g)*(c*f - d*g)) + B*(b*c - a*d)*g^3*n*(-((b^2*Log[a + b
*x]))/((b*c - a*d)*(a*f - b*g)^2)) + ((d^2*Log[c + d*x])/(b*c - a*d) + (f*((
a*f - b*g)*(c*f - d*g))/(g + f*x) + (b*c*f + a*d*f - 2*b*d*g)*Log[g + f*x]
))/((a*f - b*g)^2)/(c*f - d*g)^2 + 6*B*g*n*((Log[(f*(a + b*x))/(a*f - b*g)]
- Log[(f*(c + d*x))/(c*f - d*g)])*Log[g + f*x] + PolyLog[2, (b*(g + f*x))/
(-(a*f) + b*g)] - PolyLog[2, (d*(g + f*x))/(-(c*f) + d*g)]))/(2*f^4)
```

**Maple [F]** time = 0.084, size = 0, normalized size = 0.

$$\int \left( A + B \ln \left( e^{\left( \frac{bx+a}{dx+c} \right)^n} \right) \right) \left( f + \frac{g}{x} \right)^{-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}A \left( \frac{6fg^2x + 5g^3}{f^6x^2 + 2f^5gx + f^4g^2} - \frac{2x}{f^3} + \frac{6g \log(fx + g)}{f^4} \right) - B \int \frac{x^3 \log((bx + a)^n) - x^3 \log((dx + c)^n) + x^3 \log(e)}{f^3x^3 + 3f^2gx^2 + 3fg^2x + g^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*A*((6*f*g^2*x + 5*g^3)/(f^6*x^2 + 2*f^5*g*x + f^4*g^2) - 2*x/f^3 + 6*g
*log(f*x + g)/f^4) - B*integrate(-(x^3*log((b*x + a)^n) - x^3*log((d*x + c)
^n) + x^3*log(e))/(f^3*x^3 + 3*f^2*g*x^2 + 3*f*g^2*x + g^3), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{Bx^3 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + Ax^3}{f^3x^3 + 3f^2gx^2 + 3fg^2x + g^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(f+g/x)^3,x, algorithm="fricas")

[Out] integral((B\*x^3\*log(e\*((b\*x + a)/(d\*x + c))^n) + A\*x^3)/(f^3\*x^3 + 3\*f^2\*g\*x^2 + 3\*f\*g^2\*x + g^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(f+g/x)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A}{\left( f + \frac{g}{x} \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(f+g/x)^3,x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)/(f + g/x)^3, x)



### 3.7 $\int (a + bx)^4 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$

**Optimal.** Leaf size=201

$$-\frac{qrx(bc-ad)^4}{5d^4} + \frac{qr(a+bx)^2(bc-ad)^3}{10bd^3} - \frac{qr(a+bx)^3(bc-ad)^2}{15bd^2} + \frac{qr(bc-ad)^5 \log(c+dx)}{5bd^5} + \frac{(a+bx)^5 \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{5b}$$

```
[Out] -((b*c - a*d)^4*q*r*x)/(5*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)^2)/(10*b*d^3)
- ((b*c - a*d)^2*q*r*(a + b*x)^3)/(15*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^4)/(20*b*d)
- (p*r*(a + b*x)^5)/(25*b) - (q*r*(a + b*x)^5)/(25*b) + ((b*c - a*d)^5*q*r*Log[c + d*x])/(5*b*d^5)
+ ((a + b*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)]/(5*b)
```

**Rubi [A]** time = 0.0949281, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2495, 32, 43}

$$-\frac{qrx(bc-ad)^4}{5d^4} + \frac{qr(a+bx)^2(bc-ad)^3}{10bd^3} - \frac{qr(a+bx)^3(bc-ad)^2}{15bd^2} + \frac{qr(bc-ad)^5 \log(c+dx)}{5bd^5} + \frac{(a+bx)^5 \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], x]
```

```
[Out] -((b*c - a*d)^4*q*r*x)/(5*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)^2)/(10*b*d^3)
- ((b*c - a*d)^2*q*r*(a + b*x)^3)/(15*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^4)/(20*b*d)
- (p*r*(a + b*x)^5)/(25*b) - (q*r*(a + b*x)^5)/(25*b) + ((b*c - a*d)^5*q*r*Log[c + d*x])/(5*b*d^5)
+ ((a + b*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)]/(5*b)
```

#### Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.))*((c_.) + (d_.)*(x_.))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_.))^(m_.), x_Symbol] := Simp[((g + h*x)^(m + 1)*Log
[e*(f*(a + b*x)^p*(c + d*x)^q]^r)]/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

#### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned} \int (a + bx)^4 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(a + bx)^5 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{5b} - \frac{1}{5} (pr) \int (a + bx)^4 dx - \frac{(dqr) \int (a + bx)^4 dx}{5} \\ &= -\frac{pr(a + bx)^5}{25b} + \frac{(a + bx)^5 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{5b} - \frac{(dqr) \int \left( \frac{b(bc - ad)}{d^5} \right)}{5} \\ &= -\frac{(bc - ad)^4 qrx}{5d^4} + \frac{(bc - ad)^3 qr(a + bx)^2}{10bd^3} - \frac{(bc - ad)^2 qr(a + bx)^3}{15bd^2} + \frac{(bc - ad) qr(a + bx)^4}{20bd} \end{aligned}$$

**Mathematica [A]** time = 0.308289, size = 185, normalized size = 0.92

$$\frac{(a + bx)^5 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) - \frac{r(-60b^2(2p+5q)(c+dx)^2(bc-ad)^3 + 40b^3(3p+5q)(c+dx)^3(bc-ad)^2 - 15b^4(4p+5q)(c+dx)^4(bc-ad) + 60bdx(p+5q)(c+dx)^5(bc-ad)}{60d^5}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], x]
```

```
[Out] (-r*(60*b*d*(b*c - a*d)^4*(p + 5*q)*x - 60*b^2*(b*c - a*d)^3*(2*p + 5*q)*(c + d*x)^2 + 40*b^3*(b*c - a*d)^2*(3*p + 5*q)*(c + d*x)^3 - 15*b^4*(b*c - a*d)*(4*p + 5*q)*(c + d*x)^4 + 12*b^5*(p + q)*(c + d*x)^5 - 60*(b*c - a*d)^5*q*Log[c + d*x])/(60*d^5) + (a + b*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(5*b)
```

**Maple [F]** time = 0.734, size = 0, normalized size = 0.

$$\int (bx + a)^4 \ln \left( e \left( f(bx + a)^p (dx + c)^q \right)^r \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)`

[Out] `int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)`

**Maxima [B]** time = 1.30716, size = 533, normalized size = 2.65

$$\frac{1}{5} (b^4x^5 + 5ab^3x^4 + 10a^2b^2x^3 + 10a^3bx^2 + 5a^4x) \log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right) + \frac{\left(\frac{60a^5fp\log(bx+a)}{b} - \frac{12b^4d^4f(p+q)x^5+15(ab}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")`

[Out]  $\frac{1}{5}(b^4x^5 + 5ab^3x^4 + 10a^2b^2x^3 + 10a^3bx^2 + 5a^4x) \log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right) + \frac{1}{300} \left( 60a^5fp \log(bx+a) - (12b^4d^4f(p+q)x^5 + 15(a^2b^3d^4f(4p+5q) - b^4cd^3f^2q)x^4 + 20(2a^2b^2d^4f(3p+5q) + b^4c^2d^2f^2q - 5ab^3cd^3f^2q)x^3 + 30(2a^3bd^4f(2p+5q) - b^4c^3d^2f^2q + 5ab^3c^2d^2f^2q - 10a^2b^2cd^3f^2q)x^2 + 60(a^4d^4f(p+5q) + b^4c^4f^2q - 5ab^3c^3d^2f^2q + 10a^2b^2c^2d^2f^2q - 10a^3b^2cd^3f^2q)x) / d^4 + 60(b^4c^5f^2q - 5ab^3c^4d^2f^2q + 10a^2b^2c^3d^2f^2q - 10a^3b^2cd^3f^2q + 5a^4cd^4f^2q) \log(dx+c) / d^5 \right) r / f$

**Fricas [B]** time = 0.864394, size = 1315, normalized size = 6.54

$$\frac{12(b^5d^5p + b^5d^5q)rx^5 + 15(4ab^4d^5p - (b^5cd^4 - 5ab^4d^5)q)rx^4 + 20(6a^2b^3d^5p + (b^5c^2d^3 - 5ab^4cd^4 + 10a^2b^3d^5)q)rx^3}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")`

[Out]  $-\frac{1}{300} \left( 12(b^5d^5p + b^5d^5q)rx^5 + 15(4a^2b^4d^5p - (b^5c^2d^3 - 5ab^4cd^4 + 10a^2b^3d^5)q)rx^4 + 20(6a^2b^3d^5p + (b^5c^2d^3 - 5ab^4cd^4 + 10a^2b^3d^5)q)rx^3 + 30(4a^3b^2d^5p - (b^5c^3d^2 - 5ab^4c^2d^3 + 10a^2b^3cd^4 - 10a^3b^2d^5)q)rx^2 + 60(a^4bd^5p + ($

$$b^5c^4d - 5ab^4c^3d^2 + 10a^2b^3c^2d^3 - 10a^3b^2cd^4 + 5a^4b^2d^5)q)r^x - 60(b^5d^5p^r^x^5 + 5ab^4d^5p^r^x^4 + 10a^2b^3d^5p^r^x^3 + 10a^3b^2d^5p^r^x^2 + 5a^4b^2d^5p^r^x + a^5d^5p^r) \log(bx + a) - 60(b^5d^5q^r^x^5 + 5ab^4d^5q^r^x^4 + 10a^2b^3d^5q^r^x^3 + 10a^3b^2d^5q^r^x^2 + 5a^4b^2d^5q^r^x + (b^5c^5 - 5ab^4c^4d + 10a^2b^3c^3d^2 - 10a^3b^2c^2d^3 + 5a^4b^2cd^4)q^r) \log(dx + c) - 60(b^5d^5x^5 + 5ab^4d^5x^4 + 10a^2b^3d^5x^3 + 10a^3b^2d^5x^2 + 5a^4b^2d^5x) \log(e) - 60(b^5d^5r^x^5 + 5ab^4d^5r^x^4 + 10a^2b^3d^5r^x^3 + 10a^3b^2d^5r^x^2 + 5a^4b^2d^5r^x) \log(f)) / (b^5d^5)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*4\*ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r), x)

[Out] Timed out

**Giac [B]** time = 1.26035, size = 1002, normalized size = 4.99

$$-\frac{1}{25} (b^4pr + b^4qr - 5b^4r \log(f) - 5b^4)x^5 - \frac{(4ab^3dpr - b^4cqr + 5ab^3dqr - 20ab^3dr \log(f) - 20ab^3d)x^4}{20d} - \frac{(6a^2b^2d^2pr}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^4\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r), x, algorithm="giac")

[Out]  $-1/25*(b^4p^r + b^4q^r - 5b^4r \log(f) - 5b^4)x^5 - 1/20*(4a^2b^3d^2p^r - b^4c^2q^r + 5a^2b^3d^2q^r - 20a^2b^3d^2r \log(f) - 20a^2b^3d^2)x^4/d - 1/15*(6a^2b^2d^2p^r + b^4c^2q^r - 5a^2b^3c^2d^2q^r + 10a^2b^2d^2q^r - 30a^2b^2d^2r \log(f) - 30a^2b^2d^2)x^3/d^2 + 1/5*(b^4p^r^x^5 + 5a^2b^3p^r^x^4 + 10a^2b^2p^r^x^3 + 10a^3b^2p^r^x^2 + 5a^4p^r^x) \log(bx + a) + 1/5*(b^4q^r^x^5 + 5a^2b^3q^r^x^4 + 10a^2b^2q^r^x^3 + 10a^3b^2q^r^x^2 + 5a^4q^r^x) \log(dx + c) - 1/10*(4a^3b^2d^3p^r - b^4c^3q^r + 5a^2b^3c^2d^2q^r - 10a^2b^2c^2d^2q^r + 10a^3b^2d^3q^r - 20a^3b^2d^3r \log(f) - 20a^3b^2d^3)x^2/d^3 - 1/5*(a^4d^4p^r + b^4c^4q^r - 5a^2b^3c^3d^2q^r + 10a^2b^2c^2d^2q^r - 10a^3b^2c^2d^2q^r + 5a^4d^4q^r$

$$\begin{aligned}
& - 5a^4d^4r \log(f) - 5a^4d^4)x/d^4 + 1/10(a^5d^5p^r + b^5c^5q^r \\
& - 5ab^4c^4d^4q^r + 10a^2b^3c^3d^2q^r - 10a^3b^2c^2d^3q^r + 5a^4b^1c^1d^4q^r) \log(\text{abs}(b^2d^2x^2 + b^2c^2x + a^2d^2x + a^2c^2)) / (b^5d^5) + 1/10(a^5 \\
& *b^5c^5d^5p^r - a^6d^6p^r - b^6c^6q^r + 6a^5b^5c^5d^4q^r - 15a^4b^4c^4d^2q^r + 20a^3b^3c^3d^3q^r - 15a^4b^2c^2d^4q^r + 5a^5b^1c^1d^5q^r) \\
& * \log(\text{abs}((2b^2d^2x + b^2c^2 + a^2d^2 - \text{abs}(b^2c^2 - a^2d^2)) / (2b^2d^2x + b^2c^2 + a^2d^2 + \text{abs}(b^2c^2 - a^2d^2)))) / (b^5d^5 \text{abs}(b^2c^2 - a^2d^2))
\end{aligned}$$

### 3.8 $\int (a + bx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$

**Optimal.** Leaf size=172

$$\frac{qrx(bc - ad)^3}{4d^3} - \frac{qr(a + bx)^2(bc - ad)^2}{8bd^2} - \frac{qr(bc - ad)^4 \log(c + dx)}{4bd^4} + \frac{(a + bx)^4 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4b} + \frac{qr(a + bx)^3}{12b^2}$$

[Out]  $((b*c - a*d)^3*q*r*x)/(4*d^3) - ((b*c - a*d)^2*q*r*(a + b*x)^2)/(8*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^3)/(12*b*d) - (p*r*(a + b*x)^4)/(16*b) - (q*r*(a + b*x)^4)/(16*b) - ((b*c - a*d)^4*q*r*Log[c + d*x])/(4*b*d^4) + ((a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(4*b)$

**Rubi [A]** time = 0.0722046, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2495, 32, 43}

$$\frac{qrx(bc - ad)^3}{4d^3} - \frac{qr(a + bx)^2(bc - ad)^2}{8bd^2} - \frac{qr(bc - ad)^4 \log(c + dx)}{4bd^4} + \frac{(a + bx)^4 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4b} + \frac{qr(a + bx)^3}{12b^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x)^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r], x]$

[Out]  $((b*c - a*d)^3*q*r*x)/(4*d^3) - ((b*c - a*d)^2*q*r*(a + b*x)^2)/(8*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^3)/(12*b*d) - (p*r*(a + b*x)^4)/(16*b) - (q*r*(a + b*x)^4)/(16*b) - ((b*c - a*d)^4*q*r*Log[c + d*x])/(4*b*d^4) + ((a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(4*b)$

#### Rule 2495

$\text{Int}[\text{Log}[(e_*)*((f_*)*((a_*) + (b_*)*(x_))^{(p_*)}*((c_*) + (d_*)*(x_))^{(q_*)})^{(r_*)}]*((g_*) + (h_*)*(x_))^{(m_*)}, x\_Symbol] := \text{Simp}[(g + h*x)^{(m + 1)}*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]/(h*(m + 1)), x] + (-\text{Dist}[(b*p*r)/(h*(m + 1)), \text{Int}[(g + h*x)^{(m + 1)}/(a + b*x), x], x] - \text{Dist}[(d*q*r)/(h*(m + 1)), \text{Int}[(g + h*x)^{(m + 1)}/(c + d*x), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, p, q, r\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m, -1]$

#### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned} \int (a + bx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(a + bx)^4 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4b} - \frac{1}{4}(pr) \int (a + bx)^3 dx - \frac{(dqr)}{4b} \int (a + bx)^3 dx \\ &= -\frac{pr(a + bx)^4}{16b} + \frac{(a + bx)^4 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4b} - \frac{(dqr) \int \left( -\frac{b(bc - ad)}{4d} \right)}{4b} \\ &= \frac{(bc - ad)^3 qrx}{4d^3} - \frac{(bc - ad)^2 qr(a + bx)^2}{8bd^2} + \frac{(bc - ad)qr(a + bx)^3}{12bd} - \frac{pr(a + bx)^4}{16b} \end{aligned}$$

**Mathematica [A]** time = 0.211291, size = 154, normalized size = 0.9

$$\frac{r(-18b^2(p+2q)(c+dx)^2(bc-ad)^2+4b^3(3p+4q)(c+dx)^3(bc-ad)+12bdx(p+4q)(bc-ad)^3-12q(bc-ad)^4 \log(c+dx)-3b^4(p+q)(c+dx)^4)}{12d^4} + (a + bx)^4 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) / (4b)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], x]
```

```
[Out] ((r*(12*b*d*(b*c - a*d)^3*(p + 4*q)*x - 18*b^2*(b*c - a*d)^2*(p + 2*q)*(c + d*x)^2 + 4*b^3*(b*c - a*d)*(3*p + 4*q)*(c + d*x)^3 - 3*b^4*(p + q)*(c + d*x)^4 - 12*(b*c - a*d)^4*q*Log[c + d*x]))/(12*d^4) + (a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(4*b)
```

**Maple [F]** time = 0.422, size = 0, normalized size = 0.

$$\int (bx + a)^3 \ln \left( e \left( f(bx + a)^p (dx + c)^q \right)^r \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)`

[Out] `int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)`

**Maxima [A]** time = 1.33621, size = 385, normalized size = 2.24

$$\frac{1}{4} (b^3 x^4 + 4 a b^2 x^3 + 6 a^2 b x^2 + 4 a^3 x) \log \left( (b x + a)^p (d x + c)^q f \right)^r e + \frac{\left( \frac{12 a^4 f p \log(b x + a)}{b} - \frac{3 b^3 d^3 f (p+q) x^4 + 4 (a b^2 d^3 f (3 p+4 q) - b^3 c d^2 f}{d^3} \right)}{d^4} r / f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")`

[Out] `1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/48*(12*a^4*f*p*log(b*x + a)/b - (3*b^3*d^3*f*(p + q)*x^4 + 4*(a*b^2*d^3*f*(3*p + 4*q) - b^3*c*d^2*f*q)*x^3 + 6*(3*a^2*b*d^3*f*(p + 2*q) + b^3*c^2*d*f*q - 4*a*b^2*c*d^2*f*q)*x^2 + 12*(a^3*d^3*f*(p + 4*q) - b^3*c^3*f*q + 4*a*b^2*c^2*d*f*q - 6*a^2*b*c*d^2*f*q)*x)/d^3 - 12*(b^3*c^4*f*q - 4*a*b^2*c^3*d*f*q + 6*a^2*b*c^2*d^2*f*q - 4*a^3*c*d^3*f*q)*log(d*x + c)/d^4)*r/f`

**Fricas [B]** time = 0.956594, size = 980, normalized size = 5.7

$$\frac{3 (b^4 d^4 p + b^4 d^4 q) r x^4 + 4 (3 a b^3 d^4 p - (b^4 c d^3 - 4 a b^3 d^4) q) r x^3 + 6 (3 a^2 b^2 d^4 p + (b^4 c^2 d^2 - 4 a b^3 c d^3 + 6 a^2 b^2 d^4) q) r x^2 + 12 (a^3 b d^4 p - (b^4 c^3 d - 4 a^2 b^3 c d^2 + 6 a^2 b^2 c d^3 - 4 a^3 b d^4) q) r x - 12 (b^4 d^4 p r x^4 + 4 a b^3 d^4 p r x^3 + 6 a^2 b^2 d^4 p r x^2 + 4 a^3 b d^4 p r x + a^4 d^4 p r) \log(b x + a) - 12 (b^4 d^4 q r x^4 + 4 a b^3 d^4 q r x^3 + 6 a^2 b^2 d^4 q r x^2 + 4 a^3 b d^4 q r x + a^4 d^4 q r) \log(b x + a)}{d^4} r / f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")`

[Out] `-1/48*(3*(b^4*d^4*p + b^4*d^4*q)*r*x^4 + 4*(3*a*b^3*d^4*p - (b^4*c*d^3 - 4*a*b^3*d^4)*q)*r*x^3 + 6*(3*a^2*b^2*d^4*p + (b^4*c^2*d^2 - 4*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*q)*r*x^2 + 12*(a^3*b*d^4*p - (b^4*c^3*d - 4*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 - 4*a^3*b*d^4)*q)*r*x - 12*(b^4*d^4*p*r*x^4 + 4*a*b^3*d^4*p*r*x^3 + 6*a^2*b^2*d^4*p*r*x^2 + 4*a^3*b*d^4*p*r*x + a^4*d^4*p*r)*log(b*x + a) - 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2 + 4*a^3*b*d^4*q*r*x + a^4*d^4*q*r)*log(b*x + a)`



$$a^3 b d^4 q r x - (b^4 c^4 - 4 a b^3 c^3 d + 6 a^2 b^2 c^2 d^2 - 4 a^3 b c d^3) q r \log(d x + c) - 12 (b^4 d^4 x^4 + 4 a b^3 d^4 x^3 + 6 a^2 b^2 d^4 x^2 + 4 a^3 b d^4 x) \log(e) - 12 (b^4 d^4 r x^4 + 4 a b^3 d^4 r x^3 + 6 a^2 b^2 d^4 r x^2 + 4 a^3 b d^4 r x) \log(f) / (b d^4)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*3\*ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r),x)

[Out] Timed out

**Giac [B]** time = 1.40639, size = 774, normalized size = 4.5

$$-\frac{1}{16} (b^3 p r + b^3 q r - 4 b^3 r \log(f) - 4 b^3) x^4 - \frac{(3 a b^2 d p r - b^3 c q r + 4 a b^2 d q r - 12 a b^2 d r \log(f) - 12 a b^2 d) x^3}{12 d} + \frac{1}{4} (b^3 p r x^4 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^3\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r),x, algorithm="giac")

[Out] 
$$-1/16*(b^3*p*r + b^3*q*r - 4*b^3*r*\log(f) - 4*b^3)*x^4 - 1/12*(3*a*b^2*d*p*r - b^3*c*q*r + 4*a*b^2*d*q*r - 12*a*b^2*d*r*\log(f) - 12*a*b^2*d)*x^3/d + 1/4*(b^3*p*r*x^4 + 4*a*b^2*p*r*x^3 + 6*a^2*b*p*r*x^2 + 4*a^3*p*r*x)*\log(b*x + a) + 1/4*(b^3*q*r*x^4 + 4*a*b^2*q*r*x^3 + 6*a^2*b*q*r*x^2 + 4*a^3*q*r*x)*\log(d*x + c) - 1/8*(3*a^2*b*d^2*p*r + b^3*c^2*q*r - 4*a*b^2*c*d*q*r + 6*a^2*b*d^2*q*r - 12*a^2*b*d^2*r*\log(f) - 12*a^2*b*d^2)*x^2/d^2 - 1/4*(a^3*d^3*p*r - b^3*c^3*q*r + 4*a*b^2*c^2*d*q*r - 6*a^2*b*c*d^2*q*r + 4*a^3*d^3*q*r - 4*a^3*d^3*r*\log(f) - 4*a^3*d^3)*x/d^3 + 1/8*(a^4*d^4*p*r - b^4*c^4*q*r + 4*a*b^3*c^3*d*q*r - 6*a^2*b^2*c^2*d^2*q*r + 4*a^3*b*c*d^3*q*r)*\log(\text{abs}(b*d*x^2 + b*c*x + a*d*x + a*c))/(b*d^4) + 1/8*(a^4*b*c*d^4*p*r - a^5*d^5*p*r + b^5*c^5*q*r - 5*a*b^4*c^4*d*q*r + 10*a^2*b^3*c^3*d^2*q*r - 10*a^3*b^2*c^2*d^3*q*r + 4*a^4*b*c*d^4*q*r)*\log(\text{abs}((2*b*d*x + b*c + a*d - \text{abs}(b*c - a*d))/(2*b*d*x + b*c + a*d + \text{abs}(b*c - a*d))))/(b*d^4*\text{abs}(b*c - a*d))$$

### 3.9 $\int (a + bx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$

**Optimal.** Leaf size=143

$$-\frac{qrx(bc-ad)^2}{3d^2} + \frac{qr(bc-ad)^3 \log(c+dx)}{3bd^3} + \frac{(a+bx)^3 \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{3b} + \frac{qr(a+bx)^2(bc-ad)}{6bd} - \frac{pr(a+bx)}{9b}$$

[Out]  $-\left(\frac{(b*c - a*d)^2*q*r*x}{3*d^2}\right) + \left(\frac{(b*c - a*d)*q*r*(a + b*x)^2}{6*b*d}\right) - \left(\frac{p*r*(a + b*x)^3}{9*b}\right) - \left(\frac{q*r*(a + b*x)^3}{9*b}\right) + \left(\frac{(b*c - a*d)^3*q*r*\text{Log}[c + d*x]}{3*b*d^3}\right) + \left(\frac{(a + b*x)^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]}{3*b}\right)$

**Rubi [A]** time = 0.0608917, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2495, 32, 43}

$$-\frac{qrx(bc-ad)^2}{3d^2} + \frac{qr(bc-ad)^3 \log(c+dx)}{3bd^3} + \frac{(a+bx)^3 \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{3b} + \frac{qr(a+bx)^2(bc-ad)}{6bd} - \frac{pr(a+bx)}{9b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x)^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r], x]$

[Out]  $-\left(\frac{(b*c - a*d)^2*q*r*x}{3*d^2}\right) + \left(\frac{(b*c - a*d)*q*r*(a + b*x)^2}{6*b*d}\right) - \left(\frac{p*r*(a + b*x)^3}{9*b}\right) - \left(\frac{q*r*(a + b*x)^3}{9*b}\right) + \left(\frac{(b*c - a*d)^3*q*r*\text{Log}[c + d*x]}{3*b*d^3}\right) + \left(\frac{(a + b*x)^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]}{3*b}\right)$

#### Rule 2495

$\text{Int}[\text{Log}[(e._)*((f._)*((a._) + (b._)*(x._))^{(p._)}*((c._) + (d._)*(x._))^{(q._)})^{(r._)}]*((g._) + (h._)*(x._))^{(m._)}, x\_Symbol] \rightarrow \text{Simp}[\left(\frac{(g + h*x)^{(m+1)}*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]}{(h*(m+1))}, x\right) + \left(-\text{Dist}[(b*p*r)/(h*(m+1)), \text{Int}[(g + h*x)^{(m+1)}/(a + b*x), x], x] - \text{Dist}[(d*q*r)/(h*(m+1)), \text{Int}[(g + h*x)^{(m+1)}/(c + d*x), x], x)\right) /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, p, q, r\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m, -1]$

#### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned} \int (a + bx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(a + bx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{3b} - \frac{1}{3}(pr) \int (a + bx)^2 dx - \frac{(dqr)}{3} \int \frac{(a + bx)^2}{a + dx} dx \\ &= -\frac{pr(a + bx)^3}{9b} + \frac{(a + bx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{3b} - \frac{(dqr) \int \left( \frac{b(bc - ad)}{a^2} \right) dx}{3} \\ &= -\frac{(bc - ad)^2 qrx}{3d^2} + \frac{(bc - ad)qr(a + bx)^2}{6bd} - \frac{pr(a + bx)^3}{9b} - \frac{qr(a + bx)^3}{9b} + \frac{(dqr) \int \left( \frac{b(bc - ad)}{a^2} \right) dx}{3} \end{aligned}$$

**Mathematica [A]** time = 0.134685, size = 127, normalized size = 0.89

$$\frac{(a + bx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) - \frac{r(-3b^2(2p+3q)(c+dx)^2(bc-ad)+6bdx(p+3q)(bc-ad)^2-6q(bc-ad)^3 \log(c+dx)+2b^3(p+q)(c+dx)^3)}{6d^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], x]
```

```
[Out] (-r*(6*b*d*(b*c - a*d)^2*(p + 3*q)*x - 3*b^2*(b*c - a*d)*(2*p + 3*q)*(c + d*x)^2 + 2*b^3*(p + q)*(c + d*x)^3 - 6*(b*c - a*d)^3*q*Log[c + d*x]))/(6*d^3) + (a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]/(3*b)
```

**Maple [F]** time = 0.386, size = 0, normalized size = 0.

$$\int (bx + a)^2 \ln \left( e \left( f(bx + a)^p (dx + c)^q \right)^r \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)`

[Out] `int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)`

**Maxima [A]** time = 1.28831, size = 262, normalized size = 1.83

$$\frac{1}{3} (b^2x^3 + 3abx^2 + 3a^2x) \log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right) + \frac{\left(\frac{6a^3fp \log(bx+a)}{b} - \frac{2b^2d^2f(p+q)x^3 + 3(abd^2f(2p+3q) - b^2cdfq)x^2 + 6(a^2d^2f(p+q) - b^2cd^2f) x + 3a^2d^2f}{d^2}\right)}{18f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")`

[Out] `1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/18*(6*a^3*f*p*log(b*x + a)/b - (2*b^2*d^2*f*(p + q)*x^3 + 3*(a*b*d^2*f*(2*p + 3*q) - b^2*c*d*f*q)*x^2 + 6*(a^2*d^2*f*(p + 3*q) + b^2*c^2*f*q - 3*a*b*c*d*f*q)*x)/d^2 + 6*(b^2*c^3*f*q - 3*a*b*c^2*d*f*q + 3*a^2*c*d^2*f*q)*log(d*x + c)/d^3)*r/f`

**Fricas [B]** time = 0.880294, size = 693, normalized size = 4.85

$$\frac{2(b^3d^3p + b^3d^3q)rx^3 + 3(2ab^2d^3p - (b^3cd^2 - 3ab^2d^3)q)rx^2 + 6(a^2bd^3p + (b^3c^2d - 3ab^2cd^2 + 3a^2bd^3)q)rx - 6(b^3d^3p + b^3d^3q)r}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")`

[Out] `-1/18*(2*(b^3*d^3*p + b^3*d^3*q)*r*x^3 + 3*(2*a*b^2*d^3*p - (b^3*c*d^2 - 3*a*b^2*d^3)*q)*r*x^2 + 6*(a^2*b*d^3*p + (b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*q)*r*x - 6*(b^3*d^3*p*r*x^3 + 3*a*b^2*d^3*p*r*x^2 + 3*a^2*b*d^3*p*r*x + a^3*d^3*p*r)*log(b*x + a) - 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^3*q*r*x^2 + 3*a^2*b*d^3*q*r*x + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2)*q*r)*log(d*x + c) - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x)*log(e) - 6*(b^3*d^3*r*x^3 + 3*a*b^2*d^3*r*x^2 + 3*a^2*b*d^3*r*x)*log(f))/(b*d^3)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*2\*ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r),x)

[Out] Timed out

---

**Giac [B]** time = 1.30847, size = 566, normalized size = 3.96

$$-\frac{1}{9}(b^2pr + b^2qr - 3b^2r \log(f) - 3b^2)x^3 - \frac{(2abdpr - b^2cqr + 3abdqr - 6abdr \log(f) - 6abd)x^2}{6d} + \frac{1}{3}(b^2prx^3 + 3abprx^2 + 3abqrx^2 + 3a^2p^2r^2x^2 + 3a^2p^2r^2x) \log(bx + a) + \frac{1}{3}(b^2q^2r^2x^3 + 3a^2b^2q^2r^2x^2 + 3a^2q^2r^2x) \log(dx + c) - \frac{1}{3}(a^2d^2p^2r^2 + b^2c^2q^2r^2 - 3a^2b^2c^2d^2q^2r^2 + 3a^2d^2p^2q^2r^2 - 3a^2d^2p^2r^2 \log(f) - 3a^2d^2p^2) \frac{x}{d^2} + \frac{1}{6}(a^3d^3p^2r^2 + b^3c^3q^2r^2 - 3a^2b^2c^2d^2q^2r^2 + 3a^2b^2c^2d^2q^2r^2) \log(\text{abs}(bd^2x^2 + b^2cx + a^2dx + a^2c)) / (bd^3) + \frac{1}{6}(a^3b^2c^2d^3p^2r^2 - a^4d^4p^2r^2 - b^4c^4q^2r^2 + 4a^2b^3c^3d^2q^2r^2 - 6a^2b^2c^2d^2q^2r^2 + 3a^3b^2c^2d^3q^2r^2) \log(\text{abs}((2bd^2x + b^2c + a^2d - \text{abs}(b^2c - a^2d)) / (2bd^2x + b^2c + a^2d + \text{abs}(b^2c - a^2d)))) / (bd^3 \text{abs}(b^2c - a^2d))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r),x, algorithm="giac")

[Out] -1/9\*(b^2\*p\*r + b^2\*q\*r - 3\*b^2\*r\*log(f) - 3\*b^2)\*x^3 - 1/6\*(2\*a\*b\*d\*p\*r - b^2\*c\*q\*r + 3\*a\*b\*d\*q\*r - 6\*a\*b\*d\*r\*log(f) - 6\*a\*b\*d)\*x^2/d + 1/3\*(b^2\*p\*r\*x^3 + 3\*a\*b\*p\*r\*x^2 + 3\*a^2\*p\*r\*x)\*log(b\*x + a) + 1/3\*(b^2\*q\*r\*x^3 + 3\*a\*b\*q\*r\*x^2 + 3\*a^2\*q\*r\*x)\*log(d\*x + c) - 1/3\*(a^2\*d^2\*p\*r + b^2\*c^2\*q\*r - 3\*a\*b\*c\*d\*q\*r + 3\*a^2\*d^2\*q\*r - 3\*a^2\*d^2\*r\*log(f) - 3\*a^2\*d^2)\*x/d^2 + 1/6\*(a^3\*d^3\*p\*r + b^3\*c^3\*q\*r - 3\*a\*b^2\*c^2\*d\*q\*r + 3\*a^2\*b\*c\*d^2\*q\*r)\*log(abs(b\*d\*x^2 + b\*c\*x + a\*d\*x + a\*c))/(b\*d^3) + 1/6\*(a^3\*b\*c\*d^3\*p\*r - a^4\*d^4\*p\*r - b^4\*c^4\*q\*r + 4\*a\*b^3\*c^3\*d\*q\*r - 6\*a^2\*b^2\*c^2\*d^2\*q\*r + 3\*a^3\*b\*c\*d^3\*q\*r)\*log(abs((2\*b\*d\*x + b\*c + a\*d - abs(b\*c - a\*d))/(2\*b\*d\*x + b\*c + a\*d + abs(b\*c - a\*d))))/(b\*d^3\*abs(b\*c - a\*d))

### 3.10 $\int (a + bx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$

**Optimal.** Leaf size=116

$$-\frac{qr(bc - ad)^2 \log(c + dx)}{2bd^2} + \frac{(a + bx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2b} + \frac{qrx(bc - ad)}{2d} - \frac{qr(a + bx)^2}{4b} - \frac{1}{2}aprx - \frac{1}{4}bprx^2$$

[Out]  $-(a*p*r*x)/2 + ((b*c - a*d)*q*r*x)/(2*d) - (b*p*r*x^2)/4 - (q*r*(a + b*x)^2)/(4*b) - ((b*c - a*d)^2*q*r*Log[c + d*x])/(2*b*d^2) + ((a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/(2*b)$

**Rubi [A]** time = 0.0432106, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$ , Rules used = {2495, 43}

$$-\frac{qr(bc - ad)^2 \log(c + dx)}{2bd^2} + \frac{(a + bx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2b} + \frac{qrx(bc - ad)}{2d} - \frac{qr(a + bx)^2}{4b} - \frac{1}{2}aprx - \frac{1}{4}bprx^2$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r], x]$

[Out]  $-(a*p*r*x)/2 + ((b*c - a*d)*q*r*x)/(2*d) - (b*p*r*x^2)/4 - (q*r*(a + b*x)^2)/(4*b) - ((b*c - a*d)^2*q*r*Log[c + d*x])/(2*b*d^2) + ((a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/(2*b)$

#### Rule 2495

$\text{Int}[\text{Log}[(e._)*((f._)*((a._) + (b._)*(x._))^(p._))*((c._) + (d._)*(x._))^(q._))]^(r._)]*((g._) + (h._)*(x._))^(m._), x\_Symbol] \rightarrow \text{Simp}[(g + h*x)^(m + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]/(h*(m + 1)), x] + (-\text{Dist}[(b*p*r)/(h*(m + 1)), \text{Int}[(g + h*x)^(m + 1)/(a + b*x), x], x] - \text{Dist}[(d*q*r)/(h*(m + 1)), \text{Int}[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, p, q, r\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m, -1]$

#### Rule 43

$\text{Int}[(a._) + (b._)*(x._)]^(m._)*((c._) + (d._)*(x._))^(n._), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{Le}$

Q[7\*m + 4\*n + 4, 0] || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0]

### Rubi steps

$$\begin{aligned} \int (a + bx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(a + bx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2b} - \frac{1}{2} (pr) \int (a + bx) dx - \frac{(dqr) \int \left( \frac{(a + bx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2b} \right)}{2} \\ &= -\frac{1}{2} aprx - \frac{1}{4} bprx^2 + \frac{(a + bx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2b} - \frac{(dqr) \int \left( \frac{(a + bx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2b} \right)}{2} \\ &= -\frac{1}{2} aprx + \frac{(bc - ad)qrx}{2d} - \frac{1}{4} bprx^2 - \frac{qr(a + bx)^2}{4b} - \frac{(bc - ad)^2 qr \log(c + dx)}{2bd^2} \end{aligned}$$

**Mathematica [A]** time = 0.19288, size = 105, normalized size = 0.91

$$\frac{a^2 pr \log(a + bx)}{2b} - \frac{dx \left( r(2ad(p + 2q) - 2bcq + bdx(p + q)) - 2d(2a + bx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) \right) + 2cqr(bc - 2ad)}{4d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r], x]

[Out] (a^2\*p\*r\*Log[a + b\*x])/(2\*b) - (2\*c\*(b\*c - 2\*a\*d)\*q\*r\*Log[c + d\*x] + d\*x\*(r\*(-2\*b\*c\*q + 2\*a\*d\*(p + 2\*q) + b\*d\*(p + q)\*x) - 2\*d\*(2\*a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r))/(4\*d^2)

**Maple [F]** time = 0.174, size = 0, normalized size = 0.

$$\int (bx + a) \ln \left( e \left( f(bx + a)^p (dx + c)^q \right)^r \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r), x)

[Out] int((b\*x+a)\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r), x)

**Maxima [A]** time = 1.35446, size = 159, normalized size = 1.37

$$\frac{1}{2} (bx^2 + 2ax) \log \left( (bx + a)^p (dx + c)^q f \right)^r e + \frac{\left( \frac{2a^2fp \log(bx+a)}{b} - \frac{bdf(p+q)x^2 + 2(adf(p+2q) - bcfq)x}{d} - \frac{2(bc^2fq - 2acdfq) \log(dx+c)}{d^2} \right) r}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r),x, algorithm="maxima")

[Out] 1/2\*(b\*x^2 + 2\*a\*x)\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e) + 1/4\*(2\*a^2\*f\*p\*log(b\*x + a)/b - (b\*d\*f\*(p + q)\*x^2 + 2\*(a\*d\*f\*(p + 2\*q) - b\*c\*f\*q)\*x)/d - 2\*(b\*c^2\*f\*q - 2\*a\*c\*d\*f\*q)\*log(d\*x + c)/d^2)\*r/f

**Fricas [A]** time = 0.820055, size = 435, normalized size = 3.75

$$\frac{(b^2d^2p + b^2d^2q)rx^2 + 2(abd^2p - (b^2cd - 2abd^2)q)rx - 2(b^2d^2prx^2 + 2abd^2prx + a^2d^2pr) \log(bx + a) - 2(b^2d^2qrx^2 + \dots)}{4bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r),x, algorithm="fricas")

[Out] -1/4\*((b^2\*d^2\*p + b^2\*d^2\*q)\*r\*x^2 + 2\*(a\*b\*d^2\*p - (b^2\*c\*d - 2\*a\*b\*d^2)\*q)\*r\*x - 2\*(b^2\*d^2\*p\*r\*x^2 + 2\*a\*b\*d^2\*p\*r\*x + a^2\*d^2\*p\*r)\*log(b\*x + a) - 2\*(b^2\*d^2\*q\*r\*x^2 + 2\*a\*b\*d^2\*q\*r\*x - (b^2\*c^2 - 2\*a\*b\*c\*d)\*q\*r)\*log(d\*x + c) - 2\*(b^2\*d^2\*x^2 + 2\*a\*b\*d^2\*x)\*log(e) - 2\*(b^2\*d^2\*r\*x^2 + 2\*a\*b\*d^2\*r\*x)\*log(f))/(b\*d^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r),x)

[Out] Timed out



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**Giac [B]** time = 1.35044, size = 381, normalized size = 3.28

$$-\frac{1}{4}(bpr + bqr - 2br \log(f) - 2b)x^2 + \frac{1}{2}(bprx^2 + 2aprx) \log(bx + a) + \frac{1}{2}(bqrx^2 + 2aqr) \log(dx + c) - \frac{(adpr - bcqr)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r),x, algorithm="giac")

[Out] -1/4\*(b\*p\*r + b\*q\*r - 2\*b\*r\*log(f) - 2\*b)\*x^2 + 1/2\*(b\*p\*r\*x^2 + 2\*a\*p\*r\*x)\*log(b\*x + a) + 1/2\*(b\*q\*r\*x^2 + 2\*a\*q\*r\*x)\*log(d\*x + c) - 1/2\*(a\*d\*p\*r - b\*c\*q\*r + 2\*a\*d\*q\*r - 2\*a\*d\*r\*log(f) - 2\*a\*d)\*x/d + 1/4\*(a^2\*d^2\*p\*r - b^2\*c^2\*q\*r + 2\*a\*b\*c\*d\*q\*r)\*log(abs(b\*d\*x^2 + b\*c\*x + a\*d\*x + a\*c))/(b\*d^2) + 1/4\*(a^2\*b\*c\*d^2\*p\*r - a^3\*d^3\*p\*r + b^3\*c^3\*q\*r - 3\*a\*b^2\*c^2\*d\*q\*r + 2\*a^2\*b\*c\*d^2\*q\*r)\*log(abs((2\*b\*d\*x + b\*c + a\*d - abs(b\*c - a\*d))/(2\*b\*d\*x + b\*c + a\*d + abs(b\*c - a\*d))))/(b\*d^2\*abs(b\*c - a\*d))

$$3.11 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx} dx$$

**Optimal.** Leaf size=107

$$-\frac{qr \operatorname{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{b} + \frac{\log(a+bx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} - \frac{qr \log(a+bx) \log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} - \frac{pr \log^2(a+bx)}{2b}$$

[Out]  $-(p*r*\operatorname{Log}[a + b*x]^2)/(2*b) - (q*r*\operatorname{Log}[a + b*x]*\operatorname{Log}[(b*(c + d*x))/(b*c - a*d)]/b + (\operatorname{Log}[a + b*x]*\operatorname{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/b - (q*r*\operatorname{PolyLog}[2, -((d*(a + b*x))/(b*c - a*d))])/b$

**Rubi [A]** time = 0.0829128, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {2494, 2390, 2301, 2394, 2393, 2391}

$$-\frac{qr \operatorname{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{b} + \frac{\log(a+bx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} - \frac{qr \log(a+bx) \log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} - \frac{pr \log^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]/(a + b*x), x]$

[Out]  $-(p*r*\operatorname{Log}[a + b*x]^2)/(2*b) - (q*r*\operatorname{Log}[a + b*x]*\operatorname{Log}[(b*(c + d*x))/(b*c - a*d)]/b + (\operatorname{Log}[a + b*x]*\operatorname{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/b - (q*r*\operatorname{PolyLog}[2, -((d*(a + b*x))/(b*c - a*d))])/b$

#### Rule 2494

$\operatorname{Int}[\operatorname{Log}[(e_*)*((f_*)*((a_*) + (b_*)*(x_))^{(p_*)*((c_*) + (d_*)*(x_))^{(q_*)})^{(r_*)})]/((g_*) + (h_*)*(x_)), x\_Symbol] :> \operatorname{Simp}[(\operatorname{Log}[g + h*x]*\operatorname{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/h, x] + (-\operatorname{Dist}[(b*p*r)/h, \operatorname{Int}[\operatorname{Log}[g + h*x]/(a + b*x), x], x] - \operatorname{Dist}[(d*q*r)/h, \operatorname{Int}[\operatorname{Log}[g + h*x]/(c + d*x), x], x]) /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, h, p, q, r\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0]$

#### Rule 2390

$\operatorname{Int}[(a_*) + \operatorname{Log}[(c_*)*((d_*) + (e_*)*(x_))^{(n_*)})*(b_*)^{(p_*)*((f_*) + (g_*)*(x_))^{(q_*)}], x\_Symbol] :> \operatorname{Dist}[1/e, \operatorname{Subst}[\operatorname{Int}[(f*x)/d]^q*(a + b*\operatorname{Log}[c*x^n])^p, x], x, d + e*x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, n, p, q\}, x] \&\& E$

qQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e^n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx} dx &= \frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} - (pr) \int \frac{\log(a+bx)}{a+bx} dx - \frac{(dqr) \int \frac{\log(c+dx)}{c+dx} dx}{b} \\
 &= -\frac{qr \log(a+bx) \log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} - \frac{(pr) \log^2(a+bx)}{2b} \\
 &= -\frac{pr \log^2(a+bx)}{2b} - \frac{qr \log(a+bx) \log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} \\
 &= -\frac{pr \log^2(a+bx)}{2b} - \frac{qr \log(a+bx) \log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}
 \end{aligned}$$

**Mathematica [A]** time = 0.108976, size = 93, normalized size = 0.87

$$\frac{2qr \operatorname{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right) + \log(a+bx) \left(-2 \log\left(e \left(f(a+bx)^p (c+dx)^q\right)^r\right) + 2qr \log\left(\frac{b(c+dx)}{bc-ad}\right) + pr \log(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(a + b\*x), x]

[Out] -(Log[a + b\*x]\*(p\*r\*Log[a + b\*x] + 2\*q\*r\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] - 2\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]) + 2\*q\*r\*PolyLog[2, (d\*(a + b\*x))/(-(b\*c) + a\*d)]/(2\*b)

**Maple [F]** time = 0.436, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e \left(f(bx+a)^p (dx+c)^q\right)^r\right)}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(b\*x+a), x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(b\*x+a), x)

**Maxima [A]** time = 3.13703, size = 221, normalized size = 2.07

$$\frac{\left(\frac{2\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\operatorname{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)f q}{b} - \frac{fp \log(bx+a)^2 + 2fq \log(bx+a) \log(dx+c)}{b}\right) r}{2f} - \frac{(fp \log(bx+a) + fq \log(dx+c))r \log(bx+a)}{bf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(b\*x+a), x, algorithm="maxima")

[Out] -1/2\*(2\*(log(b\*x + a)\*log((b\*d\*x + a\*d)/(b\*c - a\*d) + 1) + dilog(-(b\*d\*x + a\*d)/(b\*c - a\*d)))\*f\*q/b - (f\*p\*log(b\*x + a)^2 + 2\*f\*q\*log(b\*x + a)\*log(d\*x + c))/b)\*r/f - (f\*p\*log(b\*x + a) + f\*q\*log(d\*x + c))\*r\*log(b\*x + a)/(b\*f)

+ log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)\*log(b\*x + a)/b

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)}{bx + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(b\*x+a),x, algorithm="fricas")

[Out] integral(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)/(b\*x + a), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r)/(b\*x+a),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)}{bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(b\*x+a),x, algorithm="giac")

[Out] integrate(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)/(b\*x + a), x)

$$3.12 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2} dx$$

**Optimal.** Leaf size=95

$$-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + \frac{dqr \log(a+bx)}{b(bc-ad)} - \frac{dqr \log(c+dx)}{b(bc-ad)} - \frac{pr}{b(a+bx)}$$

[Out] -((p\*r)/(b\*(a + b\*x))) + (d\*q\*r\*Log[a + b\*x])/(b\*(b\*c - a\*d)) - (d\*q\*r\*Log[c + d\*x])/(b\*(b\*c - a\*d)) - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]/(b\*(a + b\*x))

**Rubi [A]** time = 0.0374773, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {2495, 32, 36, 31}

$$-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + \frac{dqr \log(a+bx)}{b(bc-ad)} - \frac{dqr \log(c+dx)}{b(bc-ad)} - \frac{pr}{b(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(a + b\*x)^2,x]

[Out] -((p\*r)/(b\*(a + b\*x))) + (d\*q\*r\*Log[a + b\*x])/(b\*(b\*c - a\*d)) - (d\*q\*r\*Log[c + d\*x])/(b\*(b\*c - a\*d)) - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]/(b\*(a + b\*x))

### Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m + 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)), Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2} dx &= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + (pr) \int \frac{1}{(a+bx)^2} dx + \frac{(dqr) \int \frac{1}{(a+bx)(c+dx)} dx}{b} \\ &= -\frac{pr}{b(a+bx)} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + \frac{(dqr) \int \frac{1}{a+bx} dx}{bc-ad} - \frac{(d^2qr) \int \frac{1}{c+dx} dx}{b(bc-ad)} \\ &= -\frac{pr}{b(a+bx)} + \frac{dqr \log(a+bx)}{b(bc-ad)} - \frac{dqr \log(c+dx)}{b(bc-ad)} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} \end{aligned}$$

**Mathematica [A]** time = 0.0576862, size = 89, normalized size = 0.94

$$\frac{r\left(\frac{dq \log(a+bx)}{bc-ad} - \frac{dq \log(c+dx)}{bc-ad} - \frac{p}{a+bx}\right)}{b} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2,x]
```

```
[Out] (r*(-(p/(a + b*x)) + (d*q*Log[a + b*x])/(b*c - a*d) - (d*q*Log[c + d*x])/(b
*c - a*d))/b - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(b*(a + b*x))
```

**Maple [F]** time = 0.421, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x)`

[Out] `int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x)`

**Maxima [A]** time = 1.26499, size = 134, normalized size = 1.41

$$\frac{\left(dfq\left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right) - \frac{bfp}{b^2x+ab}\right)r}{bf} - \frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="maxima")`

[Out] `(d*f*q*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)) - b*f*p/(b^2*x + a*b))*r/(b*f) - log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x + a)*b)`

**Fricas [A]** time = 0.902385, size = 267, normalized size = 2.81

$$\frac{(bc-ad)pr + (bc-ad)r \log(f) - (bdqrx + (adq - (bc-ad)p)r) \log(bx+a) + (bdqrx + bcqr) \log(dx+c) + (bc-ad) \log(e)}{ab^2c - a^2bd + (b^3c - ab^2d)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="fricas")`

[Out] `-((b*c - a*d)*p*r + (b*c - a*d)*r*log(f) - (b*d*q*r*x + (a*d*q - (b*c - a*d)*p)*r)*log(b*x + a) + (b*d*q*r*x + b*c*q*r)*log(d*x + c) + (b*c - a*d)*log(e))/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 1.19727, size = 151, normalized size = 1.59

$$\frac{dqr \log(bx + a)}{b^2c - abd} - \frac{dqr \log(dx + c)}{b^2c - abd} - \frac{pr \log(bx + a)}{b^2x + ab} - \frac{qr \log(dx + c)}{b^2x + ab} - \frac{pr + r \log(f) + 1}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] d*q*r*log(b*x + a)/(b^2*c - a*b*d) - d*q*r*log(d*x + c)/(b^2*c - a*b*d) - p
*r*log(b*x + a)/(b^2*x + a*b) - q*r*log(d*x + c)/(b^2*x + a*b) - (p*r + r*log(f) + 1)/(b^2*x + a*b)
```

$$3.13 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3} dx$$

**Optimal.** Leaf size=135

$$-\frac{d^2qr \log(a+bx)}{2b(bc-ad)^2} + \frac{d^2qr \log(c+dx)}{2b(bc-ad)^2} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} - \frac{dqr}{2b(a+bx)(bc-ad)} - \frac{pr}{4b(a+bx)^2}$$

[Out]  $-(p*r)/(4*b*(a+b*x)^2) - (d*q*r)/(2*b*(b*c-a*d)*(a+b*x)) - (d^2*q*r*\text{Log}[a+b*x])/(2*b*(b*c-a*d)^2) + (d^2*q*r*\text{Log}[c+d*x])/(2*b*(b*c-a*d)^2) - \text{Log}[e*(f*(a+b*x)^p*(c+d*x)^q)^r]/(2*b*(a+b*x)^2)$

**Rubi [A]** time = 0.0561178, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2495, 32, 44}

$$-\frac{d^2qr \log(a+bx)}{2b(bc-ad)^2} + \frac{d^2qr \log(c+dx)}{2b(bc-ad)^2} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} - \frac{dqr}{2b(a+bx)(bc-ad)} - \frac{pr}{4b(a+bx)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Log}[e*(f*(a+b*x)^p*(c+d*x)^q)^r]/(a+b*x)^3, x]$

[Out]  $-(p*r)/(4*b*(a+b*x)^2) - (d*q*r)/(2*b*(b*c-a*d)*(a+b*x)) - (d^2*q*r*\text{Log}[a+b*x])/(2*b*(b*c-a*d)^2) + (d^2*q*r*\text{Log}[c+d*x])/(2*b*(b*c-a*d)^2) - \text{Log}[e*(f*(a+b*x)^p*(c+d*x)^q)^r]/(2*b*(a+b*x)^2)$

### Rule 2495

$\text{Int}[\text{Log}[(e._)*((f._)*((a._) + (b._)*(x._))^(p._))*((c._) + (d._)*(x._))^(q._)]^(r._)]*((g._) + (h._)*(x._))^(m._), x\_Symbol] \rightarrow \text{Simp}[(g + h*x)^(m + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1)), x] + (-\text{Dist}[(b*p*r)/(h*(m + 1)), \text{Int}[(g + h*x)^(m + 1)/(a + b*x), x], x] - \text{Dist}[(d*q*r)/(h*(m + 1)), \text{Int}[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, p, q, r\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m, -1]$

### Rule 32

$\text{Int}[(a._) + (b._)*(x._)]^(m._), x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; \text{FreeQ}\{a, b, m\}, x] \&\& \text{NeQ}[m, -1]$

Rule 44

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3} dx &= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} + \frac{1}{2}(pr) \int \frac{1}{(a+bx)^3} dx + \frac{(dqr) \int \frac{1}{(a+bx)^2(c+dx)} dx}{2b} \\ &= -\frac{pr}{4b(a+bx)^2} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} + \frac{(dqr) \int \left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{bd}{(bc-ad)^2(a+bx)}\right) dx}{2b} \\ &= -\frac{pr}{4b(a+bx)^2} - \frac{dqr}{2b(bc-ad)(a+bx)} - \frac{d^2qr \log(a+bx)}{2b(bc-ad)^2} + \frac{d^2qr \log(c+dx)}{2b(bc-ad)^2} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b} \end{aligned}$$

**Mathematica [A]** time = 0.272963, size = 116, normalized size = 0.86

$$\frac{r \left( -\frac{d^2q \log(a+bx)}{(bc-ad)^2} + \frac{d^2q \log(c+dx)}{(bc-ad)^2} - \frac{p - \frac{2dq(a+bx)}{ad-bc}}{2(a+bx)^2} \right) - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(a + b\*x)^3,x]

[Out] (r\*(-(p - (2\*d\*q\*(a + b\*x))/(-(b\*c) + a\*d))/(2\*(a + b\*x)^2) - (d^2\*q\*Log[a + b\*x]))/(b\*c - a\*d)^2 + (d^2\*q\*Log[c + d\*x])/(b\*c - a\*d)^2 - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(a + b\*x)^2)/(2\*b)

**Maple [F]** time = 0.424, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x)`

[Out] `int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x)`

**Maxima [A]** time = 1.23822, size = 223, normalized size = 1.65

$$\frac{\left(2dfq\left(\frac{d\log(bx+a)}{b^2c^2-2abcd+a^2d^2} - \frac{d\log(dx+c)}{b^2c^2-2abcd+a^2d^2} + \frac{1}{abc-a^2d+(b^2c-abd)x}\right) + \frac{bfp}{b^3x^2+2ab^2x+a^2b}\right)r - \frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)}{2(bx+a)^2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="maxima")`

[Out] `-1/4*(2*d*f*q*(d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - d*log(d*x + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x) + b*f*p/(b^3*x^2 + 2*a*b^2*x + a^2*b))*r/(b*f) - 1/2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x + a)^2*b)`

**Fricas [B]** time = 0.87075, size = 689, normalized size = 5.1

$$\frac{2(b^2cd - abd^2)qrx + 2(b^2c^2 - 2abcd + a^2d^2)r\log(f) + ((b^2c^2 - 2abcd + a^2d^2)p + 2(abcd - a^2d^2)q)r + 2(b^2d^2qrx^2 + 4(a^2b^3c^2 - 2a^3b^2cd + a^4bd^2))}{4(a^2b^3c^2 - 2a^3b^2cd + a^4bd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="fricas")`

[Out] `-1/4*(2*(b^2*c*d - a*b*d^2)*q*r*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*r*log(f) + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*p + 2*(a*b*c*d - a^2*d^2)*q)*r + 2*(b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x + (a^2*d^2*q + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*p)*r)*log(b*x + a) - 2*(b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x - (b^2*c^2 - 2*a*b*c*d)*q*r)*log(d*x + c) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(e))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r)/(b\*x+a)\*\*3,x)

[Out] Timed out

**Giac [A]** time = 1.19189, size = 332, normalized size = 2.46

$$-\frac{d^2qr \log(bx + a)}{2(b^3c^2 - 2ab^2cd + a^2bd^2)} + \frac{d^2qr \log(dx + c)}{2(b^3c^2 - 2ab^2cd + a^2bd^2)} - \frac{pr \log(bx + a)}{2(b^3x^2 + 2ab^2x + a^2b)} - \frac{qr \log(dx + c)}{2(b^3x^2 + 2ab^2x + a^2b)} - \frac{2bdqr}{2(b^3x^2 + 2ab^2x + a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(b\*x+a)^3,x, algorithm="giac")

[Out] 
$$-1/2*d^2*q*r*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + 1/2*d^2*q*r*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*p*r*log(b*x + a)/(b^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*q*r*log(d*x + c)/(b^3*x^2 + 2*a*b^2*x + a^2*b) - 1/4*(2*b*d*q*r*x + b*c*p*r - a*d*p*r + 2*a*d*q*r + 2*b*c*r*log(f) - 2*a*d*r*log(f) + 2*b*c - 2*a*d)/(b^4*c*x^2 - a*b^3*d*x^2 + 2*a*b^3*c*x - 2*a^2*b^2*d*x + a^2*b^2*c - a^3*b*d)$$

$$3.14 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4} dx$$

**Optimal.** Leaf size=164

$$\frac{d^2qr}{3b(a+bx)(bc-ad)^2} + \frac{d^3qr \log(a+bx)}{3b(bc-ad)^3} - \frac{d^3qr \log(c+dx)}{3b(bc-ad)^3} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} - \frac{dqr}{6b(a+bx)^2(bc-ad)} - \frac{dqr}{9b(a+bx)(bc-ad)}$$

[Out]  $-(p*r)/(9*b*(a+b*x)^3) - (d*q*r)/(6*b*(b*c-a*d)*(a+b*x)^2) + (d^2*q*r)/(3*b*(b*c-a*d)^2*(a+b*x)) + (d^3*q*r*Log[a+b*x])/(3*b*(b*c-a*d)^3) - (d^3*q*r*Log[c+d*x])/(3*b*(b*c-a*d)^3) - Log[e*(f*(a+b*x)^p*(c+d*x)^q]^r)/(3*b*(a+b*x)^3)$

**Rubi [A]** time = 0.0693974, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2495, 32, 44}

$$\frac{d^2qr}{3b(a+bx)(bc-ad)^2} + \frac{d^3qr \log(a+bx)}{3b(bc-ad)^3} - \frac{d^3qr \log(c+dx)}{3b(bc-ad)^3} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} - \frac{dqr}{6b(a+bx)^2(bc-ad)} - \frac{dqr}{9b(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[Log[e\*(f\*(a+b\*x)^p\*(c+d\*x)^q)^r]/(a+b\*x)^4,x]

[Out]  $-(p*r)/(9*b*(a+b*x)^3) - (d*q*r)/(6*b*(b*c-a*d)*(a+b*x)^2) + (d^2*q*r)/(3*b*(b*c-a*d)^2*(a+b*x)) + (d^3*q*r*Log[a+b*x])/(3*b*(b*c-a*d)^3) - (d^3*q*r*Log[c+d*x])/(3*b*(b*c-a*d)^3) - Log[e*(f*(a+b*x)^p*(c+d*x)^q]^r)/(3*b*(a+b*x)^3)$

### Rule 2495

Int[Log[(e.\_)\*((f.\_)\*((a.\_)+(b.\_)\*(x.\_))^(p.\_))\*((c.\_)+(d.\_)\*(x.\_))^(q.\_)]^(r.\_)]\*((g.\_)+(h.\_)\*(x.\_))^(m.\_), x\_Symbol] :> Simp[((g+h\*x)^(m+1)\*Log[e\*(f\*(a+b\*x)^p\*(c+d\*x)^q]^r]/(h\*(m+1)), x] + (-Dist[(b\*p\*r)/(h\*(m+1)), Int[(g+h\*x)^(m+1)/(a+b\*x), x], x] - Dist[(d\*q\*r)/(h\*(m+1)), Int[(g+h\*x)^(m+1)/(c+d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r}, x] && NeQ[b\*c-a\*d, 0] && NeQ[m, -1]

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

#### Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned} \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4} dx &= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} + \frac{1}{3}(pr) \int \frac{1}{(a+bx)^4} dx + \frac{(dqr) \int \frac{1}{(a+bx)^3(c+dx)} dx}{3b} \\ &= -\frac{pr}{9b(a+bx)^3} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} + \frac{(dqr) \int \left(\frac{b}{(bc-ad)(a+bx)^3} - \frac{bd}{(bc-ad)^2(a+bx)}\right) dx}{3b} \\ &= -\frac{pr}{9b(a+bx)^3} - \frac{dqr}{6b(bc-ad)(a+bx)^2} + \frac{d^2qr}{3b(bc-ad)^2(a+bx)} + \frac{d^3qr \log(a+bx)}{3b(bc-ad)^3} \end{aligned}$$

**Mathematica [A]** time = 0.391948, size = 141, normalized size = 0.86

$$\frac{r \left( \frac{6d^2q(a+bx)^2}{(bc-ad)^2} + \frac{3dq(a+bx)}{ad-bc} - 2p \right) + \frac{d^3q \log(a+bx)}{(bc-ad)^3} - \frac{d^3q \log(c+dx)}{(bc-ad)^3} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4, x]
```

```
[Out] (r*((-2*p + (3*d*q*(a + b*x)))/(-(b*c) + a*d) + (6*d^2*q*(a + b*x)^2)/(b*c - a*d)^2)/(6*(a + b*x)^3) + (d^3*q*Log[a + b*x])/(b*c - a*d)^3 - (d^3*q*Log[c + d*x])/(b*c - a*d)^3 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3)/(3*b)
```





```
[Out] 1/18*(6*(b^3*c*d^2 - a*b^2*d^3)*q*r*x^2 - 3*(b^3*c^2*d - 6*a*b^2*c*d^2 + 5*
a^2*b*d^3)*q*r*x - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*r*
log(f) - (2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*p + 3*(a*b^
2*c^2*d - 4*a^2*b*c*d^2 + 3*a^3*d^3)*q)*r + 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^
3*q*r*x^2 + 3*a^2*b*d^3*q*r*x + (a^3*d^3*q - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*p)*r)*log(b*x + a) - 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^3
*q*r*x^2 + 3*a^2*b*d^3*q*r*x + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2)*q*
r)*log(d*x + c) - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log
(e))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^
3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a
^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*
b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**4,x)
```

[Out] Timed out

**Giac [B]** time = 1.21093, size = 633, normalized size = 3.86

$$\frac{d^3qr \log(bx + a)}{3(b^4c^3 - 3ab^3c^2d + 3a^2b^2cd^2 - a^3bd^3)} - \frac{d^3qr \log(dx + c)}{3(b^4c^3 - 3ab^3c^2d + 3a^2b^2cd^2 - a^3bd^3)} - \frac{pr \log(bx + a)}{3(b^4x^3 + 3ab^3x^2 + 3a^2b^2x + a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*d^3*q*r*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b
*d^3) - 1/3*d^3*q*r*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
- a^3*b*d^3) - 1/3*p*r*log(b*x + a)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x +
a^3*b) - 1/3*q*r*log(d*x + c)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b
) + 1/18*(6*b^2*d^2*q*r*x^2 - 3*b^2*c*d*q*r*x + 15*a*b*d^2*q*r*x - 2*b^2*c^
2*p*r + 4*a*b*c*d*p*r - 2*a^2*d^2*p*r - 3*a*b*c*d*q*r + 9*a^2*d^2*q*r - 6*b
^2*c^2*r*log(f) + 12*a*b*c*d*r*log(f) - 6*a^2*d^2*r*log(f) - 6*b^2*c^2 + 12
```

$$\frac{a^5 b^5 c^2 d^2 x^2 - 6 a^4 b^4 c^2 d^2 x + 3 a^3 b^3 c^2 d^2 x^2 - 2 a^2 b^2 c^2 d^2 x + a b c^2 d^2 x^3 - 6 a^5 b^5 c^2 d^2}{(b^6 c^2 x^3 - 2 a b^5 c^2 d x^3 + a^2 b^4 c^2 d^2 x^3 + 3 a^2 b^5 c^2 x^2 - 6 a^2 b^4 c^2 d x^2 + 3 a^3 b^3 d^2 x^2 + 3 a^2 b^4 c^2 x - 6 a^3 b^3 c^2 d x + 3 a^4 b^2 d^2 x + a^3 b^3 c^2 - 2 a^4 b^2 c^2 d + a^5 b d^2)}$$

$$3.15 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5} dx$$

**Optimal.** Leaf size=193

$$-\frac{d^3qr}{4b(a+bx)(bc-ad)^3} + \frac{d^2qr}{8b(a+bx)^2(bc-ad)^2} - \frac{d^4qr \log(a+bx)}{4b(bc-ad)^4} + \frac{d^4qr \log(c+dx)}{4b(bc-ad)^4} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4}$$

[Out]  $-(p*r)/(16*b*(a+b*x)^4) - (d*q*r)/(12*b*(b*c-a*d)*(a+b*x)^3) + (d^2*q*r)/(8*b*(b*c-a*d)^2*(a+b*x)^2) - (d^3*q*r)/(4*b*(b*c-a*d)^3*(a+b*x)) - (d^4*q*r*Log[a+b*x])/(4*b*(b*c-a*d)^4) + (d^4*q*r*Log[c+d*x])/(4*b*(b*c-a*d)^4) - Log[e*(f*(a+b*x)^p*(c+d*x)^q]^r/(4*b*(a+b*x)^4)$

**Rubi [A]** time = 0.0854361, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2495, 32, 44}

$$-\frac{d^3qr}{4b(a+bx)(bc-ad)^3} + \frac{d^2qr}{8b(a+bx)^2(bc-ad)^2} - \frac{d^4qr \log(a+bx)}{4b(bc-ad)^4} + \frac{d^4qr \log(c+dx)}{4b(bc-ad)^4} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

[In] Int[Log[e\*(f\*(a+b\*x)^p\*(c+d\*x)^q)^r]/(a+b\*x)^5,x]

[Out]  $-(p*r)/(16*b*(a+b*x)^4) - (d*q*r)/(12*b*(b*c-a*d)*(a+b*x)^3) + (d^2*q*r)/(8*b*(b*c-a*d)^2*(a+b*x)^2) - (d^3*q*r)/(4*b*(b*c-a*d)^3*(a+b*x)) - (d^4*q*r*Log[a+b*x])/(4*b*(b*c-a*d)^4) + (d^4*q*r*Log[c+d*x])/(4*b*(b*c-a*d)^4) - Log[e*(f*(a+b*x)^p*(c+d*x)^q]^r/(4*b*(a+b*x)^4)$

### Rule 2495

Int[Log[(e\_.)\*((f\_.)\*((a\_.)+(b\_.)\*(x\_))^(p\_.)\*((c\_.)+(d\_.)\*(x\_))^(q\_.))^(r\_.)]\*((g\_.)+(h\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((g+h\*x)^(m+1)\*Log[e\*(f\*(a+b\*x)^p\*(c+d\*x)^q]^r]/(h\*(m+1)), x] + (-Dist[(b\*p\*r)/(h\*(m+1)), Int[(g+h\*x)^(m+1)/(a+b\*x), x], x] - Dist[(d\*q\*r)/(h\*(m+1)), Int[(g+h\*x)^(m+1)/(c+d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r}, x] && NeQ[b\*c-a\*d, 0] && NeQ[m, -1]

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5} dx &= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4} + \frac{1}{4}(pr) \int \frac{1}{(a+bx)^5} dx + \frac{(dqr) \int \frac{1}{(a+bx)^4(c+dx)} dx}{4b} \\ &= -\frac{pr}{16b(a+bx)^4} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4} + \frac{(dqr) \int \left(\frac{b}{(bc-ad)(a+bx)^4} - \frac{bd}{(bc-ad)^2(a+bx)^4}\right) dx}{4b} \\ &= -\frac{pr}{16b(a+bx)^4} - \frac{dqr}{12b(bc-ad)(a+bx)^3} + \frac{d^2qr}{8b(bc-ad)^2(a+bx)^2} - \frac{d^3qr}{4b(bc-ad)^3(a+bx)} \end{aligned}$$

**Mathematica [A]** time = 0.406553, size = 164, normalized size = 0.85

$$\frac{r \left( \frac{-\frac{12d^3q(a+bx)^3}{(bc-ad)^3} + \frac{6d^2q(a+bx)^2}{(bc-ad)^2} + \frac{4dq(a+bx)}{ad-bc} - 3p}{12(a+bx)^4} - \frac{d^4q \log(a+bx)}{(bc-ad)^4} + \frac{d^4q \log(c+dx)}{(bc-ad)^4} \right) - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^5, x]
```

```
[Out] (r*((-3*p + (4*d*q*(a + b*x)))/(-(b*c) + a*d) + (6*d^2*q*(a + b*x)^2)/(b*c - a*d)^2 - (12*d^3*q*(a + b*x)^3)/(b*c - a*d)^3)/(12*(a + b*x)^4) - (d^4*q*Log[a + b*x])/(b*c - a*d)^4 + (d^4*q*Log[c + d*x])/(b*c - a*d)^4 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4)/(4*b)
```

**Maple [F]** time = 0.424, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{(bx+a)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(b\*x+a)^5,x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(b\*x+a)^5,x)

**Maxima [B]** time = 1.27945, size = 620, normalized size = 3.21

$$\left(2\left(\frac{6d^3 \log(bx+a)}{b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4} - \frac{6d^3 \log(dx+c)}{b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4} + \frac{6b^2d}{a^3b^3c^3-3a^4b^2c^2d+3a^5bcd^2-a^6d^3+(b^6c^3-3ab^5c^2d+3a^2b^4c^3-3ab^3c^2d^2+3a^4b^2c^3-3a^5bcd^2-a^6d^3)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(b\*x+a)^5,x, algorithm="maxima")

[Out] 
$$-1/48*(2*(6*d^3*\log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 6*d^3*\log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + (6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c^3*d^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*x))*d*f*q + 3*b*f*p/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b))*r/(b*f) - 1/4*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x + a)^4*b)$$

**Fricas [B]** time = 0.848858, size = 1758, normalized size = 9.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(b\*x+a)^5,x, algorithm="fricas")

```
[Out] -1/48*(12*(b^4*c*d^3 - a*b^3*d^4)*q*r*x^3 - 6*(b^4*c^2*d^2 - 8*a*b^3*c*d^3
+ 7*a^2*b^2*d^4)*q*r*x^2 + 4*(b^4*c^3*d - 6*a*b^3*c^2*d^2 + 18*a^2*b^2*c*d^
3 - 13*a^3*b*d^4)*q*r*x + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)*r*log(f) + (3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^
2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*p + 2*(2*a*b^3*c^3*d - 9*a^2*b^2*c^2*d
^2 + 18*a^3*b*c*d^3 - 11*a^4*d^4)*q)*r + 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*
q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2 + 4*a^3*b*d^4*q*r*x + (a^4*d^4*q + (b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*p)*r)*log(b
*x + a) - 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2
+ 4*a^3*b*d^4*q*r*x - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3)*q*r)*log(d*x + c) + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d
^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(e))/(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^
6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*
a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2
*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^
2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3
*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^
3*c*d^3 + a^7*b^2*d^4)*x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

---

**Giac [B]** time = 1.34956, size = 1010, normalized size = 5.23

$$\frac{d^4 q r \log(bx + a)}{4(b^5 c^4 - 4ab^4 c^3 d + 6a^2 b^3 c^2 d^2 - 4a^3 b^2 c d^3 + a^4 b d^4)} + \frac{d^4 q r \log(dx + c)}{4(b^5 c^4 - 4ab^4 c^3 d + 6a^2 b^3 c^2 d^2 - 4a^3 b^2 c d^3 + a^4 b d^4)} - \frac{d^4 q r \log(bx + a)}{4(b^5 x^4 - 4ab^4 c^3 d + 6a^2 b^3 c^2 d^2 - 4a^3 b^2 c d^3 + a^4 b d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*d^4*q*r*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4) + 1/4*d^4*q*r*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^
```

$$\begin{aligned}
& 3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*p*r*\log(b*x + \\
& a)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*q*r* \\
& \log(d*x + c)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) \\
& - 1/48*(12*b^3*d^3*q*r*x^3 - 6*b^3*c*d^2*q*r*x^2 + 42*a*b^2*d^3*q*r*x^2 + 4 \\
& *b^3*c^2*d*q*r*x - 20*a*b^2*c*d^2*q*r*x + 52*a^2*b*d^3*q*r*x + 3*b^3*c^3*p* \\
& r - 9*a*b^2*c^2*d*p*r + 9*a^2*b*c*d^2*p*r - 3*a^3*d^3*p*r + 4*a*b^2*c^2*d*q \\
& *r - 14*a^2*b*c*d^2*q*r + 22*a^3*d^3*q*r + 12*b^3*c^3*r*\log(f) - 36*a*b^2*c \\
& ^2*d*r*\log(f) + 36*a^2*b*c*d^2*r*\log(f) - 12*a^3*d^3*r*\log(f) + 12*b^3*c^3 \\
& - 36*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 12*a^3*d^3)/(b^8*c^3*x^4 - 3*a*b^7*c^2* \\
& d*x^4 + 3*a^2*b^6*c*d^2*x^4 - a^3*b^5*d^3*x^4 + 4*a*b^7*c^3*x^3 - 12*a^2*b^6 \\
& *c^2*d*x^3 + 12*a^3*b^5*c*d^2*x^3 - 4*a^4*b^4*d^3*x^3 + 6*a^2*b^6*c^3*x^2 \\
& - 18*a^3*b^5*c^2*d*x^2 + 18*a^4*b^4*c*d^2*x^2 - 6*a^5*b^3*d^3*x^2 + 4*a^3*b^5 \\
& *c^3*x - 12*a^4*b^4*c^2*d*x + 12*a^5*b^3*c*d^2*x - 4*a^6*b^2*d^3*x + a^4* \\
& b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
\end{aligned}$$

### 3.16 $\int (a + bx)^4 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$

**Optimal.** Leaf size=920

$$\frac{q^2 r^2 \log^2(c + dx)(bc - ad)^5}{5bd^5} - \frac{137q^2 r^2 \log(c + dx)(bc - ad)^5}{150bd^5} - \frac{2pqr^2 \log(c + dx)(bc - ad)^5}{25bd^5} - \frac{2pqr^2 \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c + dx)}{5bd^5}$$

[Out]  $-(a*(b*c - a*d)^3*p*q*r^2*x)/(5*d^3) + (2*(b*c - a*d)^4*p*q*r^2*x)/(25*d^4) + (77*(b*c - a*d)^4*q^2*r^2*x)/(150*d^4) + (2*(b*c - a*d)^4*q*(p + q)*r^2*x)/(5*d^4) - (b*(b*c - a*d)^3*p*q*r^2*x^2)/(10*d^3) - ((b*c - a*d)^3*p*q*r^2*(a + b*x)^2)/(25*b*d^3) - (77*(b*c - a*d)^3*q^2*r^2*(a + b*x)^2)/(300*b*d^3) + (16*(b*c - a*d)^2*p*q*r^2*(a + b*x)^3)/(225*b*d^2) + (47*(b*c - a*d)^2*q^2*r^2*(a + b*x)^3)/(450*b*d^2) - (9*(b*c - a*d)*p*q*r^2*(a + b*x)^4)/(200*b*d) - (9*(b*c - a*d)*q^2*r^2*(a + b*x)^4)/(200*b*d) + (2*p^2*r^2*(a + b*x)^5)/(125*b) + (4*p*q*r^2*(a + b*x)^5)/(125*b) + (2*q^2*r^2*(a + b*x)^5)/(125*b) - (2*(b*c - a*d)^5*p*q*r^2*Log[c + d*x])/(25*b*d^5) - (137*(b*c - a*d)^5*q^2*r^2*Log[c + d*x])/(150*b*d^5) - (2*(b*c - a*d)^5*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(5*b*d^5) - ((b*c - a*d)^5*q^2*r^2*Log[c + d*x]^2)/(5*b*d^5) - (2*(b*c - a*d)^4*q*r*(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(5*b*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(5*b*d^3) - (2*(b*c - a*d)^2*q*r*(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(15*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(10*b*d) - (2*p*r*(a + b*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(25*b) - (2*q*r*(a + b*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(25*b) + (2*(b*c - a*d)^5*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(5*b*d^5) + ((a + b*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r^2)/(5*b) - (2*(b*c - a*d)^5*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(5*b*d^5)$

**Rubi [A]** time = 0.844897, antiderivative size = 920, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 14, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$ , Rules used = {2498, 2495, 32, 43, 2514, 2487, 31, 8, 2494, 2394, 2393, 2391, 2390, 2301}

$$\frac{q^2 r^2 \log^2(c + dx)(bc - ad)^5}{5bd^5} - \frac{137q^2 r^2 \log(c + dx)(bc - ad)^5}{150bd^5} - \frac{2pqr^2 \log(c + dx)(bc - ad)^5}{25bd^5} - \frac{2pqr^2 \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c + dx)}{5bd^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)^4\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2,x]



```
[Out] -(a*(b*c - a*d)^3*p*q*r^2*x)/(5*d^3) + (2*(b*c - a*d)^4*p*q*r^2*x)/(25*d^4)
+ (77*(b*c - a*d)^4*q^2*r^2*x)/(150*d^4) + (2*(b*c - a*d)^4*q*(p + q)*r^2*
x)/(5*d^4) - (b*(b*c - a*d)^3*p*q*r^2*x^2)/(10*d^3) - ((b*c - a*d)^3*p*q*r^
2*(a + b*x)^2)/(25*b*d^3) - (77*(b*c - a*d)^3*q^2*r^2*(a + b*x)^2)/(300*b*d
^3) + (16*(b*c - a*d)^2*p*q*r^2*(a + b*x)^3)/(225*b*d^2) + (47*(b*c - a*d)^
2*q^2*r^2*(a + b*x)^3)/(450*b*d^2) - (9*(b*c - a*d)*p*q*r^2*(a + b*x)^4)/(2
00*b*d) - (9*(b*c - a*d)*q^2*r^2*(a + b*x)^4)/(200*b*d) + (2*p^2*r^2*(a + b
*x)^5)/(125*b) + (4*p*q*r^2*(a + b*x)^5)/(125*b) + (2*q^2*r^2*(a + b*x)^5)/
(125*b) - (2*(b*c - a*d)^5*p*q*r^2*Log[c + d*x])/(25*b*d^5) - (137*(b*c - a
*d)^5*q^2*r^2*Log[c + d*x])/(150*b*d^5) - (2*(b*c - a*d)^5*p*q*r^2*Log[-((d
*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(5*b*d^5) - ((b*c - a*d)^5*q^2*r^2*
Log[c + d*x]^2)/(5*b*d^5) - (2*(b*c - a*d)^4*q*r*(a + b*x)*Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r])/(5*b*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)^2*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(5*b*d^3) - (2*(b*c - a*d)^2*q*r*(a + b*x)^3*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(15*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)
^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(10*b*d) - (2*p*r*(a + b*x)^5*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(25*b) - (2*q*r*(a + b*x)^5*Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r])/(25*b) + (2*(b*c - a*d)^5*q*r*Log[c + d*x]*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/(5*b*d^5) + ((a + b*x)^5*Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2)/(5*b) - (2*(b*c - a*d)^5*p*q*r^2*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)])/(5*b*d^5)
```

### Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
+ 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

### Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
```

1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 2514

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*(RFX\_), x\_Symbol] := With[{u = ExpandIntegrand[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]

### Rule 2487

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + (Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] - Dist[r\*s\*(p + q), Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1), x], x) /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^( -1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 2494

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[(Log[g + h\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r])/h, x] + (-Dist[(b\*p\*r)/h, Int[Log[g + h\*x]/(a + b\*x), x], x] - Dist[(d\*q\*r)/h, Int[Log[g + h\*x]/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b\*c - a\*d, 0]

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)]^(n_.)]*(b_.)/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rubi steps

$$\begin{aligned}
\int (a + bx)^4 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(a + bx)^5 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{5b} - \frac{1}{5} (2pr) \int (a + bx)^4 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx \\
&= -\frac{2pr(a + bx)^5 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{25b} + \frac{(a + bx)^5 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{5b} \\
&= \frac{2p^2 r^2 (a + bx)^5}{125b} - \frac{2pr(a + bx)^5 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{25b} + \frac{(a + bx)^5 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{5b} \\
&= \frac{2(bc - ad)^4 pqr^2 x}{25d^4} - \frac{(bc - ad)^3 pqr^2 (a + bx)^2}{25bd^3} + \frac{2(bc - ad)^2 pqr^2 (a + bx)^3}{75bd^2} \\
&= -\frac{a(bc - ad)^3 pqr^2 x}{5d^3} + \frac{2(bc - ad)^4 pqr^2 x}{25d^4} + \frac{2(bc - ad)^4 q(p + q)r^2 x}{5d^4} - \frac{b(bc - ad)^3 pqr^2 x}{5d^3} \\
&= -\frac{a(bc - ad)^3 pqr^2 x}{5d^3} + \frac{2(bc - ad)^4 pqr^2 x}{25d^4} + \frac{77(bc - ad)^4 q^2 r^2 x}{150d^4} + \frac{2(bc - ad)^3 pqr^2 x}{5d^3} \\
&= -\frac{a(bc - ad)^3 pqr^2 x}{5d^3} + \frac{2(bc - ad)^4 pqr^2 x}{25d^4} + \frac{77(bc - ad)^4 q^2 r^2 x}{150d^4} + \frac{2(bc - ad)^3 pqr^2 x}{5d^3}
\end{aligned}$$

**Mathematica [B]** time = 2.68567, size = 2508, normalized size = 2.73

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^4\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2,x]

[Out] (2\*a^5\*p\*q\*r^2)/b + (2\*a\*b^3\*c^4\*p\*q\*r^2)/(5\*d^4) - (2\*a^2\*b^2\*c^3\*p\*q\*r^2)/d^3 + (4\*a^3\*b\*c^2\*p\*q\*r^2)/d^2 - (4\*a^4\*c\*p\*q\*r^2)/d + (2\*a^4\*p^2\*r^2\*x)/25 + (197\*a^4\*p\*q\*r^2\*x)/150 + (12\*b^4\*c^4\*p\*q\*r^2\*x)/(25\*d^4) - (11\*a\*b^3\*c^3\*p\*q\*r^2\*x)/(5\*d^3) + (59\*a^2\*b^2\*c^2\*p\*q\*r^2\*x)/(15\*d^2) - (101\*a^3\*b\*c\*p\*q\*r^2\*x)/(30\*d) + 2\*a^4\*q^2\*r^2\*x + (137\*b^4\*c^4\*q^2\*r^2\*x)/(150\*d^4) - (25\*a\*b^3\*c^3\*q^2\*r^2\*x)/(6\*d^3) + (22\*a^2\*b^2\*c^2\*q^2\*r^2\*x)/(3\*d^2) - (6\*a^3\*b\*c\*q^2\*r^2\*x)/d + (4\*a^3\*b\*p^2\*r^2\*x^2)/25 + (283\*a^3\*b\*p\*q\*r^2\*x^2)/300 - (7\*b^4\*c^3\*p\*q\*r^2\*x^2)/(50\*d^3) + (19\*a\*b^3\*c^2\*p\*q\*r^2\*x^2)/(30\*d^2) - (67\*a^2\*b^2\*c\*p\*q\*r^2\*x^2)/(60\*d) + a^3\*b\*q^2\*r^2\*x^2 - (77\*b^4\*c^3\*q^2\*r^2\*x^2)/(300\*d^3) + (13\*a\*b^3\*c^2\*q^2\*r^2\*x^2)/(12\*d^2) - (5\*a^2\*b^2\*c\*q^2\*r^2\*x^2)/(3\*d) + (4\*a^2\*b^2\*p^2\*r^2\*x^3)/25 + (257\*a^2\*b^2\*p\*q\*r^2\*x^3)/45

$$\begin{aligned}
& 0 + (16*b^4*c^2*p*q*r^2*x^3)/(225*d^2) - (29*a*b^3*c*p*q*r^2*x^3)/(90*d) + \\
& (4*a^2*b^2*q^2*r^2*x^3)/9 + (47*b^4*c^2*q^2*r^2*x^3)/(450*d^2) - (7*a*b^3*c \\
& *q^2*r^2*x^3)/(18*d) + (2*a*b^3*p^2*r^2*x^4)/25 + (41*a*b^3*p*q*r^2*x^4)/20 \\
& 0 - (9*b^4*c*p*q*r^2*x^4)/(200*d) + (a*b^3*q^2*r^2*x^4)/8 - (9*b^4*c*q^2*r^ \\
& 2*x^4)/(200*d) + (2*b^4*p^2*r^2*x^5)/125 + (4*b^4*p*q*r^2*x^5)/125 + (2*b^4 \\
& *q^2*r^2*x^5)/125 - (a^5*p^2*r^2*Log[a + b*x]^2)/(5*b) + (2*a^5*p*q*r^2*Log \\
& [c + d*x])/b - (2*b^4*c^5*p*q*r^2*Log[c + d*x])/(25*d^5) + (2*a*b^3*c^4*p*q \\
& *r^2*Log[c + d*x])/(5*d^4) - (4*a^2*b^2*c^3*p*q*r^2*Log[c + d*x])/(5*d^3) + \\
& (4*a^3*b*c^2*p*q*r^2*Log[c + d*x])/(5*d^2) - (2*a^4*c*p*q*r^2*Log[c + d*x] \\
& )/(5*d) - (137*b^4*c^5*q^2*r^2*Log[c + d*x])/(150*d^5) + (25*a*b^3*c^4*q^2*r \\
& ^2*Log[c + d*x])/(6*d^4) - (22*a^2*b^2*c^3*q^2*r^2*Log[c + d*x])/(3*d^3) + \\
& (6*a^3*b*c^2*q^2*r^2*Log[c + d*x])/d^2 - (2*a^4*c*q^2*r^2*Log[c + d*x])/d \\
& - (b^4*c^5*q^2*r^2*Log[c + d*x]^2)/(5*d^5) + (a*b^3*c^4*q^2*r^2*Log[c + d*x] \\
& ]^2)/d^4 - (2*a^2*b^2*c^3*q^2*r^2*Log[c + d*x]^2)/d^3 + (2*a^3*b*c^2*q^2*r^ \\
& 2*Log[c + d*x]^2)/d^2 - (a^4*c*q^2*r^2*Log[c + d*x]^2)/d - (2*a^5*p*r*Log[e \\
& *(f*(a + b*x)^p*(c + d*x)^q]^r)/b - (2*a^4*p*r*x*Log[e*(f*(a + b*x)^p*(c + \\
& d*x)^q]^r))/5 - 2*a^4*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r] - (2*b^4*c \\
& ^4*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/(5*d^4) + (2*a*b^3*c^3*q*r*x \\
& *Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/d^3 - (4*a^2*b^2*c^2*q*r*x*Log[e*(f \\
& *(a + b*x)^p*(c + d*x)^q]^r))/d^2 + (4*a^3*b*c*q*r*x*Log[e*(f*(a + b*x)^p*( \\
& c + d*x)^q]^r))/d - (4*a^3*b*p*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/ \\
& 5 - 2*a^3*b*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r] + (b^4*c^3*q*r*x^2 \\
& *Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/(5*d^3) - (a*b^3*c^2*q*r*x^2*Log[e*( \\
& f*(a + b*x)^p*(c + d*x)^q]^r))/d^2 + (2*a^2*b^2*c*q*r*x^2*Log[e*(f*(a + b*x) \\
& )^p*(c + d*x)^q]^r))/d - (4*a^2*b^2*p*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^ \\
& q]^r))/5 - (4*a^2*b^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/3 - (2* \\
& b^4*c^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/(15*d^2) + (2*a*b^3*c \\
& *q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/(3*d) - (2*a*b^3*p*r*x^4*Log \\
& [e*(f*(a + b*x)^p*(c + d*x)^q]^r))/5 - (a*b^3*q*r*x^4*Log[e*(f*(a + b*x)^p* \\
& (c + d*x)^q]^r))/2 + (b^4*c*q*r*x^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/( \\
& 10*d) - (2*b^4*p*r*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/25 - (2*b^4*q* \\
& r*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/25 + (2*b^4*c^5*q*r*Log[c + d*x] \\
& *Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/(5*d^5) - (2*a*b^3*c^4*q*r*Log[c + \\
& d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/d^4 + (4*a^2*b^2*c^3*q*r*Log[c + \\
& d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/d^3 - (4*a^3*b*c^2*q*r*Log[c + \\
& d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/d^2 + (2*a^4*c*q*r*Log[c + d*x]* \\
& Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/d + a^4*x*Log[e*(f*(a + b*x)^p*(c + d \\
& *x)^q]^r]^2 + 2*a^3*b*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2 + 2*a^2*b^ \\
& 2*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2 + a*b^3*x^4*Log[e*(f*(a + b*x) \\
& ^p*(c + d*x)^q]^r]^2 + (b^4*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2)/5 + \\
& (p*r*Log[a + b*x]*(a*d*(a^4*d^4*(288*p - 137*q) - 60*b^4*c^4*q + 270*a*b^3 \\
& *c^3*d*q - 470*a^2*b^2*c^2*d^2*q + 385*a^3*b*c*d^3*q)*r - 60*b*c*(b^4*c^4 - \\
& 5*a*b^3*c^3*d + 10*a^2*b^2*c^2*d^2 - 10*a^3*b*c*d^3 + 5*a^4*d^4)*q*r*Log[c \\
& + d*x] + 60*(b*c - a*d)^5*q*r*Log[(b*c*(c + d*x))/(b*c - a*d)] + 60*a^5*d^5* \\
& Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/(150*b*d^5) + (2*(b*c - a*d)^5*p*q*r
\end{aligned}$$

$$\int (bx+a)^4 \ln\left(\frac{e^{(f(bx+a)^p(dx+c)^q)^r}}{(b*c+a*d)}\right) dx$$

**Maple [F]** time = 0.414, size = 0, normalized size = 0.

$$\int (bx+a)^4 \left( \ln\left( e^{(f(bx+a)^p(dx+c)^q)^r} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^4\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

[Out] int((b\*x+a)^4\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

**Maxima [A]** time = 1.57889, size = 1918, normalized size = 2.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^4\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x, algorithm="maxima")

[Out]  $\frac{1}{5}(b^4x^5 + 5ab^3x^4 + 10a^2b^2x^3 + 10a^3bx^2 + 5a^4x) \log\left(\frac{(bx+a)^p(dx+c)^q f^r e}{(b*c+a*d)}\right) + \frac{1}{150}(60a^5f^p \log(bx+a)/b - (12b^4d^4f^p(p+q)x^5 + 15(ab^3d^4f^p(4p+5q) - b^4cd^3f^p) x^4 + 20(2a^2b^2d^4f^p(3p+5q) + b^4c^2d^2f^p q - 5ab^3cd^3f^p) x^3 + 30(2a^3bd^4f^p(2p+5q) - b^4c^3d^2f^p q + 5ab^3c^2d^2f^p q - 10a^2b^2cd^3f^p) x^2 + 60(a^4d^4f^p(p+5q) + b^4c^4f^p q - 5ab^3c^3d^2f^p q + 10a^2b^2c^2d^2f^p q - 10a^3b^2cd^3f^p) x)/d^4 + 60(b^4c^5f^p q - 5ab^3c^4d^2f^p q + 10a^2b^2c^3d^2f^p q - 10a^3b^2cd^3f^p q + 5a^4c^4d^2f^p q) \log(dx+c)/d^5) r \log\left(\frac{(bx+a)^p(dx+c)^q f^r e}{(b*c+a*d)}\right) - \frac{1}{9000} r^2 (60((12p^2q + 137q^2) b^4c^5f^2 - 5(12p^2q + 125q^2) a b^3c^4d^2f^2 + 20(6p^2q + 55q^2) a^2b^2c^3d^2f^2 - 60(2p^2q + 15q^2) a^3b^2cd^3f^2 + 60(p^2q + 5q^2) a^4c^4d^2f^2) \log(dx+c)/d^5 - 3600(b^5c^5f^2p^2q - 5ab^4c^4d^2f^2p^2q + 10a^2b^3c^3d^2f^2p^2q - 10a^3b^2c^2d^3f^2p^2q + 5a^4b^2cd^4f^2p^2q - a^5d^5f^2p^2q) (\log(bx+a) \log\left(\frac{b*d*x+a*d}{b*c-a*d}\right) + 1) + \text{dilog}\left(-\frac{b*d*x+a*d}{b*c-a*d}\right)) / (b*d^5) - (144(p^2 + 2p^2q + q^2) b^5d^5f^2x^5 - 1800a^5d^5f^2p^2 \log(bx+a)^2 - 45(9(p^2q + q^2) b^5cd^4f^2 - (16p^2 + 41p^2q + 25q^2) a b^4d^5f^2) x^4 + 20((32p^2q + 47q^2) b^5c^2d^3f^2 - 5(29$

```

*p*q + 35*q^2)*a*b^4*c*d^4*f^2 + (72*p^2 + 257*p*q + 200*q^2)*a^2*b^3*d^5*f
^2)*x^3 - 30*(7*(6*p*q + 11*q^2)*b^5*c^3*d^2*f^2 - 5*(38*p*q + 65*q^2)*a*b^
4*c^2*d^3*f^2 + 5*(67*p*q + 100*q^2)*a^2*b^3*c*d^4*f^2 - (48*p^2 + 283*p*q
+ 300*q^2)*a^3*b^2*d^5*f^2)*x^2 - 3600*(b^5*c^5*f^2*p*q - 5*a*b^4*c^4*d*f^2
*p*q + 10*a^2*b^3*c^3*d^2*f^2*p*q - 10*a^3*b^2*c^2*d^3*f^2*p*q + 5*a^4*b*c*
d^4*f^2*p*q)*log(b*x + a)*log(d*x + c) - 1800*(b^5*c^5*f^2*q^2 - 5*a*b^4*c^
4*d*f^2*q^2 + 10*a^2*b^3*c^3*d^2*f^2*q^2 - 10*a^3*b^2*c^2*d^3*f^2*q^2 + 5*a
^4*b*c*d^4*f^2*q^2)*log(d*x + c)^2 + 60*((72*p*q + 137*q^2)*b^5*c^4*d*f^2 -
5*(66*p*q + 125*q^2)*a*b^4*c^3*d^2*f^2 + 10*(59*p*q + 110*q^2)*a^2*b^3*c^2
*d^3*f^2 - 5*(101*p*q + 180*q^2)*a^3*b^2*c*d^4*f^2 + (12*p^2 + 197*p*q + 30
0*q^2)*a^4*b*d^5*f^2)*x - 60*(60*a*b^4*c^4*d*f^2*p*q - 270*a^2*b^3*c^3*d^2*
f^2*p*q + 470*a^3*b^2*c^2*d^3*f^2*p*q - 385*a^4*b*c*d^4*f^2*p*q + (12*p^2 +
137*p*q)*a^5*d^5*f^2)*log(b*x + a))/(b*d^5))/f^2

```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^4x^4 + 4ab^3x^3 + 6a^2b^2x^2 + 4a^3bx + a^4\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
```

```
[Out] integral((b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^4 \log\left(\left((bx + a)^p (dx + c)^q f\right)^r e\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^4\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x, algorithm="giac")

[Out] integrate((b\*x + a)^4\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2, x)



$$3.17 \quad \int (a + bx)^3 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$$

**Optimal.** Leaf size=805

$$\frac{q^2 r^2 \log^2(c + dx)(bc - ad)^4}{4bd^4} + \frac{25q^2 r^2 \log(c + dx)(bc - ad)^4}{24bd^4} + \frac{pqr^2 \log(c + dx)(bc - ad)^4}{8bd^4} + \frac{pqr^2 \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c + dx)}{2bd^4}$$

```
[Out] (a*(b*c - a*d)^2*p*q*r^2*x)/(4*d^2) - ((b*c - a*d)^3*p*q*r^2*x)/(8*d^3) - (
13*(b*c - a*d)^3*q^2*r^2*x)/(24*d^3) - ((b*c - a*d)^3*q*(p + q)*r^2*x)/(2*d
^3) + (b*(b*c - a*d)^2*p*q*r^2*x^2)/(8*d^2) + ((b*c - a*d)^2*p*q*r^2*(a + b
*x)^2)/(16*b*d^2) + (13*(b*c - a*d)^2*q^2*r^2*(a + b*x)^2)/(48*b*d^2) - (7*
(b*c - a*d)*p*q*r^2*(a + b*x)^3)/(72*b*d) - (7*(b*c - a*d)*q^2*r^2*(a + b*x
)^3)/(72*b*d) + (p^2*r^2*(a + b*x)^4)/(32*b) + (p*q*r^2*(a + b*x)^4)/(16*b)
+ (q^2*r^2*(a + b*x)^4)/(32*b) + ((b*c - a*d)^4*p*q*r^2*Log[c + d*x])/(8*b
*d^4) + (25*(b*c - a*d)^4*q^2*r^2*Log[c + d*x])/(24*b*d^4) + ((b*c - a*d)^4
*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b*d^4) + ((b*c
- a*d)^4*q^2*r^2*Log[c + d*x]^2)/(4*b*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)*L
og[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(2*b*d^3) - ((b*c - a*d)^2*q*r*(a + b*
x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(4*b*d^2) + ((b*c - a*d)*q*r*(a
+ b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(6*b*d) - (p*r*(a + b*x)^4*L
og[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(8*b) - (q*r*(a + b*x)^4*Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r)/(8*b) - ((b*c - a*d)^4*q*r*Log[c + d*x]*Log[e*(f*(
a + b*x)^p*(c + d*x)^q]^r))/(2*b*d^4) + ((a + b*x)^4*Log[e*(f*(a + b*x)^p*(
c + d*x)^q]^r)^2)/(4*b) + ((b*c - a*d)^4*p*q*r^2*PolyLog[2, (b*(c + d*x))/(
b*c - a*d]))/(2*b*d^4)
```

---

**Rubi [A]** time = 0.66471, antiderivative size = 805, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 14, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$ , Rules used = {2498, 2495, 32, 43, 2514, 2487, 31, 8, 2494, 2394, 2393, 2391, 2390, 2301}

$$\frac{q^2 r^2 \log^2(c + dx)(bc - ad)^4}{4bd^4} + \frac{25q^2 r^2 \log(c + dx)(bc - ad)^4}{24bd^4} + \frac{pqr^2 \log(c + dx)(bc - ad)^4}{8bd^4} + \frac{pqr^2 \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c + dx)}{2bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2,x]
```

```
[Out] (a*(b*c - a*d)^2*p*q*r^2*x)/(4*d^2) - ((b*c - a*d)^3*p*q*r^2*x)/(8*d^3) - (
13*(b*c - a*d)^3*q^2*r^2*x)/(24*d^3) - ((b*c - a*d)^3*q*(p + q)*r^2*x)/(2*d
```

$$\begin{aligned} &^3) + (b*(b*c - a*d)^2*p*q*r^2*x^2)/(8*d^2) + ((b*c - a*d)^2*p*q*r^2*(a + b \\ &*x)^2)/(16*b*d^2) + (13*(b*c - a*d)^2*q^2*r^2*(a + b*x)^2)/(48*b*d^2) - (7* \\ &(b*c - a*d)*p*q*r^2*(a + b*x)^3)/(72*b*d) - (7*(b*c - a*d)*q^2*r^2*(a + b*x \\ &)^3)/(72*b*d) + (p^2*r^2*(a + b*x)^4)/(32*b) + (p*q*r^2*(a + b*x)^4)/(16*b) \\ &+ (q^2*r^2*(a + b*x)^4)/(32*b) + ((b*c - a*d)^4*p*q*r^2*Log[c + d*x])/(8*b \\ &*d^4) + (25*(b*c - a*d)^4*q^2*r^2*Log[c + d*x])/(24*b*d^4) + ((b*c - a*d)^4 \\ &*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))] * Log[c + d*x])/(2*b*d^4) + ((b*c \\ &- a*d)^4*q^2*r^2*Log[c + d*x]^2)/(4*b*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)*L \\ &og[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(2*b*d^3) - ((b*c - a*d)^2*q*r*(a + b* \\ &x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(4*b*d^2) + ((b*c - a*d)*q*r*(a \\ &+ b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(6*b*d) - (p*r*(a + b*x)^4*L \\ &og[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(8*b) - (q*r*(a + b*x)^4*Log[e*(f*(a + \\ &b*x)^p*(c + d*x)^q]^r)/(8*b) - ((b*c - a*d)^4*q*r*Log[c + d*x]*Log[e*(f*( \\ &a + b*x)^p*(c + d*x)^q]^r))/(2*b*d^4) + ((a + b*x)^4*Log[e*(f*(a + b*x)^p*( \\ &c + d*x)^q]^r)^2)/(4*b) + ((b*c - a*d)^4*p*q*r^2*PolyLog[2, (b*(c + d*x))/( \\ &b*c - a*d)])/(2*b*d^4) \end{aligned}$$

### Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
+ 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

### Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q]^r]/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
```

```
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFX_), x_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]
```

### Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s, x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s, x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

### Rule 31

```
Int[((a_.) + (b_.)*(x_))^(n_.), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

### Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(Log[g + h*x]*Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(q_.)), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int (a+bx)^3 \log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) dx &= \frac{(a+bx)^4 \log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{4b} - \frac{1}{2}(pr) \int (a+bx)^3 \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) dx \\
&= -\frac{pr(a+bx)^4 \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{8b} + \frac{(a+bx)^4 \log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{4b} \\
&= \frac{p^2 r^2 (a+bx)^4}{32b} - \frac{pr(a+bx)^4 \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{8b} + \frac{(a+bx)^4 \log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{4b} \\
&= -\frac{(bc-ad)^3 pqr^2 x}{8d^3} + \frac{(bc-ad)^2 pqr^2 (a+bx)^2}{16bd^2} - \frac{(bc-ad)pqr^2 (a+bx)^3}{24bd} \\
&= \frac{a(bc-ad)^2 pqr^2 x}{4d^2} - \frac{(bc-ad)^3 pqr^2 x}{8d^3} - \frac{(bc-ad)^3 q(p+q)r^2 x}{2d^3} + \frac{b(bc-ad)^3 q^2 r^2 x}{2d^3} \\
&= \frac{a(bc-ad)^2 pqr^2 x}{4d^2} - \frac{(bc-ad)^3 pqr^2 x}{8d^3} - \frac{13(bc-ad)^3 q^2 r^2 x}{24d^3} - \frac{(bc-ad)^3 q^2 r^2 x}{2d^3} \\
&= \frac{a(bc-ad)^2 pqr^2 x}{4d^2} - \frac{(bc-ad)^3 pqr^2 x}{8d^3} - \frac{13(bc-ad)^3 q^2 r^2 x}{24d^3} - \frac{(bc-ad)^3 q^2 r^2 x}{2d^3}
\end{aligned}$$

**Mathematica [B]** time = 1.85919, size = 1853, normalized size = 2.3

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^3\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2,x]

[Out] (2\*a^4\*p\*q\*r^2)/b - (a\*b^2\*c^3\*p\*q\*r^2)/(2\*d^3) + (2\*a^2\*b\*c^2\*p\*q\*r^2)/d^2 - (3\*a^3\*c\*p\*q\*r^2)/d + (a^3\*p^2\*r^2\*x)/8 + (37\*a^3\*p\*q\*r^2\*x)/24 - (5\*b^3\*c^3\*p\*q\*r^2\*x)/(8\*d^3) + (9\*a\*b^2\*c^2\*p\*q\*r^2\*x)/(4\*d^2) - (35\*a^2\*b\*c\*p\*q\*r^2\*x)/(12\*d) + 2\*a^3\*q^2\*r^2\*x - (25\*b^3\*c^3\*q^2\*r^2\*x)/(24\*d^3) + (11\*a\*b^2\*c^2\*q^2\*r^2\*x)/(3\*d^2) - (9\*a^2\*b\*c\*q^2\*r^2\*x)/(2\*d) + (3\*a^2\*b\*p^2\*r^2\*x^2)/16 + (41\*a^2\*b\*p\*q\*r^2\*x^2)/48 + (3\*b^3\*c^2\*p\*q\*r^2\*x^2)/(16\*d^2) - (2\*a\*b^2\*c\*p\*q\*r^2\*x^2)/(3\*d) + (3\*a^2\*b\*q^2\*r^2\*x^2)/4 + (13\*b^3\*c^2\*q^2\*r^2\*x^2)/(48\*d^2) - (5\*a\*b^2\*c\*q^2\*r^2\*x^2)/(6\*d) + (a\*b^2\*p^2\*r^2\*x^3)/8 + (25\*a\*b^2\*p\*q\*r^2\*x^3)/72 - (7\*b^3\*c\*p\*q\*r^2\*x^3)/(72\*d) + (2\*a\*b^2\*q^2\*r^2\*x^3)/9 - (7\*b^3\*c\*q^2\*r^2\*x^3)/(72\*d) + (b^3\*p^2\*r^2\*x^4)/32 + (b^3\*p\*q\*r^2\*x^4)/16 + (b^3\*q^2\*r^2\*x^4)/32 - (a^4\*p^2\*r^2\*Log[a + b\*x]^2)/(4\*b) + (2\*a

$$\begin{aligned}
& ^4*p*q*r^2*\text{Log}[c + d*x])/b + (b^3*c^4*p*q*r^2*\text{Log}[c + d*x])/(8*d^4) - (a*b^2*c^3*p*q*r^2*\text{Log}[c + d*x])/(2*d^3) + (3*a^2*b*c^2*p*q*r^2*\text{Log}[c + d*x])/(4*d^2) - (a^3*c*p*q*r^2*\text{Log}[c + d*x])/(2*d) + (25*b^3*c^4*q^2*r^2*\text{Log}[c + d*x])/(24*d^4) - (11*a*b^2*c^3*q^2*r^2*\text{Log}[c + d*x])/(3*d^3) + (9*a^2*b*c^2*q^2*r^2*\text{Log}[c + d*x])/(2*d^2) - (2*a^3*c*q^2*r^2*\text{Log}[c + d*x])/d + (b^3*c^4*q^2*r^2*\text{Log}[c + d*x]^2)/(4*d^4) - (a*b^2*c^3*q^2*r^2*\text{Log}[c + d*x]^2)/d^3 + (3*a^2*b*c^2*q^2*r^2*\text{Log}[c + d*x]^2)/(2*d^2) - (a^3*c*q^2*r^2*\text{Log}[c + d*x]^2)/d - (2*a^4*p*r*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/b - (a^3*p*r*x*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/2 - 2*a^3*q*r*x*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r] + (b^3*c^3*q*r*x*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/(2*d^3) - (2*a*b^2*c^2*q*r*x*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/d^2 + (3*a^2*b*c*q*r*x*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/d - (3*a^2*b*p*r*x^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/4 - (3*a^2*b*q*r*x^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/2 - (b^3*c^2*q*r*x^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/(4*d^2) + (a*b^2*c*q*r*x^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/d - (a*b^2*p*r*x^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/2 - (2*a*b^2*q*r*x^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/3 + (b^3*c*q*r*x^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/(6*d) - (b^3*p*r*x^4*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/8 - (b^3*q*r*x^4*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/8 - (b^3*c^4*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/(2*d^4) + (2*a*b^2*c^3*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/d^3 - (3*a^2*b*c^2*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/d^2 + (2*a^3*c*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/d + a^3*x*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2 + (3*a^2*b*x^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2)/2 + a*b^2*x^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2 + (b^3*x^4*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2)/4 + (p*r*\text{Log}[a + b*x]*(a*d*(5*a^3*d^3*(9*p - 5*q) + 12*b^3*c^3*q - 42*a*b^2*c^2*d*q + 52*a^2*b*c*d^2*q)*r + 12*b*c*(b^3*c^3 - 4*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 4*a^3*d^3)*q*r*\text{Log}[c + d*x] - 12*(b*c - a*d)^4*q*r*\text{Log}[(b*(c + d*x))/(b*c - a*d)] + 12*a^4*d^4*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]))/(24*b*d^4) - ((b*c - a*d)^4*p*q*r^2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)])/(2*b*d^4)
\end{aligned}$$

**Maple [F]** time = 0.402, size = 0, normalized size = 0.

$$\int (bx + a)^3 \left( \ln \left( e \left( f(bx + a)^p (dx + c)^q \right)^r \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^3\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

[Out] int((b\*x+a)^3\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

---

**Maxima [A]** time = 1.47114, size = 1446, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^3\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x, algorithm="maxima")

[Out]  $\frac{1}{4}(b^3x^4 + 4ab^2x^3 + 6a^2b^2x^2 + 4a^3x) \log(((b*x + a)^p(d*x + c)^q f)^r e)^2 + \frac{1}{24}(12a^4 f^p \log(b*x + a)/b - (3b^3 d^3 f^p (p + q) x^4 + 4(a b^2 d^3 f^p (3p + 4q) - b^3 c d^2 f^p q) x^3 + 6(3a^2 b d^3 f^p (p + 2q) + b^3 c^2 d f^p q - 4a b^2 c d^2 f^p q) x^2 + 12(a^3 d^3 f^p (p + 4q) - b^3 c^3 f^p q + 4a b^2 c^2 d f^p q - 6a^2 b c d^2 f^p q) x)/d^3 - 12(b^3 c^4 f^p q - 4a b^2 c^3 d f^p q + 6a^2 b c^2 d^2 f^p q - 4a^3 c d^3 f^p q) \log(d*x + c)/d^4) r \log(((b*x + a)^p(d*x + c)^q f)^r e)/f + \frac{1}{288} r^2 (12((3p^2 q + 25q^2) b^3 c^4 f^2 - 4(3p^2 q + 22q^2) a b^2 c^3 d f^2 + 18(p^2 q + 6q^2) a^2 b c^2 d^2 f^2 - 12(p^2 q + 4q^2) a^3 c d^3 f^2) \log(d*x + c)/d^4 - 144(b^4 c^4 f^2 p^2 q - 4a b^3 c^3 d f^2 p^2 q + 6a^2 b^2 c^2 d^2 f^2 p^2 q - 4a^3 b c d^3 f^2 p^2 q + a^4 d^4 f^2 p^2 q) (\log(b*x + a) \log((b*d*x + a*d)/(b*c - a*d) + 1) + \text{dilog}(-(b*d*x + a*d)/(b*c - a*d))) / (b*d^4) + (9(p^2 + 2p^2 q + q^2) b^4 d^4 f^2 x^4 - 72a^4 d^4 f^2 p^2 \log(b*x + a)^2 - 4(7(p^2 q + q^2) b^4 c d^3 f^2 - (9p^2 + 25p^2 q + 16q^2) a b^3 d^4 f^2) x^3 + 6((9p^2 q + 13q^2) b^4 c^2 d^2 f^2 - 8(4p^2 q + 5q^2) a b^3 c d^3 f^2 + (9p^2 + 41p^2 q + 36q^2) a^2 b^2 d^4 f^2) x^2 + 144(b^4 c^4 f^2 p^2 q - 4a b^3 c^3 d f^2 p^2 q + 6a^2 b^2 c^2 d^2 f^2 p^2 q - 4a^3 b c d^3 f^2 p^2 q) \log(b*x + a) \log(d*x + c) + 72(b^4 c^4 f^2 q^2 - 4a b^3 c^3 d f^2 q^2 + 6a^2 b^2 c^2 d^2 f^2 q^2 - 4a^3 b c d^3 f^2 q^2) \log(d*x + c)^2 - 12(5(3p^2 q + 5q^2) b^4 c^3 d f^2 - 2(27p^2 q + 44q^2) a b^3 c^2 d^2 f^2 + 2(35p^2 q + 54q^2) a^2 b^2 c d^3 f^2 - (3p^2 + 37p^2 q + 48q^2) a^3 b d^4 f^2) x + 12(12a b^3 c^3 d f^2 p^2 q - 42a^2 b^2 c^2 d^2 f^2 p^2 q + 52a^3 b c d^3 f^2 p^2 q - (3p^2 + 25p^2 q) a^4 d^4 f^2) \log(b*x + a) / (b*d^4) / f^2$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 x^3 + 3 a b^2 x^2 + 3 a^2 b x + a^3\right) \log\left(\left((b x + a)^p (d x + c)^q f\right)^r e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^3 \log\left(\left((bx + a)^p (dx + c)^q f\right)^r e\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```



$$3.18 \quad \int (a + bx)^2 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$$

**Optimal.** Leaf size=686

$$\frac{2pqr^2(bc - ad)^3 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right)}{3bd^3} + \frac{2qr(bc - ad)^3 \log(c + dx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{3bd^3} - \frac{2qr(a + bx)(bc - ad)}{3bd^3}$$

[Out]  $-(a*(b*c - a*d)*p*q*r^2*x)/(3*d) + (2*(b*c - a*d)^2*p*q*r^2*x)/(9*d^2) + (5*(b*c - a*d)^2*q^2*r^2*x)/(9*d^2) + (2*(b*c - a*d)^2*q*(p + q)*r^2*x)/(3*d^2) - (b*(b*c - a*d)*p*q*r^2*x^2)/(6*d) - ((b*c - a*d)*p*q*r^2*(a + b*x)^2)/(9*b*d) - (5*(b*c - a*d)*q^2*r^2*(a + b*x)^2)/(18*b*d) + (2*p^2*r^2*(a + b*x)^3)/(27*b) + (4*p*q*r^2*(a + b*x)^3)/(27*b) + (2*q^2*r^2*(a + b*x)^3)/(27*b) - (2*(b*c - a*d)^3*p*q*r^2*\text{Log}[c + d*x])/(9*b*d^3) - (11*(b*c - a*d)^3*q^2*r^2*\text{Log}[c + d*x])/(9*b*d^3) - (2*(b*c - a*d)^3*p*q*r^2*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x])/(3*b*d^3) - ((b*c - a*d)^3*q^2*r^2*\text{Log}[c + d*x]^2)/(3*b*d^3) - (2*(b*c - a*d)^2*q*r*(a + b*x)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(3*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(3*b*d) - (2*p*r*(a + b*x)^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(9*b) - (2*q*r*(a + b*x)^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(9*b) + (2*(b*c - a*d)^3*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(3*b*d^3) + ((a + b*x)^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)^2)/(3*b) - (2*(b*c - a*d)^3*p*q*r^2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(3*b*d^3)$

**Rubi [A]** time = 0.532864, antiderivative size = 686, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 14, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$ , Rules used = {2498, 2495, 32, 43, 2514, 2487, 31, 8, 2494, 2394, 2393, 2391, 2390, 2301}

$$\frac{2pqr^2(bc - ad)^3 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right)}{3bd^3} + \frac{2qr(bc - ad)^3 \log(c + dx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{3bd^3} - \frac{2qr(a + bx)(bc - ad)}{3bd^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)^2\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2,x]

[Out]  $-(a*(b*c - a*d)*p*q*r^2*x)/(3*d) + (2*(b*c - a*d)^2*p*q*r^2*x)/(9*d^2) + (5*(b*c - a*d)^2*q^2*r^2*x)/(9*d^2) + (2*(b*c - a*d)^2*q*(p + q)*r^2*x)/(3*d^2) - (b*(b*c - a*d)*p*q*r^2*x^2)/(6*d) - ((b*c - a*d)*p*q*r^2*(a + b*x)^2)/(9*b*d) - (5*(b*c - a*d)*q^2*r^2*(a + b*x)^2)/(18*b*d) + (2*p^2*r^2*(a + b$

$$\begin{aligned}
& x^3)/(27*b) + (4*p*q*r^2*(a + b*x)^3)/(27*b) + (2*q^2*r^2*(a + b*x)^3)/(27 \\
& *b) - (2*(b*c - a*d)^3*p*q*r^2*\text{Log}[c + d*x])/(9*b*d^3) - (11*(b*c - a*d)^3* \\
& q^2*r^2*\text{Log}[c + d*x])/(9*b*d^3) - (2*(b*c - a*d)^3*p*q*r^2*\text{Log}[-((d*(a + b* \\
& x))/(b*c - a*d))]*\text{Log}[c + d*x])/(3*b*d^3) - ((b*c - a*d)^3*q^2*r^2*\text{Log}[c + \\
& d*x]^2)/(3*b*d^3) - (2*(b*c - a*d)^2*q*r*(a + b*x)*\text{Log}[e*(f*(a + b*x)^p*(c \\
& + d*x)^q)^r])/(3*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^2*\text{Log}[e*(f*(a + b*x)^p \\
& *(c + d*x)^q)^r])/(3*b*d) - (2*p*r*(a + b*x)^3*\text{Log}[e*(f*(a + b*x)^p*(c + d* \\
& x)^q)^r])/(9*b) - (2*q*r*(a + b*x)^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) / \\
& (9*b) + (2*(b*c - a*d)^3*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q \\
& ^r])/(3*b*d^3) + ((a + b*x)^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(3*b) \\
& - (2*(b*c - a*d)^3*p*q*r^2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(3*b*d^3 \\
& )
\end{aligned}$$

### Rule 2498

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
+ 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]

```

### Rule 2495

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]

```

### Rule 32

```

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

```

### Rule 43

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s, RFx, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s, x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s, x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(Log[g + h*x]*Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

$(e*f - d*g), 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c\_.)*((d\_.) + (e\_.)*(x\_.)^{(n\_.)})]/(x\_.), x\_Symbol] := -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c*d, 1]$

### Rule 2390

$\text{Int}[(a\_.) + \text{Log}[(c\_.)*((d\_.) + (e\_.)*(x\_.)^{(n\_.)})]*(b\_.)^{(p\_.)}*((f\_.) + (g\_.)*(x\_.)^{(q\_.)})], x\_Symbol] := \text{Dist}[1/e, \text{Subst}[\text{Int}[(f*x)/d]^q*(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, p, q\}, x\} \ \&\& \ \text{EqQ}[e*f - d*g, 0]$

### Rule 2301

$\text{Int}[(a\_.) + \text{Log}[(c\_.)*(x\_.)^{(n\_.)}]]*(b\_.)/(x\_.), x\_Symbol] := \text{Simp}[(a + b*\text{Log}[c*x^n])^2/(2*b*n), x] /; \text{FreeQ}\{a, b, c, n\}, x]$

### Rubi steps

$$\begin{aligned} \int (a + bx)^2 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(a + bx)^3 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{3b} - \frac{1}{3} (2pr) \int (a + bx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx \\ &= -\frac{2pr(a + bx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{9b} + \frac{(a + bx)^3 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{3b} \\ &= \frac{2p^2 r^2 (a + bx)^3}{27b} - \frac{2pr(a + bx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{9b} + \frac{(a + bx)^3 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{3b} \\ &= \frac{2(bc - ad)^2 pqr^2 x}{9d^2} - \frac{(bc - ad)pqr^2 (a + bx)^2}{9bd} + \frac{2p^2 r^2 (a + bx)^3}{27b} + \frac{2pqr^2 (a + bx)^2}{27b} \\ &= -\frac{a(bc - ad)pqr^2 x}{3d} + \frac{2(bc - ad)^2 pqr^2 x}{9d^2} + \frac{2(bc - ad)^2 q(p + q)r^2 x}{3d^2} - \frac{b(bc - ad)^2 pqr^2}{3d} \\ &= -\frac{a(bc - ad)pqr^2 x}{3d} + \frac{2(bc - ad)^2 pqr^2 x}{9d^2} + \frac{5(bc - ad)^2 q^2 r^2 x}{9d^2} + \frac{2(bc - ad)^2 pqr^2}{3d} \\ &= -\frac{a(bc - ad)pqr^2 x}{3d} + \frac{2(bc - ad)^2 pqr^2 x}{9d^2} + \frac{5(bc - ad)^2 q^2 r^2 x}{9d^2} + \frac{2(bc - ad)^2 pqr^2}{3d} \end{aligned}$$

**Mathematica [A]** time = 1.12677, size = 1211, normalized size = 1.77

$$\frac{1}{54} \left( \frac{108pqr^2a^3}{b} - \frac{18p^2r^2 \log^2(a+bx)a^3}{b} + \frac{108pqr^2 \log(c+dx)a^3}{b} - \frac{108pr \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)a^3}{b} - \frac{108cpr^2a^3}{d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^2\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2,x]

[Out] ((108\*a^3\*p\*q\*r^2)/b + (36\*a\*b\*c^2\*p\*q\*r^2)/d^2 - (108\*a^2\*c\*p\*q\*r^2)/d + 12\*a^2\*p^2\*r^2\*x + 102\*a^2\*p\*q\*r^2\*x + (48\*b^2\*c^2\*p\*q\*r^2\*x)/d^2 - (126\*a\*b\*c\*p\*q\*r^2\*x)/d + 108\*a^2\*q^2\*r^2\*x + (66\*b^2\*c^2\*q^2\*r^2\*x)/d^2 - (162\*a\*b\*c\*q^2\*r^2\*x)/d + 12\*a\*b\*p^2\*r^2\*x^2 + 39\*a\*b\*p\*q\*r^2\*x^2 - (15\*b^2\*c\*p\*q\*r^2\*x^2)/d + 27\*a\*b\*q^2\*r^2\*x^2 - (15\*b^2\*c\*q^2\*r^2\*x^2)/d + 4\*b^2\*p^2\*r^2\*x^3 + 8\*b^2\*p\*q\*r^2\*x^3 + 4\*b^2\*q^2\*r^2\*x^3 - (18\*a^3\*p^2\*r^2\*Log[a + b\*x]^2)/b + (108\*a^3\*p\*q\*r^2\*Log[c + d\*x])/b - (12\*b^2\*c^3\*p\*q\*r^2\*Log[c + d\*x])/d^3 + (36\*a\*b\*c^2\*p\*q\*r^2\*Log[c + d\*x])/d^2 - (36\*a^2\*c\*p\*q\*r^2\*Log[c + d\*x])/d - (66\*b^2\*c^3\*q^2\*r^2\*Log[c + d\*x])/d^3 + (162\*a\*b\*c^2\*q^2\*r^2\*Log[c + d\*x])/d^2 - (108\*a^2\*c\*q^2\*r^2\*Log[c + d\*x])/d - (18\*b^2\*c^3\*q^2\*r^2\*Log[c + d\*x]^2)/d^3 + (54\*a\*b\*c^2\*q^2\*r^2\*Log[c + d\*x]^2)/d^2 - (54\*a^2\*c\*q^2\*r^2\*Log[c + d\*x]^2)/d - (108\*a^3\*p\*r\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r))/b - 36\*a^2\*p\*r\*x\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r] - 108\*a^2\*q\*r\*x\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r] - (36\*b^2\*c^2\*q\*r\*x\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r))/d^2 + (108\*a\*b\*c\*q\*r\*x\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r))/d - 36\*a\*b\*p\*r\*x^2\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r] - 54\*a\*b\*q\*r\*x^2\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r] + (18\*b^2\*c\*q\*r\*x^2\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r))/d - 12\*b^2\*p\*r\*x^3\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r] - 12\*b^2\*q\*r\*x^3\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r] + (36\*b^2\*c^3\*q\*r\*Log[c + d\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r))/d^3 - (108\*a\*b\*c^2\*q\*r\*Log[c + d\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r))/d^2 + (108\*a^2\*c\*q\*r\*Log[c + d\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r))/d + 54\*a^2\*x\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2 + 54\*a\*b\*x^2\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2 + 18\*b^2\*x^3\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2 + (6\*p\*r\*Log[a + b\*x]\*(a\*d\*(a^2\*d^2\*(16\*p - 11\*q) - 6\*b^2\*c^2\*q + 15\*a\*b\*c\*d\*q)\*r - 6\*b\*c\*(b^2\*c^2 - 3\*a\*b\*c\*d + 3\*a^2\*d^2)\*q\*r\*Log[c + d\*x] + 6\*(b\*c - a\*d)^3\*q\*r\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] + 6\*a^3\*d^3\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r))/((b\*d^3) + (36\*(b\*c - a\*d)^3\*p\*q\*r^2\*PolyLog[2, (d\*(a + b\*x))/(-b\*c) + a\*d]))/(b\*d^3))/54

**Maple [F]** time = 0.404, size = 0, normalized size = 0.

$$\int (bx + a)^2 \left( \ln \left( e \left( f(bx + a)^p (dx + c)^q \right)^r \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^2\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

[Out] int((b\*x+a)^2\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

**Maxima [A]** time = 1.42788, size = 1038, normalized size = 1.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x, algorithm="maxima")

[Out] 1/3\*(b^2\*x^3 + 3\*a\*b\*x^2 + 3\*a^2\*x)\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2 + 1/9\*(6\*a^3\*f\*p\*log(b\*x + a)/b - (2\*b^2\*d^2\*f\*(p + q)\*x^3 + 3\*(a\*b\*d^2\*f\*(2\*p + 3\*q) - b^2\*c\*d\*f\*q)\*x^2 + 6\*(a^2\*d^2\*f\*(p + 3\*q) + b^2\*c^2\*f\*q - 3\*a\*b\*c\*d\*f\*q)\*x)/d^2 + 6\*(b^2\*c^3\*f\*q - 3\*a\*b\*c^2\*d\*f\*q + 3\*a^2\*c\*d^2\*f\*q)\*log(d\*x + c)/d^3)\*r\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)/f - 1/54\*r^2\*(6\*((2\*p\*q + 11\*q^2)\*b^2\*c^3\*f^2 - 3\*(2\*p\*q + 9\*q^2)\*a\*b\*c^2\*d\*f^2 + 6\*(p\*q + 3\*q^2)\*a^2\*c\*d^2\*f^2)\*log(d\*x + c)/d^3 - 36\*(b^3\*c^3\*f^2\*p\*q - 3\*a\*b^2\*c^2\*d\*f^2\*p\*q + 3\*a^2\*b\*c\*d^2\*f^2\*p\*q - a^3\*d^3\*f^2\*p\*q)\*(log(b\*x + a)\*log((b\*d\*x + a\*d)/(b\*c - a\*d) + 1) + dilog(-(b\*d\*x + a\*d)/(b\*c - a\*d)))/(b\*d^3) - (4\*(p^2 + 2\*p\*q + q^2)\*b^3\*d^3\*f^2\*x^3 - 18\*a^3\*d^3\*f^2\*p^2\*log(b\*x + a)^2 - 3\*(5\*(p\*q + q^2)\*b^3\*c\*d^2\*f^2 - (4\*p^2 + 13\*p\*q + 9\*q^2)\*a\*b^2\*d^3\*f^2)\*x^2 - 36\*(b^3\*c^3\*f^2\*p\*q - 3\*a\*b^2\*c^2\*d\*f^2\*p\*q + 3\*a^2\*b\*c\*d^2\*f^2\*p\*q)\*log(b\*x + a)\*log(d\*x + c) - 18\*(b^3\*c^3\*f^2\*q^2 - 3\*a\*b^2\*c^2\*d\*f^2\*q^2 + 3\*a^2\*b\*c\*d^2\*f^2\*q^2)\*log(d\*x + c)^2 + 6\*((8\*p\*q + 11\*q^2)\*b^3\*c^2\*d\*f^2 - 3\*(7\*p\*q + 9\*q^2)\*a\*b^2\*c\*d^2\*f^2 + (2\*p^2 + 17\*p\*q + 18\*q^2)\*a^2\*b\*d^3\*f^2)\*x - 6\*(6\*a\*b^2\*c^2\*d\*f^2\*p\*q - 15\*a^2\*b\*c\*d^2\*f^2\*p\*q + (2\*p^2 + 11\*p\*q)\*a^3\*d^3\*f^2)\*log(b\*x + a))/(b\*d^3))/f^2

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( (b^2x^2 + 2abx + a^2) \log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^2 \log\left(\left((bx + a)^p(dx + c)^q f\right)^r e\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

### 3.19 $\int (a + bx) \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$

**Optimal.** Leaf size=540

$$\frac{pqr^2(bc - ad)^2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right)}{bd^2} - \frac{qr(bc - ad)^2 \log(c + dx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{bd^2} + \frac{pqr^2(bc - ad)^2 \log(c + dx)}{2bd^2}$$

[Out]  $(a^2 r^2 x)/2 + (a p q r^2 x)/2 - ((b^2 c - a^2 d) p q r^2 x)/(2 d) - ((b^2 c - a^2 d) q^2 r^2 x)/(2 d) - ((b^2 c - a^2 d) q (p + q) r^2 x)/d + (b^2 p^2 r^2 x^2)/4 + (b^2 p q r^2 x^2)/4 + (p^2 q r^2 (a + b x)^2)/(4 b) + (q^2 r^2 (a + b x)^2)/(4 b) + ((b^2 c - a^2 d)^2 p q r^2 \text{Log}[c + d x])/(2 b^2 d^2) + (3 (b^2 c - a^2 d)^2 q^2 r^2 \text{Log}[c + d x])/(2 b^2 d^2) + ((b^2 c - a^2 d)^2 p q r^2 \text{Log}[-((d (a + b x))/(b^2 c - a^2 d))] \text{Log}[c + d x])/(b^2 d^2) + ((b^2 c - a^2 d)^2 q^2 r^2 \text{Log}[c + d x]^2)/(2 b^2 d^2) + ((b^2 c - a^2 d) q r^2 (a + b x) \text{Log}[e^{(f(a + b x)^p (c + d x)^q)^r}])/(b^2 d) - (p r^2 (a + b x)^2 \text{Log}[e^{(f(a + b x)^p (c + d x)^q)^r}])/(2 b) - (q r^2 (a + b x)^2 \text{Log}[e^{(f(a + b x)^p (c + d x)^q)^r}])/(2 b) - ((b^2 c - a^2 d)^2 q r^2 \text{Log}[c + d x] \text{Log}[e^{(f(a + b x)^p (c + d x)^q)^r}])/(b^2 d^2) + ((a + b x)^2 \text{Log}[e^{(f(a + b x)^p (c + d x)^q)^r}]^2)/(2 b) + ((b^2 c - a^2 d)^2 p q r^2 \text{PolyLog}[2, (b^2 (c + d x))/(b^2 c - a^2 d)])/(b^2 d^2)$

**Rubi [A]** time = 0.390924, antiderivative size = 540, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$ , Rules used = {2498, 2495, 43, 2514, 2487, 31, 8, 2494, 2394, 2393, 2391, 2390, 2301}

$$\frac{pqr^2(bc - ad)^2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right)}{bd^2} - \frac{qr(bc - ad)^2 \log(c + dx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{bd^2} + \frac{pqr^2(bc - ad)^2 \log(c + dx)}{2bd^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)\*Log[e^(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2,x]

[Out]  $(a^2 r^2 x)/2 + (a p q r^2 x)/2 - ((b^2 c - a^2 d) p q r^2 x)/(2 d) - ((b^2 c - a^2 d) q^2 r^2 x)/(2 d) - ((b^2 c - a^2 d) q (p + q) r^2 x)/d + (b^2 p^2 r^2 x^2)/4 + (b^2 p q r^2 x^2)/4 + (p^2 q r^2 (a + b x)^2)/(4 b) + (q^2 r^2 (a + b x)^2)/(4 b) + ((b^2 c - a^2 d)^2 p q r^2 \text{Log}[c + d x])/(2 b^2 d^2) + (3 (b^2 c - a^2 d)^2 q^2 r^2 \text{Log}[c + d x])/(2 b^2 d^2) + ((b^2 c - a^2 d)^2 p q r^2 \text{Log}[-((d (a + b x))/(b^2 c - a^2 d))] \text{Log}[c + d x])/(b^2 d^2) + ((b^2 c - a^2 d)^2 q^2 r^2 \text{Log}[c + d x]^2)/(2 b^2 d^2) + ((b^2 c - a^2 d) q r^2 (a + b x) \text{Log}[e^{(f(a + b x)^p (c + d x)^q)^r}])/(b^2 d) - (p r^2 (a + b x)^2 \text{Log}[e^{(f(a + b x)^p (c + d x)^q)^r}])/(2 b) - (q r^2 (a + b x)^2 \text{Log}[e^{(f(a + b x)^p (c + d x)^q)^r}])/(2 b) - ((b^2 c - a^2 d)^2 q r^2 \text{Log}[c + d x] \text{Log}[e^{(f(a + b x)^p (c + d x)^q)^r}])/(b^2 d^2) + ((a + b x)^2 \text{Log}[e^{(f(a + b x)^p (c + d x)^q)^r}]^2)/(2 b) + ((b^2 c - a^2 d)^2 p q r^2 \text{PolyLog}[2, (b^2 (c + d x))/(b^2 c - a^2 d)])/(b^2 d^2)$



$$d)^2 q r \text{Log}[c + d x] \text{Log}[e^{(f(a + b x)^p (c + d x)^q)^r}] / (b d^2) + ((a + b x)^2 \text{Log}[e^{(f(a + b x)^p (c + d x)^q)^r}]^2) / (2 b) + ((b c - a d)^2 p q r^2 \text{PolyLog}[2, (b(c + d x)) / (b c - a d)]) / (b d^2)$$
Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
+ 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q]^r]/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s, RFx, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
```

$r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[p + q, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{LtQ}[s, 4]$

### Rule 31

$\text{Int}[(a_) + (b_)*(x_))^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

### Rule 8

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

### Rule 2494

$\text{Int}[\text{Log}[(e_)*((f_)*((a_) + (b_)*(x_))^{(p_)*((c_) + (d_)*(x_))^{(q_)})^{(r_)})]/((g_) + (h_)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[g + h*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/h, x] + (-\text{Dist}[(b*p*r)/h, \text{Int}[\text{Log}[g + h*x]/(a + b*x), x], x] - \text{Dist}[(d*q*r)/h, \text{Int}[\text{Log}[g + h*x]/(c + d*x), x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, p, q, r\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

### Rule 2394

$\text{Int}[(a_) + \text{Log}[(c_)*((d_) + (e_)*(x_))^{(n_)}]*(b_)]/((f_) + (g_)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n]))/g, x] - \text{Dist}[(b*e*n)/g, \text{Int}[\text{Log}[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n\}, x] \&\& \text{NeQ}[e*f - d*g, 0]$

### Rule 2393

$\text{Int}[(a_) + \text{Log}[(c_)*((d_) + (e_)*(x_))]*(b_)]/((f_) + (g_)*(x_)), x\_Symbol] \rightarrow \text{Dist}[1/g, \text{Subst}[\text{Int}[(a + b*\text{Log}[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{EqQ}[g + c*(e*f - d*g), 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_))^{(n_)}]]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

### Rule 2390

$\text{Int}[(a_) + \text{Log}[(c_)*((d_) + (e_)*(x_))^{(n_)}]*(b_))^{(p_)*((f_) + (g_)*(x_))^{(q_)}], x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(f*x)/d]^q*(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n, p, q\}, x] \&\& \text{EqQ}[e*f - d*g, 0]$

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rubi steps

$$\begin{aligned}
 \int (a + bx) \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(a + bx)^2 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2b} - (pr) \int (a + bx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx \\
 &= -\frac{pr(a + bx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2b} + \frac{(a + bx)^2 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2b} \\
 &= \frac{1}{2} ap^2 r^2 x + \frac{1}{4} bp^2 r^2 x^2 - \frac{pr(a + bx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2b} + \frac{(a + bx)^2 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2b} \\
 &= \frac{1}{2} ap^2 r^2 x - \frac{(bc - ad)pqr^2 x}{2d} + \frac{1}{4} bp^2 r^2 x^2 + \frac{pqr^2(a + bx)^2}{4b} + \frac{(bc - ad)^2 pqr^2 x}{2b} \\
 &= \frac{1}{2} ap^2 r^2 x + \frac{1}{2} apqr^2 x - \frac{(bc - ad)pqr^2 x}{2d} - \frac{(bc - ad)q(p + q)r^2 x}{d} + \frac{1}{4} bp^2 r^2 x^2 \\
 &= \frac{1}{2} ap^2 r^2 x + \frac{1}{2} apqr^2 x - \frac{(bc - ad)pqr^2 x}{2d} - \frac{(bc - ad)q^2 r^2 x}{2d} - \frac{(bc - ad)q(p + q)r^2 x}{d} \\
 &= \frac{1}{2} ap^2 r^2 x + \frac{1}{2} apqr^2 x - \frac{(bc - ad)pqr^2 x}{2d} - \frac{(bc - ad)q^2 r^2 x}{2d} - \frac{(bc - ad)q(p + q)r^2 x}{d}
 \end{aligned}$$

**Mathematica [A]** time = 0.570108, size = 781, normalized size = 1.45

$$-4pqr^2(bc - ad)^2 \text{PolyLog} \left( 2, \frac{d(a+bx)}{ad-bc} \right) - 8a^2 d^2 pr \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) - 2a^2 d^2 p^2 r^2 \log^2(a + bx) + 8a^2 d^2 pqr^2 \log(a + bx)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2, x]

[Out] (-4\*a\*b\*c\*d\*p\*q\*r^2 + 8\*a^2\*d^2\*p\*q\*r^2 + 2\*a\*b\*d^2\*p^2\*r^2\*x - 6\*b^2\*c\*d\*p\*q\*r^2\*x + 10\*a\*b\*d^2\*p\*q\*r^2\*x - 6\*b^2\*c\*d\*q^2\*r^2\*x + 8\*a\*b\*d^2\*q^2\*r^2\*x + b^2\*d^2\*p^2\*r^2\*x^2 + 2\*b^2\*d^2\*p\*q\*r^2\*x^2 + b^2\*d^2\*q^2\*r^2\*x^2 - 2\*a^2

$$\begin{aligned}
& 2*d^2*p^2*r^2*\text{Log}[a + b*x]^2 + 2*b^2*c^2*p*q*r^2*\text{Log}[c + d*x] - 4*a*b*c*d*p \\
& *q*r^2*\text{Log}[c + d*x] + 8*a^2*d^2*p*q*r^2*\text{Log}[c + d*x] + 6*b^2*c^2*q^2*r^2*\text{Lo} \\
& \text{g}[c + d*x] - 8*a*b*c*d*q^2*r^2*\text{Log}[c + d*x] + 2*b^2*c^2*q^2*r^2*\text{Log}[c + d*x \\
& ]^2 - 4*a*b*c*d*q^2*r^2*\text{Log}[c + d*x]^2 - 8*a^2*d^2*p*r*\text{Log}[e*(f*(a + b*x)^p \\
& *(c + d*x)^q)^r] - 4*a*b*d^2*p*r*x*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4 \\
& *b^2*c*d*q*r*x*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 8*a*b*d^2*q*r*x*\text{Log}[e \\
& *(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b^2*d^2*p*r*x^2*\text{Log}[e*(f*(a + b*x)^p*(c \\
& + d*x)^q)^r] - 2*b^2*d^2*q*r*x^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4* \\
& b^2*c^2*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 8*a*b*c*d*q \\
& *r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4*a*b*d^2*x*\text{Log}[e*(f \\
& *(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*b^2*d^2*x^2*\text{Log}[e*(f*(a + b*x)^p*(c + d* \\
& x)^q)^r]^2 + 2*p*r*\text{Log}[a + b*x]*(2*b*c*(b*c - 2*a*d)*q*r*\text{Log}[c + d*x] - 2*( \\
& b*c - a*d)^2*q*r*\text{Log}[(b*(c + d*x))/(b*c - a*d)] + a*d*(3*a*d*(p - q)*r + 2* \\
& b*c*q*r + 2*a*d*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) - 4*(b*c - a*d)^2*p* \\
& q*r^2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]/(4*b*d^2)
\end{aligned}$$

**Maple [F]** time = 0.166, size = 0, normalized size = 0.

$$\int (bx + a) \left( \ln \left( e \left( f(bx + a)^p (dx + c)^q \right)^r \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

[Out] int((b\*x+a)\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

**Maxima [A]** time = 1.5037, size = 680, normalized size = 1.26

$$\frac{1}{2} (bx^2 + 2ax) \log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2 + \frac{\left( \frac{2a^2fp \log(bx+a)}{b} - \frac{bdf(p+q)x^2 + 2(adf(p+2q) - bcfq)x}{d} - \frac{2(bc^2fq - 2acdfq) \log(dx+c)}{d^2} \right) r}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x, algorithm="maxima")

[Out] 1/2\*(b\*x^2 + 2\*a\*x)\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2 + 1/2\*(2\*a^2\*f\*p\*log(b\*x + a)/b - (b\*d\*f\*(p + q)\*x^2 + 2\*(a\*d\*f\*(p + 2\*q) - b\*c\*f\*q)\*x)/d -

$$2*(b*c^2*f*q - 2*a*c*d*f*q)*\log(d*x + c)/d^2)*r*\log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/4*r^2*(2*((p*q + 3*q^2)*b*c^2*f^2 - 2*(p*q + 2*q^2)*a*c*d*f^2)*\log(d*x + c)/d^2 - 4*(b^2*c^2*f^2*p*q - 2*a*b*c*d*f^2*p*q + a^2*d^2*f^2*p*q)*(\log(b*x + a)*\log((b*d*x + a*d)/(b*c - a*d) + 1) + \operatorname{dilog}(-(b*d*x + a*d)/(b*c - a*d)))/(b*d^2) - (2*a^2*d^2*f^2*p^2*\log(b*x + a)^2 - (p^2 + 2*p*q + q^2)*b^2*d^2*f^2*x^2 - 4*(b^2*c^2*f^2*p*q - 2*a*b*c*d*f^2*p*q)*\log(b*x + a)*\log(d*x + c) - 2*(b^2*c^2*f^2*q^2 - 2*a*b*c*d*f^2*q^2)*\log(d*x + c)^2 + 2*(3*(p*q + q^2)*b^2*c*d*f^2 - (p^2 + 5*p*q + 4*q^2)*a*b*d^2*f^2)*x - 2*(2*a*b*c*d*f^2*p*q - (p^2 + 3*p*q)*a^2*d^2*f^2)*\log(b*x + a))/(b*d^2))/f^2$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((bx + a) \log\left(\left((bx + a)^p(dx + c)^q f\right)^r e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")`

[Out] `integral((b*x + a)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a) \log\left(\left((bx + a)^p(dx + c)^q f\right)^r e\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

$$3.20 \quad \int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx} dx$$

**Optimal.** Leaf size=431

$$-\frac{1}{4} \left( -\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) + \log((a+bx)^{pr}) + \log((c+dx)^{qr}) \right) \left( 8 \left( \frac{qr \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b} \right) \log\left(-\frac{d(a+bx)}{bc-ad}\right) \right)$$

[Out]  $\text{Log}[(a + b*x)^{(p*r)}]^3/(3*b*p*r) - (q*\text{Log}[(a + b*x)^{(p*r)}]^2*\text{Log}[(b*(c + d*x))/(b*c - a*d)])/(b*p) + (\text{Log}[(a + b*x)^{(p*r)}]^2*\text{Log}[(c + d*x)^{(q*r)}])/(b*p*r) + (\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[(c + d*x)^{(q*r)}]^2)/b - (2*q*r*\text{Log}[(a + b*x)^{(p*r)}]*\text{PolyLog}[2, -((d*(a + b*x))/(b*c - a*d))])/b + (2*q*r*\text{Log}[(c + d*x)^{(q*r)}]*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/b - ((\text{Log}[(a + b*x)^{(p*r)}] + \text{Log}[(c + d*x)^{(q*r)}] - \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)) * ((\text{Log}[(a + b*x)^{(p*r)}] - \text{Log}[(c + d*x)^{(q*r)}] + \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r))^2/(b*p*r) + 8*((\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[(c + d*x)^{(q*r)}])/b + (q*r*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)]/b))/4 + (2*p*q*r^2*\text{PolyLog}[3, -((d*(a + b*x))/(b*c - a*d))])/b - (2*q^2*r^2*\text{PolyLog}[3, (b*(c + d*x))/(b*c - a*d)]/b$

**Rubi [A]** time = 0.491518, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$ , Rules used = {2496, 6742, 2390, 2302, 30, 2433, 2375, 2317, 2374, 6589, 2396, 2394, 2393, 2391, 6686}

$$-\frac{1}{4} \left( -\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) + \log((a+bx)^{pr}) + \log((c+dx)^{qr}) \right) \left( 8 \left( \frac{qr \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b} \right) \log\left(-\frac{d(a+bx)}{bc-ad}\right) \right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2/(a + b*x), x]$

[Out]  $\text{Log}[(a + b*x)^{(p*r)}]^3/(3*b*p*r) - (q*\text{Log}[(a + b*x)^{(p*r)}]^2*\text{Log}[(b*(c + d*x))/(b*c - a*d)])/(b*p) + (\text{Log}[(a + b*x)^{(p*r)}]^2*\text{Log}[(c + d*x)^{(q*r)}])/(b*p*r) + (\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[(c + d*x)^{(q*r)}]^2)/b - (2*q*r*\text{Log}[(a + b*x)^{(p*r)}]*\text{PolyLog}[2, -((d*(a + b*x))/(b*c - a*d))])/b + (2*q*r*\text{Log}[(c + d*x)^{(q*r)}]*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)]/b - ((\text{Log}[(a + b*x)^{(p*r)}] + \text{Log}[(c + d*x)^{(q*r)}] - \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r))$

$$\begin{aligned} & *((\text{Log}[(a + b*x)^{(p*r)}] - \text{Log}[(c + d*x)^{(q*r)}] + \text{Log}[e*(f*(a + b*x)^p*(c + \\ & d*x)^q]^r])^2/(b*p*r) + 8*((\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[(c + d*x) \\ & ^{(q*r)}])/b + (q*r*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)]/b))/4 + (2*p*q*r^ \\ & 2*\text{PolyLog}[3, -((d*(a + b*x))/(b*c - a*d))])/b - (2*q^2*r^2*\text{PolyLog}[3, (b*(c \\ & + d*x))/(b*c - a*d)]/b \end{aligned}$$
Rule 2496

$$\begin{aligned} & \text{Int}[\text{Log}[(e_*)*((f_*)*((a_*) + (b_*)*(x_))^{(p_*)*((c_*) + (d_*)*(x_))^{(q_*)}) \\ & ^{(r_*)}]^2/((g_*) + (h_*)*(x_)), x\_Symbol] \text{ :> } \text{Int}[(\text{Log}[(a + b*x)^{(p*r)}] + \text{Lo} \\ & g[(c + d*x)^{(q*r)}])^2/(g + h*x), x] + \text{Simp}[(\text{Log}[e*(f*(a + b*x)^p*(c + d*x) \\ & ^q]^r] - \text{Log}[(a + b*x)^{(p*r)}] - \text{Log}[(c + d*x)^{(q*r)}])*(2*\text{Int}[\text{Log}[(c + d*x) \\ & ^{(q*r)}]/(g + h*x), x] + \text{Int}[(\text{Log}[(a + b*x)^{(p*r)}] - \text{Log}[(c + d*x)^{(q*r)}] + \text{Lo} \\ & g[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(g + h*x), x]), x] \text{ /; } \text{FreeQ}\{a, b, c, d \\ & , e, f, g, h, p, q, r\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[b*g - a*h, 0] \end{aligned}$$
Rule 6742

$$\text{Int}[u_, x\_Symbol] \text{ :> } \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] \text{ /; } \text{SumQ}[v]$$
Rule 2390

$$\begin{aligned} & \text{Int}[(a_*) + \text{Log}[(c_*)*((d_*) + (e_*)*(x_))^{(n_*)}]* (b_*)^{(p_*)}*((f_*) + (g_*) \\ & )*(x_))^{(q_*)}, x\_Symbol] \text{ :> } \text{Dist}[1/e, \text{Subst}[\text{Int}[(f*x)/d]^q*(a + b*\text{Log}[c*x \\ & ^n])^p, x], x, d + e*x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, f, g, n, p, q\}, x\} \&\& \text{EqQ}[e*f - d*g, 0] \end{aligned}$$
Rule 2302

$$\begin{aligned} & \text{Int}[(a_*) + \text{Log}[(c_*)*(x_)^{(n_*)}]* (b_*)^{(p_*)}/(x_), x\_Symbol] \text{ :> } \text{Dist}[1/( \\ & b*n), \text{Subst}[\text{Int}[x^p, x], x, a + b*\text{Log}[c*x^n]], x] \text{ /; } \text{FreeQ}\{a, b, c, n, p\}, \\ & x] \end{aligned}$$
Rule 30

$$\text{Int}[(x_)^{(m_*)}, x\_Symbol] \text{ :> } \text{Simp}[x^{(m + 1)}/(m + 1), x] \text{ /; } \text{FreeQ}[m, x] \&\& \text{NeQ}[m, -1]$$
Rule 2433

$$\begin{aligned} & \text{Int}[(a_*) + \text{Log}[(c_*)*((d_*) + (e_*)*(x_))^{(n_*)}]* (b_*)^{(p_*)}*((f_*) + \text{Log} \\ & [(h_*)*((i_*) + (j_*)*(x_))^{(m_*)}]* (g_*)*((k_*) + (l_*)*(x_))^{(r_*)}, x\_Sym \\ & bol] \text{ :> } \text{Dist}[1/e, \text{Subst}[\text{Int}[(k*x)/d]^r*(a + b*\text{Log}[c*x^n])^p*(f + g*\text{Log}[h*( \\ & (e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, \end{aligned}$$



f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*1, 0]

### Rule 2375

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))]^(r\_.))\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)])\*(b\_.)^(p\_.)]/(x\_), x\_Symbol] := Simp[(Log[d\*(e + f\*x^m)^r]\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*n\*(p + 1)), x] - Dist[(f\*m\*r)/(b\*n\*(p + 1)), Int[(x^(m - 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(e + f\*x^m), x], x] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d\*e, 1]

### Rule 2317

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)^(p\_.))/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[1 + (e\*x)/d]\*(a + b\*Log[c\*x^n])^p)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[1 + (e\*x)/d]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))]^(p\_.))\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)^(p\_.)]/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.)^(p\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.)^(p\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x)

, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 6686

Int[(u\_)\*(y\_)^(m\_.), x\_Symbol] := With[{q = DerivativeDivides[y, u, x]}, Simp[(q\*y^(m + 1))/(m + 1), x] /; !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{a+bx} dx &= \int \frac{(\log((a+bx)^{pr}) + \log((c+dx)^{qr}))^2}{a+bx} dx - \left( \log((a+bx)^{pr}) + \log((c+dx)^{qr}) \right) \left( \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{a+bx} \right) \\
&= - \left( \log((a+bx)^{pr}) + \log((c+dx)^{qr}) - \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) \right) \left( \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{a+bx} \right) \\
&= 2 \int \frac{\log((a+bx)^{pr}) \log((c+dx)^{qr})}{a+bx} dx - \left( \log((a+bx)^{pr}) + \log((c+dx)^{qr}) - \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) \right) \left( \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{a+bx} \right) \\
&= \frac{\log \left( -\frac{d(a+bx)}{bc-ad} \right) \log^2((c+dx)^{qr})}{b} - \left( \log((a+bx)^{pr}) + \log((c+dx)^{qr}) - \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) \right) \left( \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{a+bx} \right) \\
&= \frac{\log^2((a+bx)^{pr}) \log((c+dx)^{qr})}{bpr} + \frac{\log \left( -\frac{d(a+bx)}{bc-ad} \right) \log^2((c+dx)^{qr})}{b} - \left( \log((a+bx)^{pr}) + \log((c+dx)^{qr}) - \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) \right) \left( \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{a+bx} \right) \\
&= \frac{\log^3((a+bx)^{pr})}{3bpr} - \frac{q \log^2((a+bx)^{pr}) \log \left( \frac{b(c+dx)}{bc-ad} \right)}{bp} + \frac{\log^2((a+bx)^{pr}) \log((c+dx)^{qr})}{bpr} \\
&= \frac{\log^3((a+bx)^{pr})}{3bpr} - \frac{q \log^2((a+bx)^{pr}) \log \left( \frac{b(c+dx)}{bc-ad} \right)}{bp} + \frac{\log^2((a+bx)^{pr}) \log((c+dx)^{qr})}{bpr} \\
&= \frac{\log^3((a+bx)^{pr})}{3bpr} - \frac{q \log^2((a+bx)^{pr}) \log \left( \frac{b(c+dx)}{bc-ad} \right)}{bp} + \frac{\log^2((a+bx)^{pr}) \log((c+dx)^{qr})}{bpr}
\end{aligned}$$

**Mathematica [A]** time = 0.17125, size = 460, normalized size = 1.07

---


$$6qr \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) \left( \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) - pr \log(a+bx) \right) + 6pqr^2 \text{PolyLog} \left( 3, \frac{d(a+bx)}{ad-bc} \right) - 6pqr^2 \log(a+bx)$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2/(a + b\*x),x]

[Out] (p^2\*r^2\*Log[a + b\*x]^3 + 6\*p\*q\*r^2\*Log[a + b\*x]^2\*Log[c + d\*x] - 6\*p\*q\*r^2\*Log[a + b\*x]\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]\*Log[c + d\*x] + 3\*q^2\*r^2\*Log[a + b\*x]\*Log[c + d\*x]^2 - 3\*q^2\*r^2\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]\*Log[c + d\*x]^2 - 3\*p\*q\*r^2\*Log[a + b\*x]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] - 3\*p\*r\*Log[a + b\*x]^2\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] - 6\*q\*r\*Log[a + b\*x]\*Log[c + d\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] + 6\*q\*r\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]\*Log[c + d\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] + 3\*Log[a + b\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2 - 6\*p\*q\*r^2\*Log[a + b\*x]\*PolyLog[2, (d\*(a + b\*x))/(-b\*c + a\*d)] + 6\*q\*r\*(-p\*r\*Log[a + b\*x] + Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r])\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)] + 6\*p\*q\*r^2\*PolyLog[3, (d\*(a + b\*x))/(-b\*c + a\*d)] - 6\*q^2\*r^2\*PolyLog[3, (b\*(c + d\*x))/(b\*c - a\*d)]/(3\*b)

**Maple [F]** time = 0.416, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)\right)^2}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a),x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\log(bx+a)\log\left(\left((dx+c)^q\right)^r\right)^2}{b} + \int \frac{\left(\log(e)^2 + 2\log(e)\log(fr) + \log(fr)^2\right)bdx + \left(\log(e)^2 + 2\log(e)\log(fr) + \log(fr)^2\right)}{b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a),x, algorithm="maxima")

```
[Out] log(b*x + a)*log(((d*x + c)^q)^r)^2/b + integrate(((log(e)^2 + 2*log(e)*log
(f^r) + log(f^r)^2)*b*d*x + (log(e)^2 + 2*log(e)*log(f^r) + log(f^r)^2)*b*c
+ (b*d*x + b*c)*log(((b*x + a)^p)^r)^2 + 2*(b*d*x*(log(e) + log(f^r)) + b*
c*(log(e) + log(f^r)))*log(((b*x + a)^p)^r) + 2*(b*d*x*(log(e) + log(f^r))
+ b*c*(log(e) + log(f^r)) - (b*d*q*r*x + a*d*q*r)*log(b*x + a) + (b*d*x + b
*c)*log(((b*x + a)^p)^r))*log(((d*x + c)^q)^r))/(b^2*d*x^2 + a*b*c + (b^2*c
+ a*b*d)*x), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2}{bx + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2}{bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a), x)
```

$$3.21 \quad \int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2} dx$$

**Optimal.** Leaf size=465

$$\frac{2dpqr^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)} - \frac{2dq^2r^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + \frac{2dqr \log(a+bx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)}$$

[Out]  $(-2*p^2*r^2)/(b*(a + b*x)) + (2*d*p*q*r^2*\text{Log}[a + b*x])/(b*(b*c - a*d)) - (d*p*q*r^2*\text{Log}[a + b*x]^2)/(b*(b*c - a*d)) - (2*d*p*q*r^2*\text{Log}[c + d*x])/(b*(b*c - a*d)) + (2*d*p*q*r^2*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x])/(b*(b*c - a*d)) + (d*q^2*r^2*\text{Log}[c + d*x]^2)/(b*(b*c - a*d)) - (2*d*q^2*r^2*\text{Log}[a + b*x]*\text{Log}[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)) - (2*p*r*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(b*(a + b*x)) + (2*d*q*r*\text{Log}[a + b*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(b*(b*c - a*d)) - (2*d*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(b*(b*c - a*d)) - \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)^2/(b*(a + b*x)) - (2*d*q^2*r^2*\text{PolyLog}[2, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)) + (2*d*p*q*r^2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d))$

**Rubi [A]** time = 0.387124, antiderivative size = 465, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$ , Rules used = {2498, 2495, 32, 36, 31, 2514, 2494, 2390, 2301, 2394, 2393, 2391}

$$\frac{2dpqr^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)} - \frac{2dq^2r^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + \frac{2dqr \log(a+bx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2/(a + b*x)^2, x]$

[Out]  $(-2*p^2*r^2)/(b*(a + b*x)) + (2*d*p*q*r^2*\text{Log}[a + b*x])/(b*(b*c - a*d)) - (d*p*q*r^2*\text{Log}[a + b*x]^2)/(b*(b*c - a*d)) - (2*d*p*q*r^2*\text{Log}[c + d*x])/(b*(b*c - a*d)) + (2*d*p*q*r^2*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x])/(b*(b*c - a*d)) + (d*q^2*r^2*\text{Log}[c + d*x]^2)/(b*(b*c - a*d)) - (2*d*q^2*r^2*\text{Log}[a + b*x]*\text{Log}[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)) - (2*p*r*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(b*(a + b*x)) + (2*d*q*r*\text{Log}[a + b*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(b*(b*c - a*d)) - (2*d*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(b*(b*c - a*d)) - \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)^2/(b*(a + b*x)) - (2*d*q^2*r^2*\text{PolyLog}[2, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)) + (2*d*p*q*r^2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d))$

$$\frac{1}{(b*c - a*d)}}{(b*(b*c - a*d)) + (2*d*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d))]}{(b*(b*c - a*d))}$$

### Rule 2498

$$\text{Int}[\text{Log}[(e_{.})*((f_{.})*((a_{.}) + (b_{.})*(x_{.}))^{(p_{.})}*((c_{.}) + (d_{.})*(x_{.}))^{(q_{.})})^{(r_{.})}]^{(s_{.})}*((g_{.}) + (h_{.})*(x_{.}))^{(m_{.})}, x\_Symbol] \rightarrow \text{Simp}[(g + h*x)^{(m+1)}*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m+1)), x] + (-\text{Dist}[(b*p*r*s)/(h*(m+1)), \text{Int}[(g + h*x)^{(m+1)}*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^{(s-1)}/(a + b*x), x], x] - \text{Dist}[(d*q*r*s)/(h*(m+1)), \text{Int}[(g + h*x)^{(m+1)}*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^{(s-1)}/(c + d*x), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, p, q, r, s\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{NeQ}[m, -1]$$

### Rule 2495

$$\text{Int}[\text{Log}[(e_{.})*((f_{.})*((a_{.}) + (b_{.})*(x_{.}))^{(p_{.})}*((c_{.}) + (d_{.})*(x_{.}))^{(q_{.})})^{(r_{.})}]*((g_{.}) + (h_{.})*(x_{.}))^{(m_{.})}, x\_Symbol] \rightarrow \text{Simp}[(g + h*x)^{(m+1)}*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m+1)), x] + (-\text{Dist}[(b*p*r)/(h*(m+1)), \text{Int}[(g + h*x)^{(m+1)}/(a + b*x), x], x] - \text{Dist}[(d*q*r)/(h*(m+1)), \text{Int}[(g + h*x)^{(m+1)}/(c + d*x), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, p, q, r\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m, -1]$$

### Rule 32

$$\text{Int}[(a_{.}) + (b_{.})*(x_{.})^{(m_{.})}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}/(b*(m+1)), x] /; \text{FreeQ}\{a, b, m\}, x\} \&\& \text{NeQ}[m, -1]$$

### Rule 36

$$\text{Int}[1/(((a_{.}) + (b_{.})*(x_{.}))*((c_{.}) + (d_{.})*(x_{.}))), x\_Symbol] \rightarrow \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[b*c - a*d, 0]$$

### Rule 31

$$\text{Int}[(a_{.}) + (b_{.})*(x_{.})^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x\}$$

### Rule 2514

$$\text{Int}[\text{Log}[(e_{.})*((f_{.})*((a_{.}) + (b_{.})*(x_{.}))^{(p_{.})}*((c_{.}) + (d_{.})*(x_{.}))^{(q_{.})})^{(r_{.})}]^{(s_{.})}*(\text{Rfx}_{.}), x\_Symbol] \rightarrow \text{With}\{u = \text{ExpandIntegrand}[\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s, \text{Rfx}, x]\}, \text{Int}[u, x] /; \text{SumQ}[u] /; \text{FreeQ}\{a, b, c$$



, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]

#### Rule 2494

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[(Log[g + h\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r])/h, x] + (-Dist[(b\*p\*r)/h, Int[Log[g + h\*x]/(a + b\*x), x], x] - Dist[(d\*q\*r)/h, Int[Log[g + h\*x]/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)^2} dx &= -\frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{b(a+bx)} + (2pr) \int \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)^2} dx + \frac{(2dqr)}{b} \int \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)^2} dx \\
&= -\frac{2pr \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{b(a+bx)} - \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{b(a+bx)} + \frac{(2dqr)}{b} \int \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)^2} dx \\
&= -\frac{2p^2 r^2}{b(a+bx)} - \frac{2pr \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{b(a+bx)} - \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{b(a+bx)} \\
&= -\frac{2p^2 r^2}{b(a+bx)} + \frac{2dpqr^2 \log(a+bx)}{b(bc-ad)} - \frac{2dpqr^2 \log(c+dx)}{b(bc-ad)} - \frac{2pr \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{b(a+bx)} \\
&= -\frac{2p^2 r^2}{b(a+bx)} + \frac{2dpqr^2 \log(a+bx)}{b(bc-ad)} - \frac{2dpqr^2 \log(c+dx)}{b(bc-ad)} + \frac{2dpqr^2 \log \left( -\frac{d(a+bx)}{bc-ad} \right)}{b(bc-ad)} \\
&= -\frac{2p^2 r^2}{b(a+bx)} + \frac{2dpqr^2 \log(a+bx)}{b(bc-ad)} - \frac{dpqr^2 \log^2(a+bx)}{b(bc-ad)} - \frac{2dpqr^2 \log(c+dx)}{b(bc-ad)} + \frac{2dpqr^2 \log \left( -\frac{d(a+bx)}{bc-ad} \right)}{b(bc-ad)} \\
&= -\frac{2p^2 r^2}{b(a+bx)} + \frac{2dpqr^2 \log(a+bx)}{b(bc-ad)} - \frac{dpqr^2 \log^2(a+bx)}{b(bc-ad)} - \frac{2dpqr^2 \log(c+dx)}{b(bc-ad)} + \frac{2dpqr^2 \log \left( -\frac{d(a+bx)}{bc-ad} \right)}{b(bc-ad)}
\end{aligned}$$

**Mathematica [A]** time = 0.836383, size = 411, normalized size = 0.88

$$\frac{-2dqr^2(p+q)(a+bx)\text{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right) - bc \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) + ad \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - 2bc \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b^2(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2/(a + b\*x)^2,x]

[Out] (-2\*b\*c\*p^2\*r^2 + 2\*a\*d\*p^2\*r^2 - d\*p\*q\*r^2\*(a + b\*x)\*Log[a + b\*x]^2 - 2\*a\*d\*p\*q\*r^2\*Log[c + d\*x] - 2\*b\*d\*p\*q\*r^2\*x\*Log[c + d\*x] + a\*d\*q^2\*r^2\*Log[c + d\*x]^2 + b\*d\*q^2\*r^2\*x\*Log[c + d\*x]^2 - 2\*b\*c\*p\*r\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] + 2\*a\*d\*p\*r\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] - 2\*a\*d\*q\*r\*Log[c + d\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] - 2\*b\*d\*q\*r\*x\*Log[c + d\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] - b\*c\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2 + a\*d\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2 + 2\*d\*q\*r\*(a + b\*x)\*Log[a + b\*x]\*(p\*r + p\*r\*Log[c + d\*x] - (p + q)\*r\*Log[(b\*(c + d\*x))/(b\*c - a\*d])

)] + Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] - 2\*d\*q\*(p + q)\*r^2\*(a + b\*x)\*PolyLog[2, (d\*(a + b\*x))/(-b\*c + a\*d)]/(b\*(b\*c - a\*d)\*(a + b\*x))

**Maple [F]** time = 0.41, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)\right)^2}{(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a)^2,x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a)^2,x)

**Maxima [A]** time = 1.34225, size = 529, normalized size = 1.14

$$\frac{2\left(\frac{dfq \log(bx+a)}{bc-ad} - \frac{dfq \log(dx+c)}{bc-ad} - \frac{fp}{bx+a}\right)r \log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{bf} - \left(\frac{2df^2pq \log(dx+c)}{bc-ad} + \frac{2(pq+q^2)\left(\log(bx+a) \log\left(\frac{bdx+ad}{bc-ad}+1\right)\right)+\text{Li}_2\left(\frac{bdx+ad}{bc-ad}\right)}{bc-ad}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a)^2,x, algorithm="maxima")

[Out] 2\*(d\*f\*q\*log(b\*x + a)/(b\*c - a\*d) - d\*f\*q\*log(d\*x + c)/(b\*c - a\*d) - f\*p/(b\*x + a))\*r\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)/(b\*f) - (2\*d\*f^2\*p\*q\*log(d\*x + c)/(b\*c - a\*d) + 2\*(p\*q + q^2)\*(log(b\*x + a)\*log((b\*d\*x + a\*d)/(b\*c - a\*d) + 1) + dilog(-(b\*d\*x + a\*d)/(b\*c - a\*d)))\*d\*f^2/(b\*c - a\*d) + (2\*b\*c\*f^2\*p^2 - 2\*a\*d\*f^2\*p^2 + (b\*d\*f^2\*p\*q\*x + a\*d\*f^2\*p\*q)\*log(b\*x + a)^2 - 2\*(b\*d\*f^2\*p\*q\*x + a\*d\*f^2\*p\*q)\*log(b\*x + a)\*log(d\*x + c) - (b\*d\*f^2\*q^2\*x + a\*d\*f^2\*q^2)\*log(d\*x + c)^2 - 2\*(b\*d\*f^2\*p\*q\*x + a\*d\*f^2\*p\*q)\*log(b\*x + a))/(a\*b\*c - a^2\*d + (b^2\*c - a\*b\*d)\*x))\*r^2/(b\*f^2) - log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2/((b\*x + a)\*b)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2}{b^2 x^2 + 2 abx + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a)^2,x, algorithm="fricas")

[Out] integral(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2/(b^2\*x^2 + 2\*a\*b\*x + a^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r)\*\*2/(b\*x+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2}{(bx + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a)^2,x, algorithm="giac")

[Out] integrate(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2/(b\*x + a)^2, x)

$$3.22 \quad \int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3} dx$$

**Optimal.** Leaf size=632

$$\frac{d^2 p q r^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2} + \frac{d^2 q^2 r^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)^2} - \frac{d^2 q r \log(a+bx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2} + \frac{d^2 q r \log(a+bx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2}$$

[Out]  $-(p^2 r^2)/(4 b^2 (a+bx)^2) - (3 d^2 p q r^2)/(2 b^2 (b^2 c - a^2 d)(a+bx)) - (d^2 p^2 q r^2 \text{Log}[a+bx])/(2 b^2 (b^2 c - a^2 d)^2) + (d^2 q^2 r^2 \text{Log}[a+bx])/(b^2 (b^2 c - a^2 d)^2) + (d^2 p^2 q r^2 \text{Log}[a+bx]^2)/(2 b^2 (b^2 c - a^2 d)^2) + (d^2 p^2 q r^2 \text{Log}[c+dx])/(2 b^2 (b^2 c - a^2 d)^2) - (d^2 q^2 r^2 \text{Log}[c+dx])/(b^2 (b^2 c - a^2 d)^2) - (d^2 p^2 q r^2 \text{Log}[-((d(a+bx))/(b^2 c - a^2 d))]*\text{Log}[c+dx])/(b^2 (b^2 c - a^2 d)^2) - (d^2 q^2 r^2 \text{Log}[c+dx]^2)/(2 b^2 (b^2 c - a^2 d)^2) + (d^2 q^2 r^2 \text{Log}[a+bx]*\text{Log}[(b^2(c+dx))/(b^2 c - a^2 d])/(b^2 (b^2 c - a^2 d)^2) - (p r \text{Log}[e*(f(a+bx))^p*(c+dx)^q]^r)/(2 b^2 (a+bx)^2) - (d^2 q r \text{Log}[e*(f(a+bx))^p*(c+dx)^q]^r)/(b^2 (b^2 c - a^2 d)(a+bx)) - (d^2 q r \text{Log}[a+bx]*\text{Log}[e*(f(a+bx))^p*(c+dx)^q]^r)/(b^2 (b^2 c - a^2 d)^2) + (d^2 q r \text{Log}[c+dx]*\text{Log}[e*(f(a+bx))^p*(c+dx)^q]^r)/(b^2 (b^2 c - a^2 d)^2) - \text{Log}[e*(f(a+bx))^p*(c+dx)^q]^r/(2 b^2 (a+bx)^2) + (d^2 q^2 r^2 \text{PolyLog}[2, -((d(a+bx))/(b^2 c - a^2 d))])/(b^2 (b^2 c - a^2 d)^2) - (d^2 p^2 q r^2 \text{PolyLog}[2, (b^2(c+dx))/(b^2 c - a^2 d)])/(b^2 (b^2 c - a^2 d)^2)$

**Rubi [A]** time = 0.48971, antiderivative size = 632, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 13, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$ , Rules used = {2498, 2495, 32, 44, 2514, 36, 31, 2494, 2390, 2301, 2394, 2393, 2391}

$$\frac{d^2 p q r^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2} + \frac{d^2 q^2 r^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)^2} - \frac{d^2 q r \log(a+bx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2} + \frac{d^2 q r \log(a+bx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Log}[e*(f(a+bx))^p*(c+dx)^q]^r/(a+bx)^3, x]$

[Out]  $-(p^2 r^2)/(4 b^2 (a+bx)^2) - (3 d^2 p q r^2)/(2 b^2 (b^2 c - a^2 d)(a+bx)) - (d^2 p^2 q r^2 \text{Log}[a+bx])/(2 b^2 (b^2 c - a^2 d)^2) + (d^2 q^2 r^2 \text{Log}[a+bx])/(b^2 (b^2 c - a^2 d)^2) + (d^2 p^2 q r^2 \text{Log}[a+bx]^2)/(2 b^2 (b^2 c - a^2 d)^2) + (d^2 p^2 q r^2 \text{Log}[c+dx])/(2 b^2 (b^2 c - a^2 d)^2) - (d^2 q^2 r^2 \text{Log}[c+dx])/(b^2 (b^2 c - a^2 d)^2) - (d^2 p^2 q r^2 \text{Log}[-((d(a+bx))/(b^2 c - a^2 d))]*\text{Log}[c+dx])/(b^2 (b^2 c - a^2 d)^2) - (d^2 q^2 r^2 \text{Log}[c+dx]^2)/(2 b^2 (b^2 c - a^2 d)^2) +$

$$\begin{aligned} & (d^2 q^2 r^2 \text{Log}[a + b*x] \text{Log}[(b*(c + d*x))/(b*c - a*d)]) / (b*(b*c - a*d)^2) \\ & - (p*r \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]) / (2*b*(a + b*x)^2) - (d*q*r \text{Log} \\ & \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]) / (b*(b*c - a*d)*(a + b*x)) - (d^2*q*r \text{Lo} \\ & \text{g}[a + b*x] \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]) / (b*(b*c - a*d)^2) + (d^2*q \\ & *r \text{Log}[c + d*x] \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]) / (b*(b*c - a*d)^2) - L \\ & \text{og}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2 / (2*b*(a + b*x)^2) + (d^2*q^2*r^2 \text{Poly} \\ & \text{Log}[2, -((d*(a + b*x))/(b*c - a*d))]) / (b*(b*c - a*d)^2) - (d^2*p*q*r^2 \text{Poly} \\ & \text{Log}[2, (b*(c + d*x))/(b*c - a*d)]) / (b*(b*c - a*d)^2) \end{aligned}$$

### Rule 2498

```
Int[Log[(e._)*((f._)*((a._) + (b._)*(x._))^(p._))*((c._) + (d._)*(x._))^(q._))
^(r._)]^(s._)*((g._) + (h._)*(x._))^(m._), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*s)
/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
+ 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

### Rule 2495

```
Int[Log[(e._)*((f._)*((a._) + (b._)*(x._))^(p._))*((c._) + (d._)*(x._))^(q._))
^(r._)]*((g._) + (h._)*(x._))^(m._), x_Symbol] := Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

### Rule 32

```
Int[((a._) + (b._)*(x._))^(m._), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 44

```
Int[((a._) + (b._)*(x._))^(m._))*((c._) + (d._)*(x._))^(n._), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

### Rule 2514

```
Int[Log[(e._)*((f._)*((a._) + (b._)*(x._))^(p._))*((c._) + (d._)*(x._))^(q._))
^(r._)]^(s._)*(RFx_), x_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a +
```

$b*x)^p*(c + d*x)^q)^r)^s, \text{RFX}, x\}$ , Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]

### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))<sup>(-1)</sup>, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 2494

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))<sup>(p\_.)</sup>\*((c\_.) + (d\_.)\*(x\_))<sup>(q\_.)</sup>)<sup>(r\_.)</sup>]/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[(Log[g + h\*x]\*Log[e\*(f\*(a + b\*x)<sup>p</sup>\*(c + d\*x)<sup>q</sup>]<sup>r</sup>)/h, x] + (-Dist[(b\*p\*r)/h, Int[Log[g + h\*x]/(a + b\*x), x], x] - Dist[(d\*q\*r)/h, Int[Log[g + h\*x]/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b\*c - a\*d, 0]

### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))<sup>(n\_.)</sup>])\*(b\_.))<sup>(p\_.)</sup>\*((f\_) + (g\_.)\*(x\_))<sup>(q\_.)</sup>, x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)<sup>q</sup>\*(a + b\*Log[c\*x<sup>n</sup>])<sup>p</sup>, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)<sup>(n\_.)</sup>])\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x<sup>n</sup>])<sup>2</sup>/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))<sup>(n\_.)</sup>])\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)<sup>n</sup>]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x]

], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)^3} dx &= -\frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{2b(a+bx)^2} + (pr) \int \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)^3} dx + \frac{(dqr) \int \left( \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)^3} \right)}{2b(a+bx)^2} \\
 &= -\frac{p^2 r^2}{4b(a+bx)^2} - \frac{pr \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{2b(a+bx)^2} - \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{2b(a+bx)^2} + \frac{(dqr) \int \left( \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)^3} \right)}{2b(bc-ad)^2} \\
 &= -\frac{p^2 r^2}{4b(a+bx)^2} - \frac{dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2 pqr^2 \log(a+bx)}{2b(bc-ad)^2} + \frac{d^2 pqr^2 \log(c+dx)}{2b(bc-ad)^2} \\
 &= -\frac{p^2 r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2 pqr^2 \log(a+bx)}{2b(bc-ad)^2} + \frac{d^2 pqr^2 \log(c+dx)}{2b(bc-ad)^2} \\
 &= -\frac{p^2 r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2 pqr^2 \log(a+bx)}{2b(bc-ad)^2} + \frac{d^2 q^2 r^2 \log(a+bx)}{b(bc-ad)^2} \\
 &= -\frac{p^2 r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2 pqr^2 \log(a+bx)}{2b(bc-ad)^2} + \frac{d^2 q^2 r^2 \log(a+bx)}{b(bc-ad)^2}
 \end{aligned}$$

**Mathematica [A]** time = 1.60547, size = 872, normalized size = 1.38

$$\frac{c^2 p^2 r^2 b^2 + 2d^2 q^2 r^2 x^2 \log^2(c+dx)b^2 + 2c^2 \log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) b^2 + 6cdpqr^2 x b^2 + 4d^2 q^2 r^2 x^2 \log(c+dx)b^2 - 2}{}$$

Antiderivative was successfully verified.



[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2/(a + b\*x)^3,x]

[Out] 
$$-(b^2*c^2*p^2*r^2 - 2*a*b*c*d*p^2*r^2 + a^2*d^2*p^2*r^2 + 6*a*b*c*d*p*q*r^2 - 6*a^2*d^2*p*q*r^2 + 6*b^2*c*d*p*q*r^2*x - 6*a*b*d^2*p*q*r^2*x - 2*d^2*p*q*r^2*(a + b*x)^2*\text{Log}[a + b*x]^2 - 2*a^2*d^2*p*q*r^2*\text{Log}[c + d*x] + 4*a^2*d^2*q^2*r^2*\text{Log}[c + d*x] - 4*a*b*d^2*p*q*r^2*x*\text{Log}[c + d*x] + 8*a*b*d^2*q^2*r^2*x*\text{Log}[c + d*x] - 2*b^2*d^2*p*q*r^2*x^2*\text{Log}[c + d*x] + 4*b^2*d^2*q^2*r^2*x^2*\text{Log}[c + d*x] + 2*a^2*d^2*q^2*r^2*\text{Log}[c + d*x]^2 + 4*a*b*d^2*q^2*r^2*x*\text{Log}[c + d*x]^2 + 2*b^2*d^2*q^2*r^2*x^2*\text{Log}[c + d*x]^2 - 2*d^2*q*r*(a + b*x)^2*\text{Log}[a + b*x]*(-(p*r) + 2*q*r - 2*p*r*\text{Log}[c + d*x] + 2*(p + q)*r*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b^2*c^2*p*r*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a*b*c*d*p*r*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*a^2*d^2*p*r*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4*a*b*c*d*q*r*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a^2*d^2*q*r*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4*b^2*c*d*q*r*x*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a*b*d^2*q*r*x*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a^2*d^2*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 8*a*b*d^2*q*r*x*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*b^2*d^2*q*r*x^2*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b^2*c^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 4*a*b*c*d*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*a^2*d^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 4*d^2*q*(p + q)*r^2*(a + b*x)^2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]/(4*b*(b*c - a*d)^2*(a + b*x)^2)$$

**Maple [F]** time = 0.412, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(e\left(f\left(bx+a\right)^p\left(dx+c\right)^q\right)^r\right)\right)^2}{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a)^3,x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a)^3,x)

**Maxima [A]** time = 1.48079, size = 1019, normalized size = 1.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*d^2*f*q*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 2*d^2*f*q*log(d*x + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + (2*b*d*f*q*x - a*d*f*(p - 2*q) + b*c*f*p)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x^2 + 2*(a*b^2*c - a^2*b*d)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) + 1/4*(4*(p*q + q^2)*log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*d^2*f^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 2*(p*q - 2*q^2)*d^2*f^2*log(d*x + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - (b^2*c^2*f^2*p^2 - 2*(p^2 - 3*p*q)*a*b*c*d*f^2 + (p^2 - 6*p*q)*a^2*d^2*f^2 - 2*(b^2*d^2*f^2*p*q*x^2 + 2*a*b*d^2*f^2*p*q*x + a^2*d^2*f^2*p*q)*log(b*x + a)^2 + 4*(b^2*d^2*f^2*p*q*x^2 + 2*a*b*d^2*f^2*p*q*x + a^2*d^2*f^2*p*q)*log(b*x + a)*log(d*x + c) + 2*(b^2*d^2*f^2*q^2*x^2 + 2*a*b*d^2*f^2*q^2*x + a^2*d^2*f^2*q^2)*log(d*x + c)^2 + 6*(b^2*c*d*f^2*p*q - a*b*d^2*f^2*p*q)*x + 2*((p*q - 2*q^2)*b^2*d^2*f^2*x^2 + 2*(p*q - 2*q^2)*a*b*d^2*f^2*x + (p*q - 2*q^2)*a^2*d^2*f^2)*log(b*x + a))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x))*r^2/(b*f^2) - 1/2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((b*x + a)^2*b)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2}{b^3 x^3 + 3 ab^2 x^2 + 3 a^2 bx + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**3,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="giac")`

[Out] `integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^3, x)`

$$3.23 \quad \int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4} dx$$

**Optimal.** Leaf size=764

$$\frac{2d^3 p q r^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3} - \frac{2d^3 q^2 r^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{3b(bc-ad)^3} + \frac{2d^3 q r \log(a+bx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(bc-ad)^3} - \frac{2d^3 q r}{3b(bc-ad)^3}$$

[Out]  $(-2p^2r^2)/(27b(a+bx)^3) - (5d^3pqr^2)/(18b(bc-ad)(a+bx)^2) + (8d^2p^2qr^2)/(9b(bc-ad)^2(a+bx)) - (d^2q^2r^2)/(3b(bc-ad)^2(a+bx)) + (2d^3p^2qr^2 \text{Log}[a+bx])/(9b(bc-ad)^3) - (d^3q^2r^2 \text{Log}[a+bx])/(b(bc-ad)^3) - (d^3pqr^2 \text{Log}[a+bx]^2)/(3b(bc-ad)^3) - (2d^3pqr^2 \text{Log}[c+dx])/(9b(bc-ad)^3) + (d^3q^2r^2 \text{Log}[c+dx])/(b(bc-ad)^3) + (2d^3pqr^2 \text{Log}[-((d(a+bx))/(bc-ad))] \text{Log}[c+dx])/(3b(bc-ad)^3) + (d^3q^2r^2 \text{Log}[c+dx]^2)/(3b(bc-ad)^3) - (2d^3q^2r^2 \text{Log}[a+bx] \text{Log}[(b(c+dx))/(bc-ad)])/(3b(bc-ad)^3) - (2p^2r \text{Log}[e(f(a+bx))^p(c+dx)^q]^r)/(9b(a+bx)^3) - (dqr \text{Log}[e(f(a+bx))^p(c+dx)^q]^r)/(3b(bc-ad)(a+bx)^2) + (2d^2q^2r \text{Log}[e(f(a+bx))^p(c+dx)^q]^r)/(3b(bc-ad)^2(a+bx)) + (2d^3q^2r \text{Log}[a+bx] \text{Log}[e(f(a+bx))^p(c+dx)^q]^r)/(3b(bc-ad)^3) - (2d^3q^2r \text{Log}[c+dx] \text{Log}[e(f(a+bx))^p(c+dx)^q]^r)/(3b(bc-ad)^3) - \text{Log}[e(f(a+bx))^p(c+dx)^q]^r^2/(3b(a+bx)^3) - (2d^3q^2r^2 \text{PolyLog}[2, -((d(a+bx))/(bc-ad))])/(3b(bc-ad)^3) + (2d^3pqr^2 \text{PolyLog}[2, (b(c+dx))/(bc-ad)])/(3b(bc-ad)^3)$

**Rubi [A]** time = 0.606462, antiderivative size = 764, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 13, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$ , Rules used = {2498, 2495, 32, 44, 2514, 36, 31, 2494, 2390, 2301, 2394, 2393, 2391}

$$\frac{2d^3 p q r^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3} - \frac{2d^3 q^2 r^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{3b(bc-ad)^3} + \frac{2d^3 q r \log(a+bx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(bc-ad)^3} - \frac{2d^3 q r}{3b(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2/(a + b\*x)^4, x]

[Out]  $(-2p^2r^2)/(27b(a+bx)^3) - (5d^3pqr^2)/(18b(bc-ad)(a+bx)^2) + (8d^2p^2qr^2)/(9b(bc-ad)^2(a+bx)) - (d^2q^2r^2)/(3b(bc-ad)^2(a+bx)) + (2d^3p^2qr^2 \text{Log}[a+bx])/(9b(bc-ad)^3) -$

$$\begin{aligned} & (d^3 q^2 r^2 \text{Log}[a + b*x]) / (b*(b*c - a*d)^3) - (d^3 p*q*r^2 \text{Log}[a + b*x]^2) / (3*b*(b*c - a*d)^3) - (2*d^3 p*q*r^2 \text{Log}[c + d*x]) / (9*b*(b*c - a*d)^3) + \\ & (d^3 q^2 r^2 \text{Log}[c + d*x]) / (b*(b*c - a*d)^3) + (2*d^3 p*q*r^2 \text{Log}[-((d*(a + b*x)) / (b*c - a*d))] * \text{Log}[c + d*x]) / (3*b*(b*c - a*d)^3) + (d^3 q^2 r^2 \text{Log}[c + d*x]^2) / (3*b*(b*c - a*d)^3) - (2*d^3 q^2 r^2 \text{Log}[a + b*x] * \text{Log}[(b*(c + d*x)) / (b*c - a*d)]) / (3*b*(b*c - a*d)^3) - (2*p*r * \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]) / (9*b*(a + b*x)^3) - (d*q*r * \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]) / (3*b*(b*c - a*d)*(a + b*x)^2) + (2*d^2 q*r * \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]) / (3*b*(b*c - a*d)^2*(a + b*x)) + (2*d^3 q*r * \text{Log}[a + b*x] * \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]) / (3*b*(b*c - a*d)^3) - (2*d^3 q*r * \text{Log}[c + d*x] * \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]) / (3*b*(b*c - a*d)^3) - \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2 / (3*b*(a + b*x)^3) - (2*d^3 q^2 r^2 \text{PolyLog}[2, -((d*(a + b*x)) / (b*c - a*d))]) / (3*b*(b*c - a*d)^3) + (2*d^3 p*q*r^2 \text{PolyLog}[2, (b*(c + d*x)) / (b*c - a*d)]) / (3*b*(b*c - a*d)^3) \end{aligned}$$
**Rule 2498**

```
Int[Log[(e._)*((f._)*((a._) + (b._)*(x._))^(p._))*((c._) + (d._)*(x._))^(q._))^(r._)]^(s._)*((g._) + (h._)*(x._))^(m._), x_Symbol] := Simp[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^(s - 1))/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && NeQ[m, -1]
```

**Rule 2495**

```
Int[Log[(e._)*((f._)*((a._) + (b._)*(x._))^(p._))*((c._) + (d._)*(x._))^(q._))^(r._)]*((g._) + (h._)*(x._))^(m._), x_Symbol] := Simp[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m + 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)), Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

**Rule 32**

```
Int[((a._) + (b._)*(x._))^(m._), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

**Rule 44**

```
Int[((a._) + (b._)*(x._))^(m._))*((c._) + (d._)*(x._))^(n._), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
```

+ n + 2, 0])

### Rule 2514

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*(RFX\_), x\_Symbol] := With[{u = ExpandIntegrand[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]

### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 31

Int[((a\_.) + (b\_.)\*(x\_))^(n\_.), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 2494

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[(Log[g + h\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r])/h, x] + (-Dist[(b\*p\*r)/h, Int[Log[g + h\*x]/(a + b\*x), x], x] - Dist[(d\*q\*r)/h, Int[Log[g + h\*x]/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b\*c - a\*d, 0]

### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x)

, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4} dx &= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} + \frac{1}{3}(2pr) \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4} dx \\
 &= -\frac{2pr \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b(a+bx)^3} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} + \frac{(2dqr)}{3b(a+bx)^3} \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4} dx \\
 &= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{2pr \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b(a+bx)^3} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} + \frac{2d^2pqr^2}{9b(bc-ad)^2(a+bx)} + \frac{2d^3pqr^2 \log(a+bx)}{9b(bc-ad)^2(a+bx)} \\
 &= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{5d^2pqr^2}{18b(bc-ad)(a+bx)^2} + \frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)} + \frac{2d^3pqr^2 \log(a+bx)}{9b(bc-ad)^2(a+bx)} \\
 &= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{5d^2pqr^2}{18b(bc-ad)(a+bx)^2} + \frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)} - \frac{d^2q^2r^2}{3b(bc-ad)^2(a+bx)} \\
 &= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{5d^2pqr^2}{18b(bc-ad)(a+bx)^2} + \frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)} - \frac{d^2q^2r^2}{3b(bc-ad)^2(a+bx)}
 \end{aligned}$$

**Mathematica [A]** time = 2.73125, size = 1407, normalized size = 1.84

$$4c^3p^2r^2b^3 + 18cd^2q^2r^2x^2b^3 - 48cd^2pqr^2x^2b^3 - 18d^3q^2r^2x^3 \log^2(c + dx)b^3 + 18c^3 \log^2\left(e\left(f(a + bx)^p(c + dx)^q\right)^r\right)b^3 + 15c$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2/(a + b\*x)^4,x]

[Out] 
$$\begin{aligned} & -(4*b^3*c^3*p^2*r^2 - 12*a*b^2*c^2*d*p^2*r^2 + 12*a^2*b*c*d^2*p^2*r^2 - 4*a \\ & ^3*d^3*p^2*r^2 + 15*a*b^2*c^2*d*p*q*r^2 - 78*a^2*b*c*d^2*p*q*r^2 + 63*a^3*d \\ & ^3*p*q*r^2 + 18*a^2*b*c*d^2*q^2*r^2 - 18*a^3*d^3*q^2*r^2 + 15*b^3*c^2*d*p*q \\ & *r^2*x - 126*a*b^2*c*d^2*p*q*r^2*x + 111*a^2*b*d^3*p*q*r^2*x + 36*a*b^2*c*d \\ & ^2*q^2*r^2*x - 36*a^2*b*d^3*q^2*r^2*x - 48*b^3*c*d^2*p*q*r^2*x^2 + 48*a*b^2 \\ & *d^3*p*q*r^2*x^2 + 18*b^3*c*d^2*q^2*r^2*x^2 - 18*a*b^2*d^3*q^2*r^2*x^2 + 18 \\ & *d^3*p*q*r^2*(a + b*x)^3*\text{Log}[a + b*x]^2 + 12*a^3*d^3*p*q*r^2*\text{Log}[c + d*x] - \\ & 54*a^3*d^3*q^2*r^2*\text{Log}[c + d*x] + 36*a^2*b*d^3*p*q*r^2*x*\text{Log}[c + d*x] - 16 \\ & 2*a^2*b*d^3*q^2*r^2*x*\text{Log}[c + d*x] + 36*a*b^2*d^3*p*q*r^2*x^2*\text{Log}[c + d*x] \\ & - 162*a*b^2*d^3*q^2*r^2*x^2*\text{Log}[c + d*x] + 12*b^3*d^3*p*q*r^2*x^3*\text{Log}[c + d \\ & *x] - 54*b^3*d^3*q^2*r^2*x^3*\text{Log}[c + d*x] - 18*a^3*d^3*q^2*r^2*\text{Log}[c + d*x] \\ & ^2 - 54*a^2*b*d^3*q^2*r^2*x*\text{Log}[c + d*x]^2 - 54*a*b^2*d^3*q^2*r^2*x^2*\text{Log}[c \\ & + d*x]^2 - 18*b^3*d^3*q^2*r^2*x^3*\text{Log}[c + d*x]^2 + 12*b^3*c^3*p*r*\text{Log}[e*(f \\ & *(a + b*x)^p*(c + d*x)^q)^r] - 36*a*b^2*c^2*d*p*r*\text{Log}[e*(f*(a + b*x)^p*(c + \\ & d*x)^q)^r] + 36*a^2*b*c*d^2*p*r*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 12* \\ & a^3*d^3*p*r*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*a*b^2*c^2*d*q*r*\text{Log}[e \\ & *(f*(a + b*x)^p*(c + d*x)^q)^r] - 72*a^2*b*c*d^2*q*r*\text{Log}[e*(f*(a + b*x)^p*( \\ & c + d*x)^q)^r] + 54*a^3*d^3*q*r*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*b \\ & ^3*c^2*d*q*r*x*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 108*a*b^2*c*d^2*q*r*x \\ & *\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 90*a^2*b*d^3*q*r*x*\text{Log}[e*(f*(a + b \\ & x)^p*(c + d*x)^q)^r] - 36*b^3*c*d^2*q*r*x^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^ \\ & q)^r] + 36*a*b^2*d^3*q*r*x^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 36*a^3* \\ & d^3*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 108*a^2*b*d^3*q \\ & *r*x*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 108*a*b^2*d^3*q*r* \\ & x^2*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 36*b^3*d^3*q*r*x^3* \\ & \text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*b^3*c^3*\text{Log}[e*(f*(a \\ & + b*x)^p*(c + d*x)^q)^r]^2 - 54*a*b^2*c^2*d*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^ \\ & q)^r]^2 + 54*a^2*b*c*d^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 18*a^3*d^3 \\ & *\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 6*d^3*q*r*(a + b*x)^3*\text{Log}[a + b \\ & x]*(2*p*r - 9*q*r + 6*p*r*\text{Log}[c + d*x] - 6*(p + q)*r*\text{Log}[(b*(c + d*x))/(b*c \\ & - a*d)] + 6*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + 36*d^3*q*(p + q)*r^2*( \\ & a + b*x)^3*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]/(54*b*(b*c - a*d)^3*(a \\ & + b*x)^3) \end{aligned}$$



**Maple [F]** time = 0.414, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)\right)^2}{(bx+a)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a)^4,x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a)^4,x)

**Maxima [A]** time = 1.76099, size = 1690, normalized size = 2.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a)^4,x, algorithm="maxima")

[Out] 1/9\*(6\*d^3\*f\*q\*log(b\*x + a)/(b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3) - 6\*d^3\*f\*q\*log(d\*x + c)/(b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3) + (6\*b^2\*d^2\*f\*q\*x^2 + a\*b\*c\*d\*f\*(4\*p - 3\*q) - a^2\*d^2\*f\*(2\*p - 9\*q) - 2\*b^2\*c^2\*f\*p - 3\*(b^2\*c\*d\*f\*q - 5\*a\*b\*d^2\*f\*q)\*x)/(a^3\*b^2\*c^2 - 2\*a^4\*b\*c\*d + a^5\*d^2 + (b^5\*c^2 - 2\*a\*b^4\*c\*d + a^2\*b^3\*d^2)\*x^3 + 3\*(a\*b^4\*c^2 - 2\*a^2\*b^3\*c\*d + a^3\*b^2\*d^2)\*x^2 + 3\*(a^2\*b^3\*c^2 - 2\*a^3\*b^2\*c\*d + a^4\*b\*d^2)\*x)\*r\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r/e)/(b\*f) - 1/54\*(36\*(p\*q + q^2)\*(log(b\*x + a)\*log((b\*d\*x + a\*d)/(b\*c - a\*d) + 1) + dilog(-(b\*d\*x + a\*d)/(b\*c - a\*d)))\*d^3\*f^2/(b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3) + 6\*(2\*p\*q - 9\*q^2)\*d^3\*f^2\*log(d\*x + c)/(b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3) + (4\*b^3\*c^3\*f^2\*p^2 - 3\*(4\*p^2 - 5\*p\*q)\*a\*b^2\*c^2\*d\*f^2 + 6\*(2\*p^2 - 13\*p\*q + 3\*q^2)\*a^2\*b\*c\*d^2\*f^2 - (4\*p^2 - 63\*p\*q + 18\*q^2)\*a^3\*d^3\*f^2 - 6\*((8\*p\*q - 3\*q^2)\*b^3\*c\*d^2\*f^2 - (8\*p\*q - 3\*q^2)\*a\*b^2\*d^3\*f^2)\*x^2 + 18\*(b^3\*d^3\*f^2\*p\*q\*x^3 + 3\*a\*b^2\*d^3\*f^2\*p\*q\*x^2 + 3\*a^2\*b\*d^3\*f^2\*p\*q\*x + a^3\*d^3\*f^2\*p\*q)\*log(b\*x + a)^2 - 36\*(b^3\*d^3\*f^2\*p\*q\*x^3 + 3\*a\*b^2\*d^3\*f^2\*p\*q\*x^2 + 3\*a^2\*b\*d^3\*f^2\*p\*q\*x + a^3\*d^3\*f^2\*p\*q)\*log(b\*x + a)\*log(d\*x + c) - 18\*(b^3\*d^3\*f^2\*q^2\*x^3 + 3\*a\*b^2\*d^3\*f^2\*q^2\*x^2 + 3\*a^2\*b\*d^3\*f^2\*q^2\*x + a^3\*d^3\*f^2\*q^2)\*log(d\*x + c)^2 + 3\*(5\*b^3\*c^2\*d\*f^2\*p\*q - 6\*(7\*p\*q - 2\*q^2)\*a\*b^2\*c\*d^2\*f^2 + (37\*p\*q - 12\*q^2)\*a^2\*b\*d^3\*f^2)\*x - 6\*((2\*p\*q - 9\*q^2)\*b^3\*d^3\*f^2\*x^3 + 3\*(2\*p\*q - 9\*q^2)\*a\*b^2\*d^3\*f^2\*x^2 + 3\*(2\*p\*q - 9\*q^2)\*a^2\*b\*d^3\*f^2\*x + (2\*p\*q - 9\*q^2)\*a^3\*d^3\*f^2)\*log(b\*x + a))

$$\frac{(a^3 b^3 c^3 - 3 a^4 b^2 c^2 d + 3 a^5 b c d^2 - a^6 d^3 + (b^6 c^3 - 3 a b^5 c^2 d + 3 a^2 b^4 c d^2 - a^3 b^3 d^3) x^3 + 3 (a b^5 c^3 - 3 a^2 b^4 c^2 d + 3 a^3 b^3 c d^2 - a^4 b^2 d^3) x^2 + 3 (a^2 b^4 c^3 - 3 a^3 b^3 c^2 d + 3 a^4 b^2 c d^2 - a^5 b d^3) x) r^2 / (b f^2) - 1/3 \log((b x + a)^p (d x + c)^q f)^r e)^2 / ((b x + a)^3 b)}{}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (b x + a)^p (d x + c)^q f \right)^r e \right)^2}{b^4 x^4 + 4 a b^3 x^3 + 6 a^2 b^2 x^2 + 4 a^3 b x + a^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**4,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( \left( (b x + a)^p (d x + c)^q f \right)^r e \right)^2}{(b x + a)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^4, x)
```

$$3.24 \quad \int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5} dx$$

**Optimal.** Leaf size=884

$$\frac{pqr^2 \log^2(a+bx)d^4}{4b(bc-ad)^4} - \frac{q^2r^2 \log^2(c+dx)d^4}{4b(bc-ad)^4} + \frac{11q^2r^2 \log(a+bx)d^4}{12b(bc-ad)^4} - \frac{pqr^2 \log(a+bx)d^4}{8b(bc-ad)^4} - \frac{11q^2r^2 \log(c+dx)d^4}{12b(bc-ad)^4} + \frac{pqr^2 \log(c+dx)d^4}{8b(bc-ad)^4}$$

```
[Out] -(p^2*r^2)/(32*b*(a + b*x)^4) - (7*d*p*q*r^2)/(72*b*(b*c - a*d)*(a + b*x)^3)
+ (3*d^2*p*q*r^2)/(16*b*(b*c - a*d)^2*(a + b*x)^2) - (d^2*q^2*r^2)/(12*b*
(b*c - a*d)^2*(a + b*x)^2) - (5*d^3*p*q*r^2)/(8*b*(b*c - a*d)^3*(a + b*x))
+ (5*d^3*q^2*r^2)/(12*b*(b*c - a*d)^3*(a + b*x)) - (d^4*p*q*r^2*Log[a + b*x
])/ (8*b*(b*c - a*d)^4) + (11*d^4*q^2*r^2*Log[a + b*x])/ (12*b*(b*c - a*d)^4)
+ (d^4*p*q*r^2*Log[a + b*x]^2)/ (4*b*(b*c - a*d)^4) + (d^4*p*q*r^2*Log[c +
d*x])/ (8*b*(b*c - a*d)^4) - (11*d^4*q^2*r^2*Log[c + d*x])/ (12*b*(b*c - a*d)^4)
- (d^4*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/ (2*b*(b*
c - a*d)^4) - (d^4*q^2*r^2*Log[c + d*x]^2)/ (4*b*(b*c - a*d)^4) + (d^4*q^2*r^
2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/ (2*b*(b*c - a*d)^4) - (p*r*
Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/ (8*b*(a + b*x)^4) - (d*q*r*Log[e*(f*(
a + b*x)^p*(c + d*x)^q]^r])/ (6*b*(b*c - a*d)*(a + b*x)^3) + (d^2*q*r*Log[e*
(f*(a + b*x)^p*(c + d*x)^q]^r])/ (4*b*(b*c - a*d)^2*(a + b*x)^2) - (d^3*q*r*
Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/ (2*b*(b*c - a*d)^3*(a + b*x)) - (d^4*
q*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/ (2*b*(b*c - a*d)^4)
+ (d^4*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/ (2*b*(b*c - a
*d)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2/ (4*b*(a + b*x)^4) + (d^4*q^
2*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/ (2*b*(b*c - a*d)^4) - (d^4*
p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/ (2*b*(b*c - a*d)^4)
```

**Rubi [A]** time = 0.737891, antiderivative size = 884, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 13, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$ , Rules used = {2498, 2495, 32, 44, 2514, 36, 31, 2494, 2390, 2301, 2394, 2393, 2391}

$$\frac{pqr^2 \log^2(a+bx)d^4}{4b(bc-ad)^4} - \frac{q^2r^2 \log^2(c+dx)d^4}{4b(bc-ad)^4} + \frac{11q^2r^2 \log(a+bx)d^4}{12b(bc-ad)^4} - \frac{pqr^2 \log(a+bx)d^4}{8b(bc-ad)^4} - \frac{11q^2r^2 \log(c+dx)d^4}{12b(bc-ad)^4} + \frac{pqr^2 \log(c+dx)d^4}{8b(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2/(a + b*x)^5,x]
```

```
[Out] -(p^2*r^2)/(32*b*(a + b*x)^4) - (7*d*p*q*r^2)/(72*b*(b*c - a*d)*(a + b*x)^3)
+ (3*d^2*p*q*r^2)/(16*b*(b*c - a*d)^2*(a + b*x)^2) - (d^2*q^2*r^2)/(12*b*
```

$$\begin{aligned}
& (b*c - a*d)^2*(a + b*x)^2) - (5*d^3*p*q*r^2)/(8*b*(b*c - a*d)^3*(a + b*x)) \\
& + (5*d^3*q^2*r^2)/(12*b*(b*c - a*d)^3*(a + b*x)) - (d^4*p*q*r^2*Log[a + b*x] \\
& ])/(8*b*(b*c - a*d)^4) + (11*d^4*q^2*r^2*Log[a + b*x])/(12*b*(b*c - a*d)^4) \\
& + (d^4*p*q*r^2*Log[a + b*x]^2)/(4*b*(b*c - a*d)^4) + (d^4*p*q*r^2*Log[c + \\
& d*x])/(8*b*(b*c - a*d)^4) - (11*d^4*q^2*r^2*Log[c + d*x])/(12*b*(b*c - a*d) \\
& ^4) - (d^4*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b*(b* \\
& c - a*d)^4) - (d^4*q^2*r^2*Log[c + d*x]^2)/(4*b*(b*c - a*d)^4) + (d^4*q^2*r \\
& ^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*b*(b*c - a*d)^4) - (p*r* \\
& Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)]/(8*b*(a + b*x)^4) - (d*q*r*Log[e*(f*( \\
& a + b*x)^p*(c + d*x)^q]^r)]/(6*b*(b*c - a*d)*(a + b*x)^3) + (d^2*q*r*Log[e* \\
& (f*(a + b*x)^p*(c + d*x)^q]^r)]/(4*b*(b*c - a*d)^2*(a + b*x)^2) - (d^3*q*r* \\
& Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)]/(2*b*(b*c - a*d)^3*(a + b*x)) - (d^4* \\
& q*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)]/(2*b*(b*c - a*d)^4) \\
& + (d^4*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)]/(2*b*(b*c - a \\
& *d)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2/(4*b*(a + b*x)^4) + (d^4*q^ \\
& 2*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(2*b*(b*c - a*d)^4) - (d^4* \\
& p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(2*b*(b*c - a*d)^4)
\end{aligned}$$

#### Rule 2498

$$\begin{aligned}
& \text{Int}[\text{Log}[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.)) \\
& ^{(r_.)}]^{(s_.)*((g_.) + (h_.)*(x_.))^{(m_.)}, x\_Symbol] \text{ :> } \text{Simp}[(g + h*x)^{(m + \\
& 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] + (-\text{Dist}[(b*p*r* \\
& s)/(h*(m + 1)), \text{Int}[(g + h*x)^{(m + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r} \\
& ^{(s - 1))}/(a + b*x), x], x] - \text{Dist}[(d*q*r*s)/(h*(m + 1)), \text{Int}[(g + h*x)^{(m \\
& + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(c + d*x), x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, m, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IG} \\
& \text{tQ}[s, 0] \&\& \text{NeQ}[m, -1]
\end{aligned}$$

#### Rule 2495

$$\begin{aligned}
& \text{Int}[\text{Log}[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.)) \\
& ^{(r_.)}]^{(s_.)*((g_.) + (h_.)*(x_.))^{(m_.)}, x\_Symbol] \text{ :> } \text{Simp}[(g + h*x)^{(m + 1)*\text{Lo} \\
& g[e*(f*(a + b*x)^p*(c + d*x)^q]^r)]/(h*(m + 1)), x] + (-\text{Dist}[(b*p*r)/(h*(m \\
& + 1)), \text{Int}[(g + h*x)^{(m + 1)}/(a + b*x), x], x] - \text{Dist}[(d*q*r)/(h*(m + 1)), \\
& \text{Int}[(g + h*x)^{(m + 1)}/(c + d*x), x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, \\
& m, p, q, r\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m, -1]
\end{aligned}$$

#### Rule 32

$$\text{Int}[(a + b*x)^m, x\_Symbol] \text{ :> } \text{Simp}[(a + b*x)^{(m + 1)}/(b*(m + 1)), x] /; \text{FreeQ}[\{a, b, m\}, x] \&\& \text{NeQ}[m, -1]$$

#### Rule 44

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

### Rule 2514

```
Int[Log[(e_)*((f_)*((a_) + (b_)*(x_))^(p_)*((c_) + (d_)*(x_))^(q_))
^(r_)]^(s_)*(RFx_), x_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

### Rule 36

```
Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

### Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 2494

```
Int[Log[(e_)*((f_)*((a_) + (b_)*(x_))^(p_)*((c_) + (d_)*(x_))^(q_))
^(r_)]/((g_) + (h_)*(x_)), x_Symbol] := Simp[(Log[g + h*x]*Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

### Rule 2390

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_))^(p_)*((f_) + (g_
)*(x_))^(q_), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_) + Log[(c_)*(x_)]^(n_)]*(b_)/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)^5} dx &= -\frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{4b(a+bx)^4} + \frac{1}{2} (pr) \int \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)^5} dx + \\
 &= -\frac{pr \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{8b(a+bx)^4} - \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{4b(a+bx)^4} + \frac{(dqr) \int \left( \dots \right)}{\dots} \\
 &= -\frac{p^2 r^2}{32b(a+bx)^4} - \frac{pr \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{8b(a+bx)^4} - \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{4b(a+bx)^4} \\
 &= -\frac{p^2 r^2}{32b(a+bx)^4} - \frac{dpqr^2}{24b(bc-ad)(a+bx)^3} + \frac{d^2 pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{d^3 pq}{8b(bc-ad)} \\
 &= -\frac{p^2 r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2 pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{5d^3 pq}{8b(bc-ad)} \\
 &= -\frac{p^2 r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2 pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{d^2 q}{12b(bc-ad)} \\
 &= -\frac{p^2 r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2 pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{d^2 q}{12b(bc-ad)}
 \end{aligned}$$

**Mathematica [B]** time = 7.02508, size = 14573, normalized size = 16.49

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2/(a + b\*x)^5,x]

[Out] Result too large to show

**Maple [F]** time = 0.418, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)\right)^2}{(bx+a)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a)^5,x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a)^5,x)

**Maxima [B]** time = 2.1752, size = 2452, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a)^5,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/24*(12*d^4*f*q*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 \\ & - 4*a^3*b*c*d^3 + a^4*d^4) - 12*d^4*f*q*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d \\ & + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + (12*b^3*d^3*f*q*x^3 - \\ & a*b^2*c^2*d*f*(9*p - 4*q) + a^2*b*c*d^2*f*(9*p - 14*q) - a^3*d^3*f*(3*p - 2 \\ & 2*q) + 3*b^3*c^3*f*p - 6*(b^3*c*d^2*f*q - 7*a*b^2*d^3*f*q)*x^2 + 4*(b^3*c^2 \\ & *d*f*q - 5*a*b^2*c*d^2*f*q + 13*a^2*b*d^3*f*q)*x)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d \\ & + 3*a^6*b*c*d^2 - a^7*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 \\ & - a^3*b^4*d^3)*x^4 + 4*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a \end{aligned}$$



$$\begin{aligned}
&^4b^3d^3)x^3 + 6*(a^2b^5c^3 - 3a^3b^4c^2d + 3a^4b^3c^2d^2 - a^5b^2d^3)x^2 + 4*(a^3b^4c^3 - 3a^4b^3c^2d + 3a^5b^2c^2d^2 - a^6b^1d^3)x) \\
& *r*\log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) + 1/288*(144*(p*q + q^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + \text{dilog}(-(b*d*x + a*d)/(b*c - a*d))) \\
& *d^4*f^2/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 12*(3*p*q - 22*q^2)*d^4*f^2*\log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - (9*b^4*c^4*f^2*p^2 - 4*(9*p^2 - 7*p*q)*a*b^3*c^3*d*f^2 + 6*(9*p^2 - 23*p*q + 4*q^2)*a^2*b^2*c^2*d^2*f^2 - 12*(3*p^2 - 31*p*q + 14*q^2)*a^3*b*c*d^3*f^2 + (9*p^2 - 26*2*p*q + 144*q^2)*a^4*d^4*f^2 + 60*((3*p*q - 2*q^2)*b^4*c*d^3*f^2 - (3*p*q - 2*q^2)*a*b^3*d^4*f^2)x^3 - 6*((9*p*q - 4*q^2)*b^4*c^2*d^2*f^2 - 4*(27*p*q - 17*q^2)*a*b^3*c*d^3*f^2 + (99*p*q - 64*q^2)*a^2*b^2*d^4*f^2)x^2 - 72*(b^4*d^4*f^2*p*q*x^4 + 4*a*b^3*d^4*f^2*p*q*x^3 + 6*a^2*b^2*d^4*f^2*p*q*x^2 + 4*a^3*b*d^4*f^2*p*q*x + a^4*d^4*f^2*p*q) * \log(b*x + a)^2 + 144*(b^4*d^4*f^2*p*q*x^4 + 4*a*b^3*d^4*f^2*p*q*x^3 + 6*a^2*b^2*d^4*f^2*p*q*x^2 + 4*a^3*b*d^4*f^2*p*q*x + a^4*d^4*f^2*p*q) * \log(b*x + a) * \log(d*x + c) + 72*(b^4*d^4*f^2*q^2*x^4 + 4*a*b^3*d^4*f^2*q^2*x^3 + 6*a^2*b^2*d^4*f^2*q^2*x^2 + 4*a^3*b*d^4*f^2*q^2*x + a^4*d^4*f^2*q^2) * \log(d*x + c)^2 + 4*(7*b^4*c^3*d*f^2*p*q - 12*(4*p*q - q^2)*a*b^3*c^2*d^2*f^2 + 6*(35*p*q - 19*q^2)*a^2*b^2*c*d^3*f^2 - (169*p*q - 102*q^2)*a^3*b*d^4*f^2)x + 12*((3*p*q - 22*q^2)*b^4*d^4*f^2*x^4 + 4*(3*p*q - 22*q^2)*a*b^3*d^4*f^2*x^3 + 6*(3*p*q - 22*q^2)*a^2*b^2*d^4*f^2*x^2 + 4*(3*p*q - 22*q^2)*a^3*b*d^4*f^2*x + (3*p*q - 22*q^2)*a^4*d^4*f^2) * \log(b*x + a) / (a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)x^4 + 4*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)x) * r^2 / (b*f^2) - 1/4*\log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((b*x + a)^4*b)
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2}{b^5 x^5 + 5 a b^4 x^4 + 10 a^2 b^3 x^3 + 10 a^3 b^2 x^2 + 5 a^4 b x + a^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(b\*x+a)^5,x, algorithm="fricas")

[Out] integral(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2/(b^5\*x^5 + 5\*a\*b^4\*x^4 + 10

$*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5), x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**5,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="giac")`

[Out] `integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^5, x)`

### 3.25 $\int (g + hx)^4 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$

**Optimal.** Leaf size=334

$$\frac{prx(bg - ah)^4}{5b^4} - \frac{pr(g + hx)^2(bg - ah)^3}{10b^3h} - \frac{pr(g + hx)^3(bg - ah)^2}{15b^2h} - \frac{pr(bg - ah)^5 \log(a + bx)}{5b^5h} + \frac{(g + hx)^5 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{5h}$$

[Out]  $-\frac{(b*g - a*h)^4*p*r*x}{5*b^4} - \frac{(d*g - c*h)^4*q*r*x}{5*d^4} - \frac{(b*g - a*h)^3*p*r*(g + h*x)^2}{10*b^3*h} - \frac{(d*g - c*h)^3*q*r*(g + h*x)^2}{10*d^3*h} - \frac{(b*g - a*h)^2*p*r*(g + h*x)^3}{15*b^2*h} - \frac{(d*g - c*h)^2*q*r*(g + h*x)^3}{15*d^2*h} - \frac{(b*g - a*h)*p*r*(g + h*x)^4}{20*b*h} - \frac{(d*g - c*h)*q*r*(g + h*x)^4}{20*d*h} - \frac{p*r*(g + h*x)^5}{25*h} - \frac{q*r*(g + h*x)^5}{25*h} - \frac{(b*g - a*h)^5*p*r*\text{Log}[a + b*x]}{5*b^5*h} - \frac{(d*g - c*h)^5*q*r*\text{Log}[c + d*x]}{5*d^5*h} + \frac{(g + h*x)^5*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]}{5*h}$

**Rubi [A]** time = 0.185563, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {2495, 43}

$$\frac{prx(bg - ah)^4}{5b^4} - \frac{pr(g + hx)^2(bg - ah)^3}{10b^3h} - \frac{pr(g + hx)^3(bg - ah)^2}{15b^2h} - \frac{pr(bg - ah)^5 \log(a + bx)}{5b^5h} + \frac{(g + hx)^5 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{5h}$$

Antiderivative was successfully verified.

[In] Int[(g + h\*x)^4\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r], x]

[Out]  $-\frac{(b*g - a*h)^4*p*r*x}{5*b^4} - \frac{(d*g - c*h)^4*q*r*x}{5*d^4} - \frac{(b*g - a*h)^3*p*r*(g + h*x)^2}{10*b^3*h} - \frac{(d*g - c*h)^3*q*r*(g + h*x)^2}{10*d^3*h} - \frac{(b*g - a*h)^2*p*r*(g + h*x)^3}{15*b^2*h} - \frac{(d*g - c*h)^2*q*r*(g + h*x)^3}{15*d^2*h} - \frac{(b*g - a*h)*p*r*(g + h*x)^4}{20*b*h} - \frac{(d*g - c*h)*q*r*(g + h*x)^4}{20*d*h} - \frac{p*r*(g + h*x)^5}{25*h} - \frac{q*r*(g + h*x)^5}{25*h} - \frac{(b*g - a*h)^5*p*r*\text{Log}[a + b*x]}{5*b^5*h} - \frac{(d*g - c*h)^5*q*r*\text{Log}[c + d*x]}{5*d^5*h} + \frac{(g + h*x)^5*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]}{5*h}$

#### Rule 2495

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.))\*((c\_.) + (d\_.)\*(x\_.))^(q\_.))]^(r\_.)]\*((g\_.) + (h\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((g + h\*x)^(m + 1)\*Lo

```
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m + 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)), Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned} \int (g + hx)^4 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(g + hx)^5 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{5h} - \frac{(bpr) \int \frac{(g+hx)^5}{a+bx} dx}{5h} - \frac{(dqr) \int \frac{(g+hx)^5}{c+dx} dx}{5h} \\ &= \frac{(g + hx)^5 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{5h} - \frac{(bpr) \int \left( \frac{h(bg-ah)^4}{b^5} + \frac{(bg-ah)^5}{b^5(a+bx)} + \frac{h^2}{b^5} \right) dx}{5h} \\ &= -\frac{(bg-ah)^4 prx}{5b^4} - \frac{(dg-ch)^4 qrx}{5d^4} - \frac{(bg-ah)^3 pr(g+hx)^2}{10b^3h} - \frac{(dg-ch)^3 qrx}{10d^3h} \end{aligned}$$

**Mathematica [A]** time = 0.351812, size = 275, normalized size = 0.82

$$\frac{pr(30b^2(g+hx)^2(bg-ah)^3 + 20b^3(g+hx)^3(bg-ah)^2 + 15b^4(g+hx)^4(bg-ah) + 60bhx(bg-ah)^4 + 60(bg-ah)^5 \log(a+bx) + 12b^5(g+hx)^5)}{60b^5} + \frac{(g+hx)^5 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{5h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-(p*r*(60*b*h*(b*g - a*h)^4*x + 30*b^2*(b*g - a*h)^3*(g + h*x)^2 + 20*b^3*(b*g - a*h)^2*(g + h*x)^3 + 15*b^4*(b*g - a*h)*(g + h*x)^4 + 12*b^5*(g + h*x)^5 + 60*(b*g - a*h)^5*Log[a + b*x]))/(60*b^5) - (q*r*(60*d*h*(d*g - c*h)^4*x + 30*d^2*(d*g - c*h)^3*(g + h*x)^2 + 20*d^3*(d*g - c*h)^2*(g + h*x)^3 + 15*d^4*(d*g - c*h)*(g + h*x)^4 + 12*d^5*(g + h*x)^5 + 60*(d*g - c*h)^5*Log[c + d*x]))/(60*d^5) + (g + h*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(5*h)
```

---

**Maple [F]** time = 0.393, size = 0, normalized size = 0.

$$\int (hx + g)^4 \ln\left(e\left(f(bx + a)^p(dx + c)^q\right)^r\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)^4\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r), x)

[Out] int((h\*x+g)^4\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r), x)

---

**Maxima [B]** time = 1.20345, size = 842, normalized size = 2.52

$$\frac{1}{5} (h^4x^5 + 5gh^3x^4 + 10g^2h^2x^3 + 10g^3hx^2 + 5g^4x) \log\left(\left((bx + a)^p(dx + c)^qf\right)^r e\right) + \frac{r\left(\frac{60(5ab^4fg^4p - 10a^2b^3fg^3hp + 10a^3b^2fg^2h^2p}{b^5}\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^4\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r), x, algorithm="maxima")

[Out] 1/5\*(h^4\*x^5 + 5\*g\*h^3\*x^4 + 10\*g^2\*h^2\*x^3 + 10\*g^3\*h\*x^2 + 5\*g^4\*x)\*log((  
 (b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e) + 1/300\*r\*(60\*(5\*a\*b^4\*f\*g^4\*p - 10\*a^2\*b^3  
 \*f\*g^3\*h\*p + 10\*a^3\*b^2\*f\*g^2\*h^2\*p - 5\*a^4\*b\*f\*g\*h^3\*p + a^5\*f\*h^4\*p)\*log(  
 b\*x + a)/b^5 + 60\*(5\*c\*d^4\*f\*g^4\*q - 10\*c^2\*d^3\*f\*g^3\*h\*q + 10\*c^3\*d^2\*f\*g^2  
 \*h^2\*q - 5\*c^4\*d\*f\*g\*h^3\*q + c^5\*f\*h^4\*q)\*log(d\*x + c)/d^5 - (12\*b^4\*d^4\*f  
 \*h^4\*(p + q)\*x^5 - 15\*(a\*b^3\*d^4\*f\*h^4\*p - (5\*d^4\*f\*g\*h^3\*(p + q) - c\*d^3\*f  
 \*h^4\*q)\*b^4)\*x^4 - 20\*(5\*a\*b^3\*d^4\*f\*g\*h^3\*p - a^2\*b^2\*d^4\*f\*h^4\*p - (10\*d^4  
 \*f\*g^2\*h^2\*(p + q) - 5\*c\*d^3\*f\*g\*h^3\*q + c^2\*d^2\*f\*h^4\*q)\*b^4)\*x^3 - 30\*(1  
 0\*a\*b^3\*d^4\*f\*g^2\*h^2\*p - 5\*a^2\*b^2\*d^4\*f\*g\*h^3\*p + a^3\*b\*d^4\*f\*h^4\*p - (10  
 \*d^4\*f\*g^3\*h\*(p + q) - 10\*c\*d^3\*f\*g^2\*h^2\*q + 5\*c^2\*d^2\*f\*g\*h^3\*q - c^3\*d\*f  
 \*h^4\*q)\*b^4)\*x^2 - 60\*(10\*a\*b^3\*d^4\*f\*g^3\*h\*p - 10\*a^2\*b^2\*d^4\*f\*g^2\*h^2\*p  
 + 5\*a^3\*b\*d^4\*f\*g\*h^3\*p - a^4\*d^4\*f\*h^4\*p - (5\*d^4\*f\*g^4\*(p + q) - 10\*c\*d^3  
 \*f\*g^3\*h\*q + 10\*c^2\*d^2\*f\*g^2\*h^2\*q - 5\*c^3\*d\*f\*g\*h^3\*q + c^4\*f\*h^4\*q)\*b^4  
 \*x)/(b^4\*d^4))/f

---

**Fricas [B]** time = 1.54248, size = 1932, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/300*(12*(b^5*d^5*h^4*p + b^5*d^5*h^4*q)*r*x^5 + 15*((5*b^5*d^5*g*h^3 - a
*b^4*d^5*h^4)*p + (5*b^5*d^5*g*h^3 - b^5*c*d^4*h^4)*q)*r*x^4 + 20*((10*b^5*
d^5*g^2*h^2 - 5*a*b^4*d^5*g*h^3 + a^2*b^3*d^5*h^4)*p + (10*b^5*d^5*g^2*h^2
- 5*b^5*c*d^4*g*h^3 + b^5*c^2*d^3*h^4)*q)*r*x^3 + 30*((10*b^5*d^5*g^3*h - 1
0*a*b^4*d^5*g^2*h^2 + 5*a^2*b^3*d^5*g*h^3 - a^3*b^2*d^5*h^4)*p + (10*b^5*d^
5*g^3*h - 10*b^5*c*d^4*g^2*h^2 + 5*b^5*c^2*d^3*g*h^3 - b^5*c^3*d^2*h^4)*q)*
r*x^2 + 60*((5*b^5*d^5*g^4 - 10*a*b^4*d^5*g^3*h + 10*a^2*b^3*d^5*g^2*h^2 -
5*a^3*b^2*d^5*g*h^3 + a^4*b*d^5*h^4)*p + (5*b^5*d^5*g^4 - 10*b^5*c*d^4*g^3*
h + 10*b^5*c^2*d^3*g^2*h^2 - 5*b^5*c^3*d^2*g*h^3 + b^5*c^4*d*h^4)*q)*r*x -
60*(b^5*d^5*h^4*p*r*x^5 + 5*b^5*d^5*g*h^3*p*r*x^4 + 10*b^5*d^5*g^2*h^2*p*r*
x^3 + 10*b^5*d^5*g^3*h*p*r*x^2 + 5*b^5*d^5*g^4*p*r*x + (5*a*b^4*d^5*g^4 - 1
0*a^2*b^3*d^5*g^3*h + 10*a^3*b^2*d^5*g^2*h^2 - 5*a^4*b*d^5*g*h^3 + a^5*d^5*
h^4)*p*r)*log(b*x + a) - 60*(b^5*d^5*h^4*q*r*x^5 + 5*b^5*d^5*g*h^3*q*r*x^4
+ 10*b^5*d^5*g^2*h^2*q*r*x^3 + 10*b^5*d^5*g^3*h*q*r*x^2 + 5*b^5*d^5*g^4*q*r
*x + (5*b^5*c*d^4*g^4 - 10*b^5*c^2*d^3*g^3*h + 10*b^5*c^3*d^2*g^2*h^2 - 5*b
^5*c^4*d*g*h^3 + b^5*c^5*h^4)*q*r)*log(d*x + c) - 60*(b^5*d^5*h^4*x^5 + 5*b
^5*d^5*g*h^3*x^4 + 10*b^5*d^5*g^2*h^2*x^3 + 10*b^5*d^5*g^3*h*x^2 + 5*b^5*d^
5*g^4*x)*log(e) - 60*(b^5*d^5*h^4*r*x^5 + 5*b^5*d^5*g*h^3*r*x^4 + 10*b^5*d^
5*g^2*h^2*r*x^3 + 10*b^5*d^5*g^3*h*r*x^2 + 5*b^5*d^5*g^4*r*x)*log(f))/(b^5*
d^5)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

**Giac [B]** time = 1.52229, size = 1715, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^4\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/25*(h^4*p*r + h^4*q*r - 5*h^4*r*\log(f) - 5*h^4)*x^5 - 1/20*(5*b*d*g*h^3* \\ & p*r - a*d*h^4*p*r + 5*b*d*g*h^3*q*r - b*c*h^4*q*r - 20*b*d*g*h^3*r*\log(f) - \\ & 20*b*d*g*h^3)*x^4/(b*d) + 1/5*(h^4*p*r*x^5 + 5*g*h^3*p*r*x^4 + 10*g^2*h^2* \\ & p*r*x^3 + 10*g^3*h*p*r*x^2 + 5*g^4*p*r*x)*\log(b*x + a) + 1/5*(h^4*q*r*x^5 + \\ & 5*g*h^3*q*r*x^4 + 10*g^2*h^2*q*r*x^3 + 10*g^3*h*q*r*x^2 + 5*g^4*q*r*x)*\log \\ & (d*x + c) - 1/15*(10*b^2*d^2*g^2*h^2*p*r - 5*a*b*d^2*g*h^3*p*r + a^2*d^2*h^ \\ & 4*p*r + 10*b^2*d^2*g^2*h^2*q*r - 5*b^2*c*d*g*h^3*q*r + b^2*c^2*h^4*q*r - 30 \\ & *b^2*d^2*g^2*h^2*r*\log(f) - 30*b^2*d^2*g^2*h^2)*x^3/(b^2*d^2) - 1/10*(10*b^ \\ & 3*d^3*g^3*h*p*r - 10*a*b^2*d^3*g^2*h^2*p*r + 5*a^2*b*d^3*g*h^3*p*r - a^3*d^ \\ & 3*h^4*p*r + 10*b^3*d^3*g^3*h*q*r - 10*b^3*c*d^2*g^2*h^2*q*r + 5*b^3*c^2*d*g \\ & *h^3*q*r - b^3*c^3*h^4*q*r - 20*b^3*d^3*g^3*h*r*\log(f) - 20*b^3*d^3*g^3*h) \\ & *x^2/(b^3*d^3) - 1/5*(5*b^4*d^4*g^4*p*r - 10*a*b^3*d^4*g^3*h*p*r + 10*a^2*b^ \\ & 2*d^4*g^2*h^2*p*r - 5*a^3*b*d^4*g*h^3*p*r + a^4*d^4*h^4*p*r + 5*b^4*d^4*g^4 \\ & *q*r - 10*b^4*c*d^3*g^3*h*q*r + 10*b^4*c^2*d^2*g^2*h^2*q*r - 5*b^4*c^3*d*g* \\ & h^3*q*r + b^4*c^4*h^4*q*r - 5*b^4*d^4*g^4*r*\log(f) - 5*b^4*d^4*g^4)*x/(b^4* \\ & d^4) + 1/10*(5*a*b^4*d^5*g^4*p*r - 10*a^2*b^3*d^5*g^3*h*p*r + 10*a^3*b^2*d^ \\ & 5*g^2*h^2*p*r - 5*a^4*b*d^5*g*h^3*p*r + a^5*d^5*h^4*p*r + 5*b^5*c*d^4*g^4*q \\ & *r - 10*b^5*c^2*d^3*g^3*h*q*r + 10*b^5*c^3*d^2*g^2*h^2*q*r - 5*b^5*c^4*d*g* \\ & h^3*q*r + b^5*c^5*h^4*q*r)*\log(\text{abs}(b*d*x^2 + b*c*x + a*d*x + a*c))/(b^5*d^5 \\ & ) + 1/10*(5*a*b^5*c*d^5*g^4*p*r - 5*a^2*b^4*d^6*g^4*p*r - 10*a^2*b^4*c*d^5* \\ & g^3*h*p*r + 10*a^3*b^3*d^6*g^3*h*p*r + 10*a^3*b^3*c*d^5*g^2*h^2*p*r - 10*a^ \\ & 4*b^2*d^6*g^2*h^2*p*r - 5*a^4*b^2*c*d^5*g*h^3*p*r + 5*a^5*b*d^6*g*h^3*p*r + \\ & a^5*b*c*d^5*h^4*p*r - a^6*d^6*h^4*p*r - 5*b^6*c^2*d^4*g^4*q*r + 5*a*b^5*c* \\ & d^5*g^4*q*r + 10*b^6*c^3*d^3*g^3*h*q*r - 10*a*b^5*c^2*d^4*g^3*h*q*r - 10*b^ \\ & 6*c^4*d^2*g^2*h^2*q*r + 10*a*b^5*c^3*d^3*g^2*h^2*q*r + 5*b^6*c^5*d*g*h^3*q* \\ & r - 5*a*b^5*c^4*d^2*g*h^3*q*r - b^6*c^6*h^4*q*r + a*b^5*c^5*d*h^4*q*r)*\log( \\ & \text{abs}((2*b*d*x + b*c + a*d - \text{abs}(-b*c + a*d))/(2*b*d*x + b*c + a*d + \text{abs}(-b*c \\ & + a*d))))/(b^5*d^5*\text{abs}(-b*c + a*d)) \end{aligned}$$

### 3.26 $\int (g + hx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$

**Optimal.** Leaf size=276

$$\frac{prx(bg - ah)^3}{4b^3} - \frac{pr(g + hx)^2(bg - ah)^2}{8b^2h} - \frac{pr(bg - ah)^4 \log(a + bx)}{4b^4h} + \frac{(g + hx)^4 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4h} - \frac{pr(g + h$$

[Out]  $-\frac{(b^3g - a^3h)^3 p r x}{4b^3} - \frac{(d^3g - c^3h)^3 q r x}{4d^3} - \frac{(b^2g - a^2h)^2 p r (g + hx)^2}{8b^2h} - \frac{(d^2g - c^2h)^2 q r (g + hx)^2}{8d^2h} - \frac{(b^2g - a^2h) p r (g + hx)^3}{12bh} - \frac{(d^2g - c^2h) q r (g + hx)^3}{12dh} - \frac{p r (g + hx)^4}{16h} - \frac{q r (g + hx)^4}{16h} - \frac{(b^4g - a^4h)^4 p r \text{Log}[a + bx]}{4b^4h} - \frac{(d^4g - c^4h)^4 q r \text{Log}[c + dx]}{4d^4h} + \frac{(g + hx)^4 \text{Log}[e (f(a + bx)^p (c + dx)^q)^r]}{4h}$

**Rubi [A]** time = 0.127419, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {2495, 43}

$$\frac{prx(bg - ah)^3}{4b^3} - \frac{pr(g + hx)^2(bg - ah)^2}{8b^2h} - \frac{pr(bg - ah)^4 \log(a + bx)}{4b^4h} + \frac{(g + hx)^4 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4h} - \frac{pr(g + h$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(g + hx)^3 \text{Log}[e (f(a + bx)^p (c + dx)^q)^r], x]$

[Out]  $-\frac{(b^3g - a^3h)^3 p r x}{4b^3} - \frac{(d^3g - c^3h)^3 q r x}{4d^3} - \frac{(b^2g - a^2h)^2 p r (g + hx)^2}{8b^2h} - \frac{(d^2g - c^2h)^2 q r (g + hx)^2}{8d^2h} - \frac{(b^2g - a^2h) p r (g + hx)^3}{12bh} - \frac{(d^2g - c^2h) q r (g + hx)^3}{12dh} - \frac{p r (g + hx)^4}{16h} - \frac{q r (g + hx)^4}{16h} - \frac{(b^4g - a^4h)^4 p r \text{Log}[a + bx]}{4b^4h} - \frac{(d^4g - c^4h)^4 q r \text{Log}[c + dx]}{4d^4h} + \frac{(g + hx)^4 \text{Log}[e (f(a + bx)^p (c + dx)^q)^r]}{4h}$

**Rule 2495**

$\text{Int}[\text{Log}[(e \cdot (f \cdot (a \cdot (b \cdot (x))^p) + (c \cdot (d \cdot (x))^q))]^r) \cdot (g \cdot (h \cdot (x))^m), x\_Symbol] \rightarrow \text{Simp}[(g + hx)^{m+1} \text{Log}[e (f(a + bx)^p (c + dx)^q)^r] / (h(m+1)), x] + (-\text{Dist}[(b^p r) / (h(m+1)), \text{Int}[(g + hx)^{m+1} / (a + bx), x], x] - \text{Dist}[(d^q r) / (h(m+1)), \text{Int}[(g + hx)^{m+1} / (c + dx), x], x]) /;$  FreeQ[{a, b, c, d, e, f, g, h,



$m, p, q, r\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m, -1]$

### Rule 43

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$

### Rubi steps

$$\begin{aligned} \int (g + hx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(g + hx)^4 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4h} - \frac{(bpr) \int \frac{(g+hx)^4}{a+bx} dx}{4h} - \frac{(dqr) \int \frac{(g+hx)^4}{c+dx} dx}{4h} \\ &= \frac{(g + hx)^4 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4h} - \frac{(bpr) \int \left( \frac{h(bg-ah)^3}{b^4} + \frac{(bg-ah)^4}{b^4(a+bx)} + \frac{(bg-ah)^5}{b^4(a+bx)^2} \right) dx}{4h} \\ &= -\frac{(bg-ah)^3 prx}{4b^3} - \frac{(dg-ch)^3 qrx}{4d^3} - \frac{(bg-ah)^2 pr(g+hx)^2}{8b^2h} - \frac{(dg-ch)^2 q(g+hx)^2}{8d^2h} \end{aligned}$$

**Mathematica [A]** time = 0.310036, size = 231, normalized size = 0.84

$$\frac{1}{12} r \left( -\frac{p(6b^2(g+hx)^2(bg-ah)^2+4b^3(g+hx)^3(bg-ah)+12bhx(bg-ah)^3+12(bg-ah)^4 \log(a+bx)+3b^4(g+hx)^4)}{b^4} - \frac{q(6d^2(g+hx)^2(dg-ch)^2+4d^3(g+hx)^3(dg-ch)+12d^4(g+hx)^4 \log(c+dx))}{d^4} \right) / 4h$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)^3\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r], x]

[Out] ((r\*(-((p\*(12\*b\*h\*(b\*g - a\*h)^3\*x + 6\*b^2\*(b\*g - a\*h)^2\*(g + h\*x)^2 + 4\*b^3\*(b\*g - a\*h)\*(g + h\*x)^3 + 3\*b^4\*(g + h\*x)^4 + 12\*(b\*g - a\*h)^4\*Log[a + b\*x]))/b^4) - (q\*(12\*d\*h\*(d\*g - c\*h)^3\*x + 6\*d^2\*(d\*g - c\*h)^2\*(g + h\*x)^2 + 4\*d^3\*(d\*g - c\*h)\*(g + h\*x)^3 + 3\*d^4\*(g + h\*x)^4 + 12\*(d\*g - c\*h)^4\*Log[c + d\*x]))/d^4))/12 + (g + h\*x)^4\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)/(4\*h)

**Maple [F]** time = 0.397, size = 0, normalized size = 0.

$$\int (hx + g)^3 \ln \left( e \left( f(bx + a)^p (dx + c)^q \right)^r \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)`

[Out] `int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)`

**Maxima [A]** time = 1.21627, size = 582, normalized size = 2.11

$$\frac{1}{4} (h^3 x^4 + 4 g h^2 x^3 + 6 g^2 h x^2 + 4 g^3 x) \log \left( (b x + a)^p (d x + c)^q f \right)^r e + \frac{r \left( \frac{12 (4 a b^3 f g^3 p - 6 a^2 b^2 f g^2 h p + 4 a^3 b f g h^2 p - a^4 f h^3 p) \log(b x + a)}{b^4} + \dots \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")`

[Out] `1/4*(h^3*x^4 + 4*g*h^2*x^3 + 6*g^2*h*x^2 + 4*g^3*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/48*r*(12*(4*a*b^3*f*g^3*p - 6*a^2*b^2*f*g^2*h*p + 4*a^3*b*f*g*h^2*p - a^4*f*h^3*p)*log(b*x + a)/b^4 + 12*(4*c*d^3*f*g^3*q - 6*c^2*d^2*f*g^2*h*q + 4*c^3*d*f*g*h^2*q - c^4*f*h^3*q)*log(d*x + c)/d^4 - (3*b^3*d^3*f*h^3*(p + q)*x^4 - 4*(a*b^2*d^3*f*h^3*p - (4*d^3*f*g*h^2*(p + q) - c*d^2*f*h^3*q)*b^3)*x^3 - 6*(4*a*b^2*d^3*f*g*h^2*p - a^2*b*d^3*f*h^3*p - (6*d^3*f*g^2*h*(p + q) - 4*c*d^2*f*g*h^2*q + c^2*d*f*h^3*q)*b^3)*x^2 - 12*(6*a*b^2*d^3*f*g^2*h*p - 4*a^2*b*d^3*f*g*h^2*p + a^3*d^3*f*h^3*p - (4*d^3*f*g^3*(p + q) - 6*c*d^2*f*g^2*h*q + 4*c^2*d*f*g*h^2*q - c^3*f*h^3*q)*b^3)*x)/(b^3*d^3))/f`

**Fricas [B]** time = 1.24885, size = 1377, normalized size = 4.99

$$\frac{3 (b^4 d^4 h^3 p + b^4 d^4 h^3 q) r x^4 + 4 \left( (4 b^4 d^4 g h^2 - a b^3 d^4 h^3) p + (4 b^4 d^4 g h^2 - b^4 c d^3 h^3) q \right) r x^3 + 6 \left( (6 b^4 d^4 g^2 h - 4 a b^3 d^4 g h^2 + a^2 b^3 d^4 h^3) p + (6 b^4 d^4 g^2 h - 4 a b^3 d^4 g h^2 + a^2 b^3 d^4 h^3) q \right) r x^2 + \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")`

[Out] `-1/48*(3*(b^4*d^4*h^3*p + b^4*d^4*h^3*q)*r*x^4 + 4*((4*b^4*d^4*g*h^2 - a*b^3*d^4*h^3)*p + (4*b^4*d^4*g*h^2 - b^4*c*d^3*h^3)*q)*r*x^3 + 6*((6*b^4*d^4*g^2*h - 4*a*b^3*d^4*g*h^2 + a^2*b^3*d^4*h^3)*p + (6*b^4*d^4*g^2*h - 4*a*b^3*d^4*g*h^2 + a^2*b^3*d^4*h^3)*q)*r*x^2 + \dots`

$$d^3*g*h^2 + b^4*c^2*d^2*h^3)*q)*r*x^2 + 12*((4*b^4*d^4*g^3 - 6*a*b^3*d^4*g^2*h + 4*a^2*b^2*d^4*g*h^2 - a^3*b*d^4*h^3)*p + (4*b^4*d^4*g^3 - 6*b^4*c*d^3*g^2*h + 4*b^4*c^2*d^2*g*h^2 - b^4*c^3*d*h^3)*q)*r*x - 12*(b^4*d^4*h^3*p*r*x^4 + 4*b^4*d^4*g*h^2*p*r*x^3 + 6*b^4*d^4*g^2*h*p*r*x^2 + 4*b^4*d^4*g^3*p*r*x + (4*a*b^3*d^4*g^3 - 6*a^2*b^2*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a^4*d^4*h^3)*p*r)*\log(b*x + a) - 12*(b^4*d^4*h^3*q*r*x^4 + 4*b^4*d^4*g*h^2*q*r*x^3 + 6*b^4*d^4*g^2*h*q*r*x^2 + 4*b^4*d^4*g^3*q*r*x + (4*b^4*c*d^3*g^3 - 6*b^4*c^2*d^2*g^2*h + 4*b^4*c^3*d*g*h^2 - b^4*c^4*h^3)*q*r)*\log(d*x + c) - 12*(b^4*d^4*h^3*x^4 + 4*b^4*d^4*g*h^2*x^3 + 6*b^4*d^4*g^2*h*x^2 + 4*b^4*d^4*g^3*x)*\log(e) - 12*(b^4*d^4*h^3*r*x^4 + 4*b^4*d^4*g*h^2*r*x^3 + 6*b^4*d^4*g^2*h*r*x^2 + 4*b^4*d^4*g^3*r*x)*\log(f))/(b^4*d^4)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*3\*ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r),x)

[Out] Timed out

**Giac [B]** time = 1.32799, size = 1254, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r),x, algorithm="giac")

[Out] 
$$-1/16*(h^3*p*r + h^3*q*r - 4*h^3*r*\log(f) - 4*h^3)*x^4 - 1/12*(4*b*d*g*h^2*p*r - a*d*h^3*p*r + 4*b*d*g*h^2*q*r - b*c*h^3*q*r - 12*b*d*g*h^2*r*\log(f) - 12*b*d*g*h^2)*x^3/(b*d) + 1/4*(h^3*p*r*x^4 + 4*g*h^2*p*r*x^3 + 6*g^2*h*p*r*x^2 + 4*g^3*p*r*x)*\log(b*x + a) + 1/4*(h^3*q*r*x^4 + 4*g*h^2*q*r*x^3 + 6*g^2*h*q*r*x^2 + 4*g^3*q*r*x)*\log(d*x + c) - 1/8*(6*b^2*d^2*g^2*h*p*r - 4*a*b*d^2*g*h^2*p*r + a^2*d^2*h^3*p*r + 6*b^2*d^2*g^2*h*q*r - 4*b^2*c*d*g*h^2*q*r + b^2*c^2*h^3*q*r - 12*b^2*d^2*g^2*h*r*\log(f) - 12*b^2*d^2*g^2*h)*x^2/(b^2*d^2) - 1/4*(4*b^3*d^3*g^3*p*r - 6*a*b^2*d^3*g^2*h*p*r + 4*a^2*b*d^3*g*h^2*p*r - a^3*d^3*h^3*p*r + 4*b^3*d^3*g^3*q*r - 6*b^3*c*d^2*g^2*h*q*r + 4*b^3*$$

$$\begin{aligned}
& c^2*d*g*h^2*q*r - b^3*c^3*h^3*q*r - 4*b^3*d^3*g^3*r*\log(f) - 4*b^3*d^3*g^3) \\
& *x/(b^3*d^3) + 1/8*(4*a*b^3*d^4*g^3*p*r - 6*a^2*b^2*d^4*g^2*h*p*r + 4*a^3*b \\
& *d^4*g*h^2*p*r - a^4*d^4*h^3*p*r + 4*b^4*c*d^3*g^3*q*r - 6*b^4*c^2*d^2*g^2* \\
& h*q*r + 4*b^4*c^3*d*g*h^2*q*r - b^4*c^4*h^3*q*r)*\log(\text{abs}(b*d*x^2 + b*c*x + \\
& a*d*x + a*c))/(b^4*d^4) + 1/8*(4*a*b^4*c*d^4*g^3*p*r - 4*a^2*b^3*d^5*g^3*p* \\
& r - 6*a^2*b^3*c*d^4*g^2*h*p*r + 6*a^3*b^2*d^5*g^2*h*p*r + 4*a^3*b^2*c*d^4*g \\
& *h^2*p*r - 4*a^4*b*d^5*g*h^2*p*r - a^4*b*c*d^4*h^3*p*r + a^5*d^5*h^3*p*r - \\
& 4*b^5*c^2*d^3*g^3*q*r + 4*a*b^4*c*d^4*g^3*q*r + 6*b^5*c^3*d^2*g^2*h*q*r - 6 \\
& *a*b^4*c^2*d^3*g^2*h*q*r - 4*b^5*c^4*d*g*h^2*q*r + 4*a*b^4*c^3*d^2*g*h^2*q* \\
& r + b^5*c^5*h^3*q*r - a*b^4*c^4*d*h^3*q*r)*\log(\text{abs}((2*b*d*x + b*c + a*d - a \\
& \text{bs}(-b*c + a*d))/(2*b*d*x + b*c + a*d + \text{abs}(-b*c + a*d))))/(b^4*d^4*\text{abs}(-b*c \\
& + a*d))
\end{aligned}$$

$$3.27 \quad \int (g + hx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$$

**Optimal.** Leaf size=218

$$\frac{prx(bg - ah)^2}{3b^2} - \frac{pr(bg - ah)^3 \log(a + bx)}{3b^3h} + \frac{(g + hx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{3h} - \frac{pr(g + hx)^2 (bg - ah)}{6bh} - \frac{qrx(dg - ah)}{3d}$$

[Out]  $-\frac{(b^2 g - a^2 h)^2 p r x}{3 b^2} - \frac{(d^2 g - c^2 h)^2 q r x}{3 d^2} - \frac{(b^2 g - a^2 h) p r (g + h x)^2}{6 b^2 h} - \frac{(d^2 g - c^2 h) q r (g + h x)^2}{6 d^2 h} - \frac{p r (g + h x)^3}{9 h} - \frac{q r (g + h x)^3}{9 h} - \frac{(b^2 g - a^2 h)^3 p r \operatorname{Log}[a + b x]}{3 b^3 h} - \frac{(d^2 g - c^2 h)^3 q r \operatorname{Log}[c + d x]}{3 d^3 h} + \frac{(g + h x)^3 \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]}{3 h}$

**Rubi [A]** time = 0.0987066, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {2495, 43}

$$\frac{prx(bg - ah)^2}{3b^2} - \frac{pr(bg - ah)^3 \log(a + bx)}{3b^3h} + \frac{(g + hx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{3h} - \frac{pr(g + hx)^2 (bg - ah)}{6bh} - \frac{qrx(dg - ah)}{3d}$$

Antiderivative was successfully verified.

[In] Int[(g + h\*x)^2\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r], x]

[Out]  $-\frac{(b^2 g - a^2 h)^2 p r x}{3 b^2} - \frac{(d^2 g - c^2 h)^2 q r x}{3 d^2} - \frac{(b^2 g - a^2 h) p r (g + h x)^2}{6 b^2 h} - \frac{(d^2 g - c^2 h) q r (g + h x)^2}{6 d^2 h} - \frac{p r (g + h x)^3}{9 h} - \frac{q r (g + h x)^3}{9 h} - \frac{(b^2 g - a^2 h)^3 p r \operatorname{Log}[a + b x]}{3 b^3 h} - \frac{(d^2 g - c^2 h)^3 q r \operatorname{Log}[c + d x]}{3 d^3 h} + \frac{(g + h x)^3 \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]}{3 h}$

### Rule 2495

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.))\*((c\_.) + (d\_.)\*(x\_.))^(q\_.))^(r\_.)]\*((g\_.) + (h\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)]/(h\*(m + 1)), x] + (-Dist[(b\*p\*r)/(h\*(m + 1)), Int[(g + h\*x)^(m + 1)/(a + b\*x), x], x] - Dist[(d\*q\*r)/(h\*(m + 1)), Int[(g + h\*x)^(m + 1)/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1]

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned} \int (g + hx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(g + hx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{3h} - \frac{(bpr) \int \frac{(g+hx)^3}{a+bx} dx}{3h} - \frac{(dqr) \int \frac{(g+hx)^3}{c+dx} dx}{3h} \\ &= \frac{(g + hx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{3h} - \frac{(bpr) \int \left( \frac{h(bg-ah)^2}{b^3} + \frac{(bg-ah)^3}{b^3(a+bx)} + \frac{h^3}{b^3} \right) dx}{3h} \\ &= -\frac{(bg-ah)^2 prx}{3b^2} - \frac{(dg-ch)^2 qrx}{3d^2} - \frac{(bg-ah)pr(g+hx)^2}{6bh} - \frac{(dg-ch)qr(g+hx)^2}{6dh} \end{aligned}$$

**Mathematica [A]** time = 0.253959, size = 209, normalized size = 0.96

$$\frac{(g + hx)^3 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) - \frac{r(b(6a^2d^3h^3px - 3abd^3hp(g^2 + 6ghx + h^2x^2) + b^2d(6c^2h^3qx - 3cdhq(g^2 + 6ghx + h^2x^2) + d^2(p+q)(18g^2hx + 5g^3 + 9g^2h^2 + 6gh^2 + h^3)))}{6b^3d^3}}{3h}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], x]
```

```
[Out] (-r*(6*d^3*(b*g - a*h)^3*p*Log[a + b*x] + b*(6*a^2*d^3*h^3*p*x - 3*a*b*d^3
*h*p*(g^2 + 6*g*h*x + h^2*x^2) + b^2*d*(6*c^2*h^3*q*x - 3*c*d*h*q*(g^2 + 6*
g*h*x + h^2*x^2) + d^2*(p + q)*(5*g^3 + 18*g^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^
3)) + 6*b^2*(d*g - c*h)^3*q*Log[c + d*x]))/(6*b^3*d^3) + (g + h*x)^3*Log[e
*(f*(a + b*x)^p*(c + d*x)^q]^r)/(3*h)
```

**Maple [F]** time = 0.431, size = 0, normalized size = 0.

$$\int (hx + g)^2 \ln \left( e \left( f(bx + a)^p (dx + c)^q \right)^r \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r), x)
```

[Out]  $\int (h*x+g)^2*\ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r), x$

**Maxima [A]** time = 1.25796, size = 363, normalized size = 1.67

$$\frac{1}{3} (h^2x^3 + 3ghx^2 + 3g^2x) \log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right) + \frac{r\left(\frac{6(3ab^2fg^2p-3a^2bfg hp+a^3fh^2p)\log(bx+a)}{b^3} + \frac{6(3cd^2fg^2q-3c^2dfghq+c^3f)}{d^3}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((h*x+g)^2*\log(e*(f*(b*x+a)^p*(d*x+c)^q)^r), x, \text{algorithm}="maxima")$

[Out]  $\frac{1}{3}(h^2x^3 + 3g*hx^2 + 3g^2x)*\log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + \frac{1}{18}*r*(6*(3*a*b^2*f*g^2*p - 3*a^2*b*f*g*h*p + a^3*f*h^2*p)*\log(b*x + a)/b^3 + 6*(3*c*d^2*f*g^2*q - 3*c^2*d*f*g*h*q + c^3*f*h^2*q)*\log(d*x + c)/d^3 - (2*b^2*d^2*f*h^2*(p + q)*x^3 - 3*(a*b*d^2*f*h^2*p - (3*d^2*f*g*h*(p + q) - c*d*f*h^2*q)*b^2)*x^2 - 6*(3*a*b*d^2*f*g*h*p - a^2*d^2*f*h^2*p - (3*d^2*f*g^2*(p + q) - 3*c*d*f*g*h*q + c^2*f*h^2*q)*b^2)*x)/(b^2*d^2))/f$

**Fricas [B]** time = 1.16108, size = 917, normalized size = 4.21

$$\frac{2(b^3d^3h^2p + b^3d^3h^2q)rx^3 + 3((3b^3d^3gh - ab^2d^3h^2)p + (3b^3d^3gh - b^3cd^2h^2)q)rx^2 + 6((3b^3d^3g^2 - 3ab^2d^3gh + a^2bd^3h^2)p + (3b^3d^3g^2 - 3ab^2d^3gh + a^2bd^3h^2)q)rx + 6((3b^3d^3g^2 - 3ab^2d^3gh + a^2bd^3h^2)p + (3b^3d^3g^2 - 3ab^2d^3gh + a^2bd^3h^2)q)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((h*x+g)^2*\log(e*(f*(b*x+a)^p*(d*x+c)^q)^r), x, \text{algorithm}="fricas")$

[Out]  $-1/18*(2*(b^3*d^3*h^2*p + b^3*d^3*h^2*q)*r*x^3 + 3*((3*b^3*d^3*g*h - a*b^2*d^3*h^2)*p + (3*b^3*d^3*g*h - b^3*c*d^2*h^2)*q)*r*x^2 + 6*((3*b^3*d^3*g^2 - 3*a*b^2*d^3*g*h + a^2*b*d^3*h^2)*p + (3*b^3*d^3*g^2 - 3*b^3*c*d^2*g*h + b^3*c^2*d*h^2)*q)*r*x - 6*(b^3*d^3*h^2*p*r*x^3 + 3*b^3*d^3*g*h*p*r*x^2 + 3*b^3*d^3*g^2*p*r*x + (3*a*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h + a^3*d^3*h^2)*p*r)*\log(b*x + a) - 6*(b^3*d^3*h^2*q*r*x^3 + 3*b^3*d^3*g*h*q*r*x^2 + 3*b^3*d^3*g^2*q*r*x + (3*b^3*c*d^2*g^2 - 3*b^3*c^2*d*g*h + b^3*c^3*h^2)*q*r)*\log(d*x + c) - 6*(b^3*d^3*h^2*x^3 + 3*b^3*d^3*g*h*x^2 + 3*b^3*d^3*g^2*x)*\log(e) - 6*(b^3*d^3*h^2*r*x^3 + 3*b^3*d^3*g*h*r*x^2 + 3*b^3*d^3*g^2*r*x)*\log(f))/(b^3*d^3)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*2\*ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r),x)

[Out] Timed out

---

**Giac [B]** time = 1.30521, size = 837, normalized size = 3.84

$$-\frac{1}{9}(h^2pr + h^2qr - 3h^2r \log(f) - 3h^2)x^3 + \frac{1}{3}(h^2prx^3 + 3ghprx^2 + 3g^2prx) \log(bx + a) + \frac{1}{3}(h^2qrx^3 + 3ghqrx^2 + 3g^2qr$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r),x, algorithm="giac")

[Out] 
$$-1/9*(h^2*p*r + h^2*q*r - 3*h^2*r*\log(f) - 3*h^2)*x^3 + 1/3*(h^2*p*r*x^3 + 3*g*h*p*r*x^2 + 3*g^2*p*r*x)*\log(b*x + a) + 1/3*(h^2*q*r*x^3 + 3*g*h*q*r*x^2 + 3*g^2*q*r*x)*\log(d*x + c) - 1/6*(3*b*d*g*h*p*r - a*d*h^2*p*r + 3*b*d*g*h*q*r - b*c*h^2*q*r - 6*b*d*g*h*r*\log(f) - 6*b*d*g*h)*x^2/(b*d) - 1/3*(3*b^2*d^2*g^2*p*r - 3*a*b*d^2*g*h*p*r + a^2*d^2*h^2*p*r + 3*b^2*d^2*g^2*q*r - 3*b^2*c*d*g*h*q*r + b^2*c^2*h^2*q*r - 3*b^2*d^2*g^2*r*\log(f) - 3*b^2*d^2*g^2)*x/(b^2*d^2) + 1/6*(3*a*b^2*d^3*g^2*p*r - 3*a^2*b*d^3*g*h*p*r + a^3*d^3*h^2*p*r + 3*b^3*c*d^2*g^2*q*r - 3*b^3*c^2*d*g*h*q*r + b^3*c^3*h^2*q*r)*\log(\text{abs}(b*d*x^2 + b*c*x + a*d*x + a*c))/(b^3*d^3) + 1/6*(3*a*b^3*c*d^3*g^2*p*r - 3*a^2*b^2*d^4*g^2*p*r - 3*a^2*b^2*c*d^3*g*h*p*r + 3*a^3*b*d^4*g*h*p*r + a^3*b*c*d^3*h^2*p*r - a^4*d^4*h^2*p*r - 3*b^4*c^2*d^2*g^2*q*r + 3*a*b^3*c*d^3*g^2*q*r + 3*b^4*c^3*d*g*h*q*r - 3*a*b^3*c^2*d^2*g*h*q*r - b^4*c^4*h^2*q*r + a*b^3*c^3*d*h^2*q*r)*\log(\text{abs}((2*b*d*x + b*c + a*d - \text{abs}(-b*c + a*d))/(2*b*d*x + b*c + a*d + \text{abs}(-b*c + a*d))))/(b^3*d^3*\text{abs}(-b*c + a*d))$$



### 3.28 $\int (g + hx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$

**Optimal.** Leaf size=160

$$\frac{pr(bg - ah)^2 \log(a + bx)}{2b^2h} + \frac{(g + hx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2h} - \frac{prx(bg - ah)}{2b} - \frac{qr(dg - ch)^2 \log(c + dx)}{2d^2h} - \frac{qrx(a + bx)}{2d}$$

[Out]  $-\frac{((b*g - a*h)*p*r*x)}{(2*b)} - \frac{((d*g - c*h)*q*r*x)}{(2*d)} - \frac{(p*r*(g + h*x)^2)}{(4*h)} - \frac{(q*r*(g + h*x)^2)}{(4*h)} - \frac{((b*g - a*h)^2*p*r*Log[a + b*x])}{(2*b^2*h)} - \frac{((d*g - c*h)^2*q*r*Log[c + d*x])}{(2*d^2*h)} + \frac{((g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)}{(2*h)}$

**Rubi [A]** time = 0.0687969, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$ , Rules used = {2495, 43}

$$\frac{pr(bg - ah)^2 \log(a + bx)}{2b^2h} + \frac{(g + hx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2h} - \frac{prx(bg - ah)}{2b} - \frac{qr(dg - ch)^2 \log(c + dx)}{2d^2h} - \frac{qrx(a + bx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(g + h\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r], x]

[Out]  $-\frac{((b*g - a*h)*p*r*x)}{(2*b)} - \frac{((d*g - c*h)*q*r*x)}{(2*d)} - \frac{(p*r*(g + h*x)^2)}{(4*h)} - \frac{(q*r*(g + h*x)^2)}{(4*h)} - \frac{((b*g - a*h)^2*p*r*Log[a + b*x])}{(2*b^2*h)} - \frac{((d*g - c*h)^2*q*r*Log[c + d*x])}{(2*d^2*h)} + \frac{((g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)}{(2*h)}$

#### Rule 2495

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_.))^p\_.)\*((c\_.) + (d\_.)\*(x\_.))^q\_.)]^(r\_.)]\*((g\_.) + (h\_.)\*(x\_.))^m\_.], x\_Symbol] := Simp[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)/(h\*(m + 1)), x] + (-Dist[(b\*p\*r)/(h\*(m + 1)), Int[(g + h\*x)^(m + 1)/(a + b\*x), x], x] - Dist[(d\*q\*r)/(h\*(m + 1)), Int[(g + h\*x)^(m + 1)/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_.))^m\_.)\*((c\_.) + (d\_.)\*(x\_.))^n\_.], x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n},

x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned} \int (g + hx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(g + hx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2h} - \frac{(bpr) \int \frac{(g+hx)^2}{a+bx} dx}{2h} - \frac{(dqr) \int \frac{(g+hx)^2}{c+dx} dx}{2h} \\ &= \frac{(g + hx)^2 \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2h} - \frac{(bpr) \int \left( \frac{h(bg-ah)}{b^2} + \frac{(bg-ah)^2}{b^2(a+bx)} + \frac{hg}{c+dx} \right) dx}{2h} \\ &= -\frac{(bg-ah)prx}{2b} - \frac{(dg-ch)qrx}{2d} - \frac{pr(g+hx)^2}{4h} - \frac{qr(g+hx)^2}{4h} - \frac{(bg-ah)^2 pr}{2b} \end{aligned}$$

**Mathematica [A]** time = 0.196375, size = 120, normalized size = 0.75

$$\frac{b \left( dx \left( r(-2adh p - 2bch q + bd(p + q)(4g + hx)) - 2bd(2g + hx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) \right) + 2bcqr(ch - 2dg) \log(c + dx) \right)}{4b^2 d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r], x]

[Out] -(2\*a\*d^2\*(-2\*b\*g + a\*h)\*p\*r\*Log[a + b\*x] + b\*(2\*b\*c\*(-2\*d\*g + c\*h)\*q\*r\*Log[c + d\*x] + d\*x\*(r\*(-2\*a\*d\*h\*p - 2\*b\*c\*h\*q + b\*d\*(p + q)\*(4\*g + h\*x)) - 2\*b\*d\*(2\*g + h\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)))/(4\*b^2\*d^2)

**Maple [F]** time = 0.174, size = 0, normalized size = 0.

$$\int (hx + g) \ln \left( e \left( f(bx + a)^p (dx + c)^q \right)^r \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r), x)

[Out] int((h\*x+g)\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r), x)

---

**Maxima [A]** time = 1.20219, size = 193, normalized size = 1.21

$$\frac{1}{2} (hx^2 + 2gx) \log\left(\left((bx + a)^p(dx + c)^q f\right)^r e\right) + \frac{r\left(\frac{2(2abfgp - a^2fhp)\log(bx+a)}{b^2} + \frac{2(2cdfgq - c^2fhq)\log(dx+c)}{d^2} - \frac{bdfh(p+q)x^2 - 2(adflp - bd^2fhp)}{bd}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r),x, algorithm="maxima")

[Out] 1/2\*(h\*x^2 + 2\*g\*x)\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e) + 1/4\*r\*(2\*(2\*a\*b\*f\*g\*p - a^2\*f\*h\*p)\*log(b\*x + a)/b^2 + 2\*(2\*c\*d\*f\*g\*q - c^2\*f\*h\*q)\*log(d\*x + c)/d^2 - (b\*d\*f\*h\*(p + q)\*x^2 - 2\*(a\*d\*f\*h\*p - (2\*d\*f\*g\*(p + q) - c\*f\*h\*q)\*b)\*x)/(b\*d))/f

---

**Fricas [A]** time = 1.20831, size = 524, normalized size = 3.28

$$\frac{(b^2d^2hp + b^2d^2hq)rx^2 + 2((2b^2d^2g - abd^2h)p + (2b^2d^2g - b^2cdh)q)rx - 2(b^2d^2hprx^2 + 2b^2d^2gprx + (2abd^2g - a^2d^2h)pr)}{(b^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r),x, algorithm="fricas")

[Out] -1/4\*((b^2\*d^2\*h\*p + b^2\*d^2\*h\*q)\*r\*x^2 + 2\*((2\*b^2\*d^2\*g - a\*b\*d^2\*h)\*p + (2\*b^2\*d^2\*g - b^2\*c\*d\*h)\*q)\*r\*x - 2\*(b^2\*d^2\*h\*p\*r\*x^2 + 2\*b^2\*d^2\*g\*p\*r\*x + (2\*a\*b\*d^2\*g - a^2\*d^2\*h)\*p\*r)\*log(b\*x + a) - 2\*(b^2\*d^2\*h\*q\*r\*x^2 + 2\*b^2\*d^2\*g\*q\*r\*x + (2\*b^2\*c\*d\*g - b^2\*c^2\*h)\*q\*r)\*log(d\*x + c) - 2\*(b^2\*d^2\*h\*x^2 + 2\*b^2\*d^2\*g\*x)\*log(e) - 2\*(b^2\*d^2\*h\*r\*x^2 + 2\*b^2\*d^2\*g\*r\*x)\*log(f))/(b^2\*d^2)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r),x)

[Out] Timed out

**Giac [B]** time = 1.31039, size = 479, normalized size = 2.99

$$-\frac{1}{4}(hpr + hqr - 2hr \log(f) - 2h)x^2 + \frac{1}{2}(hprx^2 + 2gprx) \log(bx + a) + \frac{1}{2}(hqr x^2 + 2gqrx) \log(dx + c) - \frac{(2bdgpr - ac)}{...}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r),x, algorithm="giac")

[Out] 
$$-1/4*(h*p*r + h*q*r - 2*h*r*\log(f) - 2*h)*x^2 + 1/2*(h*p*r*x^2 + 2*g*p*r*x) * \log(b*x + a) + 1/2*(h*q*r*x^2 + 2*g*q*r*x) * \log(d*x + c) - 1/2*(2*b*d*g*p*r - a*d*h*p*r + 2*b*d*g*q*r - b*c*h*q*r - 2*b*d*g*r*\log(f) - 2*b*d*g)*x/(b*d) + 1/4*(2*a*b*d^2*g*p*r - a^2*d^2*h*p*r + 2*b^2*c*d*g*q*r - b^2*c^2*h*q*r) * \log(\text{abs}(b*d*x^2 + b*c*x + a*d*x + a*c))/(b^2*d^2) + 1/4*(2*a*b^2*c*d^2*g*p*r - 2*a^2*b*d^3*g*p*r - a^2*b*c*d^2*h*p*r + a^3*d^3*h*p*r - 2*b^3*c^2*d*g*q*r + 2*a*b^2*c*d^2*g*q*r + b^3*c^3*h*q*r - a*b^2*c^2*d*h*q*r) * \log(\text{abs}((2*b*d*x + b*c + a*d - \text{abs}(-b*c + a*d))/(2*b*d*x + b*c + a*d + \text{abs}(-b*c + a*d))))/(b^2*d^2*\text{abs}(-b*c + a*d))$$

$$3.29 \quad \int \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$$

**Optimal.** Leaf size=61

$$\frac{(a + bx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{b} + \frac{qr(bc - ad) \log(c + dx)}{bd} + rx(-(p + q))$$

[Out]  $-\left((p + q)*r*x\right) + \left((b*c - a*d)*q*r*\text{Log}[c + d*x]\right)/(b*d) + \left((a + b*x)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]\right)/b$

**Rubi [A]** time = 0.0148794, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2487, 31, 8}

$$\frac{(a + bx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{b} + \frac{qr(bc - ad) \log(c + dx)}{bd} + rx(-(p + q))$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r], x]$

[Out]  $-\left((p + q)*r*x\right) + \left((b*c - a*d)*q*r*\text{Log}[c + d*x]\right)/(b*d) + \left((a + b*x)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]\right)/b$

### Rule 2487

$\text{Int}[\text{Log}[(e_*)*((f_*)*((a_*) + (b_*)*(x_))^{(p_*)}*((c_*) + (d_*)*(x_))^{(q_*)})^{(r_*)}]^{(s_*)}, x\_Symbol] \rightarrow \text{Simp}[\left((a + b*x)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r\right)^s/b, x] + (\text{Dist}[(q*r*s*(b*c - a*d))/b, \text{Int}[\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^{(s - 1)}/(c + d*x), x], x] - \text{Dist}[r*s*(p + q), \text{Int}[\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^{(s - 1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, p, q, r, s\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[p + q, 0] \ \&\& \ \text{IGtQ}[s, 0] \ \&\& \ \text{LtQ}[s, 4]$

### Rule 31

$\text{Int}[\left((a_*) + (b_*)*(x_)\right)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx &= \frac{(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} + \frac{((bc-ad)qr)\int\frac{1}{c+dx}dx}{b} - ((p+q)r)\int 1 dx \\ &= -(p+q)rx + \frac{(bc-ad)qr\log(c+dx)}{bd} + \frac{(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0628468, size = 57, normalized size = 0.93

$$x\left(\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - r(p+q)\right) + \frac{apr\log(a+bx)}{b} + \frac{cqr\log(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r], x]

[Out] (a\*p\*r\*Log[a + b\*x])/b + (c\*q\*r\*Log[c + d\*x])/d + x\*(-((p + q)\*r) + Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r])

**Maple [A]** time = 0.071, size = 61, normalized size = 1.

$$\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)x - rpx - rqx + \frac{rap\ln(bx+a)}{b} + \frac{rqc\ln(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r), x)

[Out] ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)\*x-r\*p\*x-r\*q\*x+r\*a\*p/b\*ln(b\*x+a)+r\*q\*c/d\*ln(d\*x+c)

**Maxima [A]** time = 1.15706, size = 101, normalized size = 1.66

$$x\log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right) - \frac{\left(bfp\left(\frac{x}{b} - \frac{a\log(bx+a)}{b^2}\right) + dfq\left(\frac{x}{d} - \frac{c\log(dx+c)}{d^2}\right)\right)r}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r),x, algorithm="maxima")

[Out] x\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e) - (b\*f\*p\*(x/b - a\*log(b\*x + a)/b^2) + d\*f\*q\*(x/d - c\*log(d\*x + c)/d^2))\*r/f

**Fricas [A]** time = 1.08855, size = 182, normalized size = 2.98

$$\frac{bdrx \log(f) + bdx \log(e) - (bdp + bdq)rx + (bdprx + adpr) \log(bx + a) + (bdqrx + bcqr) \log(dx + c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r),x, algorithm="fricas")

[Out] (b\*d\*r\*x\*log(f) + b\*d\*x\*log(e) - (b\*d\*p + b\*d\*q)\*r\*x + (b\*d\*p\*r\*x + a\*d\*p\*r)\*log(b\*x + a) + (b\*d\*q\*r\*x + b\*c\*q\*r)\*log(d\*x + c))/(b\*d)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r),x)

[Out] Timed out

**Giac [B]** time = 1.23847, size = 242, normalized size = 3.97

$$prx \log(bx + a) + qrx \log(dx + c) - (pr + qr - r \log(f) - 1)x + \frac{(adpr + bcqr) \log(|bdx^2 + bcx + adx + ac|)}{2bd} + \frac{(abcdpr)}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] p*r*x*log(b*x + a) + q*r*x*log(d*x + c) - (p*r + q*r - r*log(f) - 1)*x + 1/2*(a*d*p*r + b*c*q*r)*log(abs(b*d*x^2 + b*c*x + a*d*x + a*c))/(b*d) + 1/2*(a*b*c*d*p*r - a^2*d^2*p*r - b^2*c^2*q*r + a*b*c*d*q*r)*log(abs((2*b*d*x + b*c + a*d - abs(-b*c + a*d))/(2*b*d*x + b*c + a*d + abs(-b*c + a*d))))/(b*d*abs(-b*c + a*d))
```



$$3.30 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx} dx$$

**Optimal.** Leaf size=148

$$\frac{pr \operatorname{PolyLog}\left(2, \frac{b(g+hx)}{bg-ah}\right)}{h} - \frac{qr \operatorname{PolyLog}\left(2, \frac{d(g+hx)}{dg-ch}\right)}{h} + \frac{\log(g+hx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h} - \frac{pr \log(g+hx) \log\left(-\left(\frac{h(a+bx)}{b^2g-ah^2}\right)\right)}{h}$$

[Out]  $-\left(\frac{p*r*\operatorname{Log}\left[-\left(\frac{h*(a+b*x)}{b^2*g-a*h}\right)\right]*\operatorname{Log}[g+h*x]}{h}\right) - \left(\frac{q*r*\operatorname{Log}\left[-\left(\frac{h*(c+d*x)}{d*g-c*h}\right)\right]*\operatorname{Log}[g+h*x]}{h}\right) + \frac{\operatorname{Log}[e*(f*(a+b*x)^p*(c+d*x)^q]^r]*\operatorname{Log}[g+h*x]}{h} - \frac{p*r*\operatorname{PolyLog}\left[2, \left(\frac{b*(g+h*x)}{b^2*g-a*h}\right)\right]}{h} - \frac{q*r*\operatorname{PolyLog}\left[2, \left(\frac{d*(g+h*x)}{d*g-c*h}\right)\right]}{h}$

**Rubi [A]** time = 0.120007, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {2494, 2394, 2393, 2391}

$$\frac{pr \operatorname{PolyLog}\left(2, \frac{b(g+hx)}{bg-ah}\right)}{h} - \frac{qr \operatorname{PolyLog}\left(2, \frac{d(g+hx)}{dg-ch}\right)}{h} + \frac{\log(g+hx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h} - \frac{pr \log(g+hx) \log\left(-\left(\frac{h(a+bx)}{b^2g-ah^2}\right)\right)}{h}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\operatorname{Log}\left[e*(f*(a+b*x)^p*(c+d*x)^q)^r\right]/(g+h*x), x\right]$

[Out]  $-\left(\frac{p*r*\operatorname{Log}\left[-\left(\frac{h*(a+b*x)}{b^2*g-a*h}\right)\right]*\operatorname{Log}[g+h*x]}{h}\right) - \left(\frac{q*r*\operatorname{Log}\left[-\left(\frac{h*(c+d*x)}{d*g-c*h}\right)\right]*\operatorname{Log}[g+h*x]}{h}\right) + \frac{\operatorname{Log}[e*(f*(a+b*x)^p*(c+d*x)^q]^r]*\operatorname{Log}[g+h*x]}{h} - \frac{p*r*\operatorname{PolyLog}\left[2, \left(\frac{b*(g+h*x)}{b^2*g-a*h}\right)\right]}{h} - \frac{q*r*\operatorname{PolyLog}\left[2, \left(\frac{d*(g+h*x)}{d*g-c*h}\right)\right]}{h}$

#### Rule 2494

$\operatorname{Int}\left[\operatorname{Log}\left[\left(e_{.}\right)*\left(\left(f_{.}\right)*\left(\left(a_{.}\right)+\left(b_{.}\right)*\left(x_{.}\right)\right)^{\left(p_{.}\right)}*\left(\left(c_{.}\right)+\left(d_{.}\right)*\left(x_{.}\right)\right)^{\left(q_{.}\right)}\right]^{\left(r_{.}\right)}\right]/\left(\left(g_{.}\right)+\left(h_{.}\right)*\left(x_{.}\right)\right), x_{\text{Symbol}}] := \operatorname{Simp}\left[\frac{\operatorname{Log}[g+h*x]*\operatorname{Log}\left[e*(f*(a+b*x)^p*(c+d*x)^q)^r\right]}{h}, x\right] + \left(-\operatorname{Dist}\left[\frac{b*p*r}{h}, \operatorname{Int}\left[\frac{\operatorname{Log}[g+h*x]}{a+b*x}, x\right], x\right] - \operatorname{Dist}\left[\frac{d*q*r}{h}, \operatorname{Int}\left[\frac{\operatorname{Log}[g+h*x]}{c+d*x}, x\right], x\right]\right) /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, h, p, q, r\}, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0]$

#### Rule 2394

$\operatorname{Int}\left[\left(\left(a_{.}\right)+\operatorname{Log}\left[\left(c_{.}\right)*\left(\left(d_{.}\right)+\left(e_{.}\right)*\left(x_{.}\right)\right)^{\left(n_{.}\right)}\right)*\left(b_{.}\right)\right]/\left(\left(f_{.}\right)+\left(g_{.}\right)*\left(x_{.}\right)\right), x_{\text{Symbol}}] := \operatorname{Simp}\left[\frac{\operatorname{Log}\left[\frac{e*(f+g*x)}{e*f-d*g}\right]*(a+b*\operatorname{Log}[c*(d+e*x)])}{(f+g*x)}$

)^n))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_.)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned} \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx} dx &= \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} - \frac{(bpr) \int \frac{\log(g+hx)}{a+bx} dx}{h} - \frac{(dqr) \int \frac{\log(g+hx)}{c+dx} dx}{h} \\ &= -\frac{pr \log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr \log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} \\ &= -\frac{pr \log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr \log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} \\ &= -\frac{pr \log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr \log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} \end{aligned}$$

**Mathematica [A]** time = 0.0834048, size = 163, normalized size = 1.1

$$\frac{pr \operatorname{PolyLog}\left(2, \frac{h(a+bx)}{ah-bg}\right) + qr \operatorname{PolyLog}\left(2, \frac{h(c+dx)}{ch-dg}\right) + \log(g+hx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - pr \log(a+bx) \log(g+hx)}{h}$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(g + h\*x), x]

[Out] (-(p\*r\*Log[a + b\*x]\*Log[g + h\*x]) - q\*r\*Log[c + d\*x]\*Log[g + h\*x] + Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]\*Log[g + h\*x] + p\*r\*Log[a + b\*x]\*Log[(b\*(g + h

$*x))/(b*g - a*h)] + q*r*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + p*r*PolyLog[2, (h*(a + b*x))/(-b*g) + a*h)] + q*r*PolyLog[2, (h*(c + d*x))/(-d*g) + c*h)]/h$

**Maple [F]** time = 0.622, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{hx+g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g), x)`

[Out] `int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g), x)`

**Maxima [A]** time = 1.21998, size = 251, normalized size = 1.7

$$\frac{\left(\frac{\left(\log(bx+a)\log\left(\frac{bhx+ah}{bg-ah}+1\right)+\text{Li}_2\left(-\frac{bhx+ah}{bg-ah}\right)\right)fp}{h} + \frac{\left(\log(dx+c)\log\left(\frac{dhx+ch}{dg-ch}+1\right)+\text{Li}_2\left(-\frac{dhx+ch}{dg-ch}\right)\right)fq}{h}\right)r}{f} - \frac{(fp \log(bx+a) + fq \log(dx+c))r \log(hx+g)}{fh}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g), x, algorithm="maxima")`

[Out] `((log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))*f*p/h + (log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilog(-(d*h*x + c*h)/(d*g - c*h)))*f*q/h)*r/f - (f*p*log(b*x + a) + f*q*log(d*x + c))*r*log(h*x + g)/(f*h) + log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(h*x + g)/h`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hx+g}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*x + g), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hx+g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*x + g), x)
```

$$3.31 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2} dx$$

**Optimal.** Leaf size=128

$$\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} + \frac{bpr \log(a+bx)}{h(bg-ah)} - \frac{bpr \log(g+hx)}{h(bg-ah)} + \frac{dqr \log(c+dx)}{h(dg-ch)} - \frac{dqr \log(g+hx)}{h(dg-ch)}$$

[Out] (b\*p\*r\*Log[a + b\*x])/(h\*(b\*g - a\*h)) + (d\*q\*r\*Log[c + d\*x])/(h\*(d\*g - c\*h)) - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]/(h\*(g + h\*x)) - (b\*p\*r\*Log[g + h\*x])/(h\*(b\*g - a\*h)) - (d\*q\*r\*Log[g + h\*x])/(h\*(d\*g - c\*h))

**Rubi [A]** time = 0.0524397, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2495, 36, 31}

$$\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} + \frac{bpr \log(a+bx)}{h(bg-ah)} - \frac{bpr \log(g+hx)}{h(bg-ah)} + \frac{dqr \log(c+dx)}{h(dg-ch)} - \frac{dqr \log(g+hx)}{h(dg-ch)}$$

Antiderivative was successfully verified.

[In] Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(g + h\*x)^2, x]

[Out] (b\*p\*r\*Log[a + b\*x])/(h\*(b\*g - a\*h)) + (d\*q\*r\*Log[c + d\*x])/(h\*(d\*g - c\*h)) - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]/(h\*(g + h\*x)) - (b\*p\*r\*Log[g + h\*x])/(h\*(b\*g - a\*h)) - (d\*q\*r\*Log[g + h\*x])/(h\*(d\*g - c\*h))

### Rule 2495

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_.))^p\_.)\*((c\_.) + (d\_.)\*(x\_.))^q\_.)]^(r\_.)]\*((g\_.) + (h\_.)\*(x\_.))^m\_.], x\_Symbol] := Simp[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]/(h\*(m + 1)), x] + (-Dist[(b\*p\*r)/(h\*(m + 1)), Int[(g + h\*x)^(m + 1)/(a + b\*x), x], x] - Dist[(d\*q\*r)/(h\*(m + 1)), Int[(g + h\*x)^(m + 1)/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1]

### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x],

x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))<sup>(-1)</sup>, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rubi steps

$$\begin{aligned} \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2} dx &= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} + \frac{(bpr) \int \frac{1}{(a+bx)(g+hx)} dx}{h} + \frac{(dqr) \int \frac{1}{(c+dx)(g+hx)} dx}{h} \\ &= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} - \frac{(bpr) \int \frac{1}{g+hx} dx}{bg-ah} + \frac{(b^2pr) \int \frac{1}{a+bx} dx}{h(bg-ah)} - \frac{(dqr) \int \frac{1}{g+dx} dx}{dg-ch} \\ &= \frac{bpr \log(a+bx)}{h(bg-ah)} + \frac{dqr \log(c+dx)}{h(dg-ch)} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} - \frac{bpr \log(g+hx)}{h(bg-ah)} \end{aligned}$$

**Mathematica [A]** time = 0.232556, size = 93, normalized size = 0.73

$$\frac{-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx} + \frac{bpr(\log(a+bx)-\log(g+hx))}{bg-ah} + \frac{dqr(\log(c+dx)-\log(g+hx))}{dg-ch}}{h}$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(g + h\*x)^2,x]

[Out] (-(Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(g + h\*x)) + (b\*p\*r\*(Log[a + b\*x] - Log[g + h\*x]))/(b\*g - a\*h) + (d\*q\*r\*(Log[c + d\*x] - Log[g + h\*x]))/(d\*g - c\*h))/h

**Maple [F]** time = 0.493, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{(hx+g)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x)`

[Out] `int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x)`

**Maxima [A]** time = 1.25269, size = 166, normalized size = 1.3

$$\frac{\left( bfp \left( \frac{\log(bx+a)}{bg-ah} - \frac{\log(hx+g)}{bg-ah} \right) + dfq \left( \frac{\log(dx+c)}{dg-ch} - \frac{\log(hx+g)}{dg-ch} \right) \right)^r}{fh} - \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{(hx+g)h}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="maxima")`

[Out] `(b*f*p*(log(b*x + a)/(b*g - a*h) - log(h*x + g)/(b*g - a*h)) + d*f*q*(log(d*x + c)/(d*g - c*h) - log(h*x + g)/(d*g - c*h)))*r/(f*h) - log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*x + g)*h)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**2,x)`

[Out] Timed out

---

**Giac [A]** time = 1.46868, size = 257, normalized size = 2.01

$$\frac{b^2 p r \log(|-b x - a|)}{b^2 g h - a b h^2} + \frac{d^2 q r \log(|d x + c|)}{d^2 g h - c d h^2} - \frac{p r \log(b x + a)}{h^2 x + g h} - \frac{q r \log(d x + c)}{h^2 x + g h} - \frac{(b d g p r - b c h p r + b d g q r - a d h q r) \log(h x + g)}{b d g^2 h - b c g h^2 - a d g h^2 + a c h^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(h\*x+g)^2,x, algorithm="giac")

[Out]  $b^2 p r \log(\text{abs}(-b x - a)) / (b^2 g h - a b h^2) + d^2 q r \log(\text{abs}(d x + c)) / (d^2 g h - c d h^2) - p r \log(b x + a) / (h^2 x + g h) - q r \log(d x + c) / (h^2 x + g h) - (b d g p r - b c h p r + b d g q r - a d h q r) * \log(h x + g) / (b d g^2 h - b c g h^2 - a d g h^2 + a c h^3) - (r \log(f) + 1) / (h^2 x + g h)$



$$3.32 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^3} dx$$

**Optimal.** Leaf size=202

$$\frac{b^2pr \log(a+bx)}{2h(bg-ah)^2} - \frac{b^2pr \log(g+hx)}{2h(bg-ah)^2} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2} + \frac{bpr}{2h(g+hx)(bg-ah)} + \frac{d^2qr \log(c+dx)}{2h(dg-ch)^2} - \frac{d^2qr}{2h(dg-ch)}$$

[Out] (b\*p\*r)/(2\*h\*(b\*g - a\*h)\*(g + h\*x)) + (d\*q\*r)/(2\*h\*(d\*g - c\*h)\*(g + h\*x)) + (b^2\*p\*r\*Log[a + b\*x])/(2\*h\*(b\*g - a\*h)^2) + (d^2\*q\*r\*Log[c + d\*x])/(2\*h\*(d\*g - c\*h)^2) - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]/(2\*h\*(g + h\*x)^2) - (b^2\*p\*r\*Log[g + h\*x])/(2\*h\*(b\*g - a\*h)^2) - (d^2\*q\*r\*Log[g + h\*x])/(2\*h\*(d\*g - c\*h)^2)

**Rubi [A]** time = 0.111538, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {2495, 44}

$$\frac{b^2pr \log(a+bx)}{2h(bg-ah)^2} - \frac{b^2pr \log(g+hx)}{2h(bg-ah)^2} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2} + \frac{bpr}{2h(g+hx)(bg-ah)} + \frac{d^2qr \log(c+dx)}{2h(dg-ch)^2} - \frac{d^2qr}{2h(dg-ch)}$$

Antiderivative was successfully verified.

[In] Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(g + h\*x)^3, x]

[Out] (b\*p\*r)/(2\*h\*(b\*g - a\*h)\*(g + h\*x)) + (d\*q\*r)/(2\*h\*(d\*g - c\*h)\*(g + h\*x)) + (b^2\*p\*r\*Log[a + b\*x])/(2\*h\*(b\*g - a\*h)^2) + (d^2\*q\*r\*Log[c + d\*x])/(2\*h\*(d\*g - c\*h)^2) - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]/(2\*h\*(g + h\*x)^2) - (b^2\*p\*r\*Log[g + h\*x])/(2\*h\*(b\*g - a\*h)^2) - (d^2\*q\*r\*Log[g + h\*x])/(2\*h\*(d\*g - c\*h)^2)

### Rule 2495

Int[Log[(e.\_)\*((f.\_)\*((a.\_) + (b.\_)\*(x.\_))^(p.\_))\*((c.\_) + (d.\_)\*(x.\_))^(q.\_)]^(r.\_)]\*((g.\_) + (h.\_)\*(x.\_))^(m.\_), x\_Symbol] := Simp[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]/(h\*(m + 1)), x] + (-Dist[(b\*p\*r)/(h\*(m + 1)), Int[(g + h\*x)^(m + 1)/(a + b\*x), x], x] - Dist[(d\*q\*r)/(h\*(m + 1)), Int[(g + h\*x)^(m + 1)/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1]

**Rule 44**

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] & & NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^3} dx &= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2} + \frac{(bpr) \int \frac{1}{(a+bx)(g+hx)^2} dx}{2h} + \frac{(dqr) \int \frac{1}{(c+dx)(g+hx)^2} dx}{2h} \\ &= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2} + \frac{(bpr) \int \left(\frac{b^2}{(bg-ah)^2(a+bx)} - \frac{h}{(bg-ah)(g+hx)^2} - \frac{bh}{(bg-ah)^2(g+hx)}\right) dx}{2h} \\ &= \frac{bpr}{2h(bg-ah)(g+hx)} + \frac{dqr}{2h(dg-ch)(g+hx)} + \frac{b^2pr \log(a+bx)}{2h(bg-ah)^2} + \frac{d^2qr \log(c+dx)}{2h(dg-ch)^2} \end{aligned}$$

**Mathematica [A]** time = 0.549866, size = 206, normalized size = 1.02

$$\frac{r(g+hx)\left((bc-ad)(bg-ah)(dg-ch)(bdg(p+q)-h(adq+bc p))-(g+hx)\left(d^2q(ad-bc)(bg-ah)^2(\log(c+dx)-\log(g+hx))-b^2p(bc-ad)(dg-ch)^2(\log(a+bx)-\log(g+hx))\right)\right)}{(bc-ad)(bg-ah)^2(dg-ch)^2} - \log\left(\frac{e\left(f(a+bx)^p(c+dx)^q\right)^r}{(g+hx)^3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(g + h\*x)^3,x]

[Out] (-Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] + (r\*(g + h\*x)\*((b\*c - a\*d)\*(b\*g - a\*h)\*(d\*g - c\*h)\*(b\*d\*g\*(p + q) - h\*(b\*c\*p + a\*d\*q)) - (g + h\*x)\*(-(b^2\*(b\*c - a\*d)\*(d\*g - c\*h)^2\*p\*(Log[a + b\*x] - Log[g + h\*x])) + d^2\*(-(b\*c) + a\*d)\*(b\*g - a\*h)^2\*q\*(Log[c + d\*x] - Log[g + h\*x]))))/((b\*c - a\*d)\*(b\*g - a\*h)^2\*(d\*g - c\*h)^2)/(2\*h\*(g + h\*x)^2)

**Maple [F]** time = 0.496, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{(hx+g)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x)`

[Out] `int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x)`

**Maxima [A]** time = 1.25929, size = 313, normalized size = 1.55

$$\frac{\left( bfp \left( \frac{b \log(bx+a)}{b^2g^2-2abgh+a^2h^2} - \frac{b \log(hx+g)}{b^2g^2-2abgh+a^2h^2} + \frac{1}{bg^2-agh+(bgh-ah^2)x} \right) + dfq \left( \frac{d \log(dx+c)}{d^2g^2-2cdgh+c^2h^2} - \frac{d \log(hx+g)}{d^2g^2-2cdgh+c^2h^2} + \frac{1}{dg^2-cgh+(dgh-ch^2)x} \right) \right)}{2fh}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="maxima")`

[Out]  $\frac{1}{2} * (b * f * p * (b * \log(b * x + a) / (b^2 * g^2 - 2 * a * b * g * h + a^2 * h^2) - b * \log(h * x + g) / (b^2 * g^2 - 2 * a * b * g * h + a^2 * h^2) + 1 / (b * g^2 - a * g * h + (b * g * h - a * h^2) * x)) + d * f * q * (d * \log(d * x + c) / (d^2 * g^2 - 2 * c * d * g * h + c^2 * h^2) - d * \log(h * x + g) / (d^2 * g^2 - 2 * c * d * g * h + c^2 * h^2) + 1 / (d * g^2 - c * g * h + (d * g * h - c * h^2) * x))) * r / (f * h) - 1/2 * \log(((b * x + a)^p * (d * x + c)^q * f)^r * e) / ((h * x + g)^2 * h)$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**3,x)
```

```
[Out] Timed out
```

**Giac [B]** time = 1.57893, size = 1413, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] -1/2*p*r*log(b*x + a)/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*q*r*log(d*x + c)/
(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*(b^2*d^2*g^2*p*r - 2*b^2*c*d*g*h*p*r +
b^2*c^2*h^2*p*r + b^2*d^2*g^2*q*r - 2*a*b*d^2*g*h*q*r + a^2*d^2*h^2*q*r)*lo
g(h*x + g)/(b^2*d^2*g^4*h - 2*b^2*c*d*g^3*h^2 - 2*a*b*d^2*g^3*h^2 + b^2*c^2
*g^2*h^3 + 4*a*b*c*d*g^2*h^3 + a^2*d^2*g^2*h^3 - 2*a*b*c^2*g*h^4 - 2*a^2*c*
d*g*h^4 + a^2*c^2*h^5) + 1/4*(b^2*d^2*g^2*p*r - 2*b^2*c*d*g*h*p*r + b^2*c^2
*h^2*p*r + b^2*d^2*g^2*q*r - 2*a*b*d^2*g*h*q*r + a^2*d^2*h^2*q*r)*log(abs(b
*d*x^2 + b*c*x + a*d*x + a*c))/(b^2*d^2*g^4*h - 2*b^2*c*d*g^3*h^2 - 2*a*b*d
^2*g^3*h^2 + b^2*c^2*g^2*h^3 + 4*a*b*c*d*g^2*h^3 + a^2*d^2*g^2*h^3 - 2*a*b*
c^2*g*h^4 - 2*a^2*c*d*g*h^4 + a^2*c^2*h^5) + 1/2*(b*d*g*h*p*r*x - b*c*h^2*p
*r*x + b*d*g*h*q*r*x - a*d*h^2*q*r*x + b*d*g^2*p*r - b*c*g*h*p*r + b*d*g^2*
q*r - a*d*g*h*q*r - b*d*g^2*r*log(f) + b*c*g*h*r*log(f) + a*d*g*h*r*log(f)
- a*c*h^2*r*log(f) - b*d*g^2 + b*c*g*h + a*d*g*h - a*c*h^2)/(b*d*g^2*h^3*x^
2 - b*c*g*h^4*x^2 - a*d*g*h^4*x^2 + a*c*h^5*x^2 + 2*b*d*g^3*h^2*x - 2*b*c*g
^2*h^3*x - 2*a*d*g^2*h^3*x + 2*a*c*g*h^4*x + b*d*g^4*h - b*c*g^3*h^2 - a*d*
g^3*h^2 + a*c*g^2*h^3) + 1/4*(b^3*c*d^2*g^2*p*r - a*b^2*d^3*g^2*p*r - 2*b^3
*c^2*d*g*h*p*r + 2*a*b^2*c*d^2*g*h*p*r + b^3*c^3*h^2*p*r - a*b^2*c^2*d*h^2*
p*r - b^3*c*d^2*g^2*q*r + a*b^2*d^3*g^2*q*r + 2*a*b^2*c*d^2*g*h*q*r - 2*a^2
*b*d^3*g*h*q*r - a^2*b*c*d^2*h^2*q*r + a^3*d^3*h^2*q*r)*log(abs((2*b*d*x +
b*c + a*d - abs(-b*c + a*d))/(2*b*d*x + b*c + a*d + abs(-b*c + a*d))))/((b^
2*d^2*g^4*h - 2*b^2*c*d*g^3*h^2 - 2*a*b*d^2*g^3*h^2 + b^2*c^2*g^2*h^3 + 4*a
*b*c*d*g^2*h^3 + a^2*d^2*g^2*h^3 - 2*a*b*c^2*g*h^4 - 2*a^2*c*d*g*h^4 + a^2*
c^2*h^5)*abs(-b*c + a*d))
```

$$3.33 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^4} dx$$

**Optimal.** Leaf size=260

$$\frac{b^2pr}{3h(g+hx)(bg-ah)^2} + \frac{b^3pr \log(a+bx)}{3h(bg-ah)^3} - \frac{b^3pr \log(g+hx)}{3h(bg-ah)^3} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{bpr}{6h(g+hx)^2(bg-ah)} +$$

[Out] (b\*p\*r)/(6\*h\*(b\*g - a\*h)\*(g + h\*x)^2) + (d\*q\*r)/(6\*h\*(d\*g - c\*h)\*(g + h\*x)^2) + (b^2\*p\*r)/(3\*h\*(b\*g - a\*h)^2\*(g + h\*x)) + (d^2\*q\*r)/(3\*h\*(d\*g - c\*h)^2\*(g + h\*x)) + (b^3\*p\*r\*Log[a + b\*x])/(3\*h\*(b\*g - a\*h)^3) + (d^3\*q\*r\*Log[c + d\*x])/(3\*h\*(d\*g - c\*h)^3) - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)/(3\*h\*(g + h\*x)^3) - (b^3\*p\*r\*Log[g + h\*x])/(3\*h\*(b\*g - a\*h)^3) - (d^3\*q\*r\*Log[g + h\*x])/(3\*h\*(d\*g - c\*h)^3)

**Rubi [A]** time = 0.151959, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {2495, 44}

$$\frac{b^2pr}{3h(g+hx)(bg-ah)^2} + \frac{b^3pr \log(a+bx)}{3h(bg-ah)^3} - \frac{b^3pr \log(g+hx)}{3h(bg-ah)^3} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{bpr}{6h(g+hx)^2(bg-ah)} +$$

Antiderivative was successfully verified.

[In] Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(g + h\*x)^4, x]

[Out] (b\*p\*r)/(6\*h\*(b\*g - a\*h)\*(g + h\*x)^2) + (d\*q\*r)/(6\*h\*(d\*g - c\*h)\*(g + h\*x)^2) + (b^2\*p\*r)/(3\*h\*(b\*g - a\*h)^2\*(g + h\*x)) + (d^2\*q\*r)/(3\*h\*(d\*g - c\*h)^2\*(g + h\*x)) + (b^3\*p\*r\*Log[a + b\*x])/(3\*h\*(b\*g - a\*h)^3) + (d^3\*q\*r\*Log[c + d\*x])/(3\*h\*(d\*g - c\*h)^3) - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)/(3\*h\*(g + h\*x)^3) - (b^3\*p\*r\*Log[g + h\*x])/(3\*h\*(b\*g - a\*h)^3) - (d^3\*q\*r\*Log[g + h\*x])/(3\*h\*(d\*g - c\*h)^3)

### Rule 2495

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]\*((g\_.) + (h\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]/(h\*(m + 1)), x] + (-Dist[(b\*p\*r)/(h\*(m + 1)), Int[(g + h\*x)^(m + 1)/(a + b\*x), x], x] - Dist[(d\*q\*r)/(h\*(m + 1)), Int[(g + h\*x)^(m + 1)/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,

$m, p, q, r\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m, -1]$

#### Rule 44

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{ILtQ}[m, 0] \&\& \text{IntegerQ}[n] \&\& !(\text{IGtQ}[n, 0] \&\& \text{LtQ}[m + n + 2, 0])$

#### Rubi steps

$$\begin{aligned} \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^4} dx &= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{(bpr) \int \frac{1}{(a+bx)(g+hx)^3} dx}{3h} + \frac{(dqr) \int \frac{1}{(c+dx)(g+hx)^3} dx}{3h} \\ &= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{(bpr) \int \left(\frac{b^3}{(bg-ah)^3(a+bx)} - \frac{h}{(bg-ah)(g+hx)^3} - \frac{bh}{(bg-ah)^2(g+hx)}\right) dx}{3h} \\ &= \frac{bpr}{6h(bg-ah)(g+hx)^2} + \frac{dqr}{6h(dg-ch)(g+hx)^2} + \frac{b^2pr}{3h(bg-ah)^2(g+hx)} + \frac{d}{3h(dg-ch)} \end{aligned}$$

**Mathematica [A]** time = 0.959729, size = 254, normalized size = 0.98

$$\frac{r(g+hx)((bg-ah)^2(dg-ch)^2(bdg(p+q)-h(adq+bc p))-(g+hx)((bg-ah)(dg-ch)(-2a^2d^2h^2q+4abd^2ghq-2b^2(c^2h^2p-2cdghp+d^2g^2(p+q)))-2(g+hx)(b^3p(dg-ch)^3(\log(a+bx)-\log(g+hx))))}{(bg-ah)^3(dg-ch)^3} + \frac{d}{6h(g+hx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(g + h\*x)^4,x]

[Out]  $(-2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*((b*g - a*h)^2*(d*g - c*h)^2*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*((b*g - a*h)*(d*g - c*h)*(4*a*b*d^2*g*h*q - 2*a^2*d^2*h^2*q - 2*b^2*(-2*c*d*g*h*p + c^2*h^2*p + d^2*g^2*(p + q))) - 2*(g + h*x)*(b^3*(d*g - c*h)^3*p*(\text{Log}[a + b*x] - \text{Log}[g + h*x]) + d^3*(b*g - a*h)^3*q*(\text{Log}[c + d*x] - \text{Log}[g + h*x])))))/((b*g - a*h)^3*(d*g - c*h)^3))/(6*h*(g + h*x)^3)$

**Maple [F]** time = 0.519, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{(hx+g)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(h\*x+g)^4,x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(h\*x+g)^4,x)

**Maxima [A]** time = 1.29783, size = 616, normalized size = 2.37

$$\left( \left( \frac{2b^2 \log(bx+a)}{b^3g^3-3ab^2g^2h+3a^2bg^2h^2-a^3h^3} - \frac{2b^2 \log(hx+g)}{b^3g^3-3ab^2g^2h+3a^2bg^2h^2-a^3h^3} + \frac{2bhx+3bg-ah}{b^2g^4-2abg^3h+a^2g^2h^2+(b^2g^2h^2-2abgh^3+a^2h^4)x^2+2(b^2g^3h-2abg^2h^2+a^2gh^3)x} \right) b f r \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(h\*x+g)^4,x, algorithm="maxima")

[Out] 1/6\*((2\*b^2\*log(b\*x + a)/(b^3\*g^3 - 3\*a\*b^2\*g^2\*h + 3\*a^2\*b\*g\*h^2 - a^3\*h^3) - 2\*b^2\*log(h\*x + g)/(b^3\*g^3 - 3\*a\*b^2\*g^2\*h + 3\*a^2\*b\*g\*h^2 - a^3\*h^3) + (2\*b\*h\*x + 3\*b\*g - a\*h)/(b^2\*g^4 - 2\*a\*b\*g^3\*h + a^2\*g^2\*h^2 + (b^2\*g^2\*h^2 - 2\*a\*b\*g^3\*h + a^2\*g^2\*h^2 + a^2\*g\*h^3)\*x))\*b\*f\*p + (2\*d^2\*log(d\*x + c)/(d^3\*g^3 - 3\*c\*d^2\*g^2\*h + 3\*c^2\*d\*g\*h^2 - c^3\*h^3) - 2\*d^2\*log(h\*x + g)/(d^3\*g^3 - 3\*c\*d^2\*g^2\*h + 3\*c^2\*d\*g\*h^2 - c^3\*h^3) + (2\*d\*h\*x + 3\*d\*g - c\*h)/(d^2\*g^4 - 2\*c\*d\*g^3\*h + c^2\*g^2\*h^2 + (d^2\*g^2\*h^2 - 2\*c\*d\*g^3\*h + c^2\*h^4)\*x^2 + 2\*(d^2\*g^3\*h - 2\*c\*d\*g^2\*h^2 + c^2\*g\*h^3)\*x))\*d\*f\*q)\*r/(f\*h) - 1/3\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)/((h\*x + g)^3\*h)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**4,x)
```

```
[Out] Timed out
```

**Giac [B]** time = 1.86354, size = 2383, normalized size = 9.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/3*b^4*p*r*log(abs(b*x + a))/(b^4*g^3*h - 3*a*b^3*g^2*h^2 + 3*a^2*b^2*g*h^3 - a^3*b*h^4) + 1/3*d^4*q*r*log(abs(d*x + c))/(d^4*g^3*h - 3*c*d^3*g^2*h^2 + 3*c^2*d^2*g*h^3 - c^3*d*h^4) - 1/3*p*r*log(b*x + a)/(h^4*x^3 + 3*g*h^3*x^2 + 3*g^2*h^2*x + g^3*h) - 1/3*q*r*log(d*x + c)/(h^4*x^3 + 3*g*h^3*x^2 + 3*g^2*h^2*x + g^3*h) - 1/3*(b^3*d^3*g^3*p*r - 3*b^3*c*d^2*g^2*h*p*r + 3*b^3*c^2*d*g*h^2*p*r - b^3*c^3*h^3*p*r + b^3*d^3*g^3*q*r - 3*a*b^2*d^3*g^2*h*q*r + 3*a^2*b*d^3*g^2*h^2*q*r - a^3*d^3*h^3*q*r)*log(h*x + g)/(b^3*d^3*g^6*h - 3*b^3*c*d^2*g^5*h^2 - 3*a*b^2*d^3*g^5*h^2 + 3*b^3*c^2*d*g^4*h^3 + 9*a*b^2*c*d^2*g^4*h^3 + 3*a^2*b*d^3*g^4*h^3 - b^3*c^3*g^3*h^4 - 9*a*b^2*c^2*d*g^3*h^4 - 9*a^2*b*c*d^2*g^3*h^4 - a^3*d^3*g^3*h^4 + 3*a*b^2*c^3*g^2*h^5 + 9*a^2*b*c^2*d*g^2*h^5 + 3*a^3*c*d^2*g^2*h^5 - 3*a^2*b*c^3*g*h^6 - 3*a^3*c^2*d*g*h^6 + a^3*c^3*h^7) + 1/6*(2*b^2*d^2*g^2*h^2*p*r*x^2 - 4*b^2*c*d*g*h^3*p*r*x^2 + 2*b^2*c^2*h^4*p*r*x^2 + 2*b^2*d^2*g^2*h^2*q*r*x^2 - 4*a*b*d^2*g*h^3*q*r*x^2 + 2*a^2*d^2*h^4*q*r*x^2 + 5*b^2*d^2*g^3*h*p*r*x - 10*b^2*c*d*g^2*h^2*p*r*x - a*b*d^2*g^2*h^2*p*r*x + 5*b^2*c^2*g*h^3*p*r*x + 2*a*b*c*d*g*h^3*p*r*x - a*b*c^2*h^4*p*r*x + 5*b^2*d^2*g^3*h*q*r*x - b^2*c*d*g^2*h^2*q*r*x - 10*a*b*d^2*g^2*h^2*q*r*x + 2*a*b*c*d*g*h^3*q*r*x + 5*a^2*d^2*g*h^3*q*r*x - a^2*c
```



$$\begin{aligned}
& *d^4*qr*x + 3*b^2*d^2*g^4*p*r - 6*b^2*c*d*g^3*h*p*r - a*b*d^2*g^3*h*p*r \\
& + 3*b^2*c^2*g^2*h^2*p*r + 2*a*b*c*d*g^2*h^2*p*r - a*b*c^2*g*h^3*p*r + 3*b^2 \\
& *d^2*g^4*qr - b^2*c*d*g^3*h*qr - 6*a*b*d^2*g^3*h*qr + 2*a*b*c*d*g^2*h^2* \\
& qr + 3*a^2*d^2*g^2*h^2*qr - a^2*c*d*g*h^3*qr - 2*b^2*d^2*g^4*r*log(f) + \\
& 4*b^2*c*d*g^3*h*r*log(f) + 4*a*b*d^2*g^3*h*r*log(f) - 2*b^2*c^2*g^2*h^2*r*log(f) - 8*a*b*c*d*g^2*h^2*r*log(f) - 2*a^2*d^2*g^2*h^2*r*log(f) + 4*a*b*c^2 \\
& *g*h^3*r*log(f) + 4*a^2*c*d*g*h^3*r*log(f) - 2*a^2*c^2*h^4*r*log(f) - 2*b^2 \\
& *d^2*g^4 + 4*b^2*c*d*g^3*h + 4*a*b*d^2*g^3*h - 2*b^2*c^2*g^2*h^2 - 8*a*b*c \\
& *d*g^2*h^2 - 2*a^2*d^2*g^2*h^2 + 4*a*b*c^2*g*h^3 + 4*a^2*c*d*g*h^3 - 2*a^2*c \\
& ^2*h^4)/(b^2*d^2*g^4*h^4*x^3 - 2*b^2*c*d*g^3*h^5*x^3 - 2*a*b*d^2*g^3*h^5*x^ \\
& 3 + b^2*c^2*g^2*h^6*x^3 + 4*a*b*c*d*g^2*h^6*x^3 + a^2*d^2*g^2*h^6*x^3 - 2*a \\
& *b*c^2*g*h^7*x^3 - 2*a^2*c*d*g*h^7*x^3 + a^2*c^2*h^8*x^3 + 3*b^2*d^2*g^5*h^ \\
& 3*x^2 - 6*b^2*c*d*g^4*h^4*x^2 - 6*a*b*d^2*g^4*h^4*x^2 + 3*b^2*c^2*g^3*h^5*x \\
& ^2 + 12*a*b*c*d*g^3*h^5*x^2 + 3*a^2*d^2*g^3*h^5*x^2 - 6*a*b*c^2*g^2*h^6*x^2 \\
& - 6*a^2*c*d*g^2*h^6*x^2 + 3*a^2*c^2*g*h^7*x^2 + 3*b^2*d^2*g^6*h^2*x - 6*b^ \\
& 2*c*d*g^5*h^3*x - 6*a*b*d^2*g^5*h^3*x + 3*b^2*c^2*g^4*h^4*x + 12*a*b*c*d*g^ \\
& 4*h^4*x + 3*a^2*d^2*g^4*h^4*x - 6*a*b*c^2*g^3*h^5*x - 6*a^2*c*d*g^3*h^5*x + \\
& 3*a^2*c^2*g^2*h^6*x + b^2*d^2*g^7*h - 2*b^2*c*d*g^6*h^2 - 2*a*b*d^2*g^6*h^ \\
& 2 + b^2*c^2*g^5*h^3 + 4*a*b*c*d*g^5*h^3 + a^2*d^2*g^5*h^3 - 2*a*b*c^2*g^4*h \\
& ^4 - 2*a^2*c*d*g^4*h^4 + a^2*c^2*g^3*h^5)
\end{aligned}$$

$$3.34 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^5} dx$$

**Optimal.** Leaf size=318

$$\frac{b^3pr}{4h(g+hx)(bg-ah)^3} + \frac{b^2pr}{8h(g+hx)^2(bg-ah)^2} + \frac{b^4pr \log(a+bx)}{4h(bg-ah)^4} - \frac{b^4pr \log(g+hx)}{4h(bg-ah)^4} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4} +$$

[Out] (b\*p\*r)/(12\*h\*(b\*g - a\*h)\*(g + h\*x)^3) + (d\*q\*r)/(12\*h\*(d\*g - c\*h)\*(g + h\*x)^3) + (b^2\*p\*r)/(8\*h\*(b\*g - a\*h)^2\*(g + h\*x)^2) + (d^2\*q\*r)/(8\*h\*(d\*g - c\*h)^2\*(g + h\*x)^2) + (b^3\*p\*r)/(4\*h\*(b\*g - a\*h)^3\*(g + h\*x)) + (d^3\*q\*r)/(4\*h\*(d\*g - c\*h)^3\*(g + h\*x)) + (b^4\*p\*r\*Log[a + b\*x])/(4\*h\*(b\*g - a\*h)^4) + (d^4\*q\*r\*Log[c + d\*x])/(4\*h\*(d\*g - c\*h)^4) - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)/(4\*h\*(g + h\*x)^4) - (b^4\*p\*r\*Log[g + h\*x])/(4\*h\*(b\*g - a\*h)^4) - (d^4\*q\*r\*Log[g + h\*x])/(4\*h\*(d\*g - c\*h)^4)

**Rubi [A]** time = 0.194835, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {2495, 44}

$$\frac{b^3pr}{4h(g+hx)(bg-ah)^3} + \frac{b^2pr}{8h(g+hx)^2(bg-ah)^2} + \frac{b^4pr \log(a+bx)}{4h(bg-ah)^4} - \frac{b^4pr \log(g+hx)}{4h(bg-ah)^4} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4} +$$

Antiderivative was successfully verified.

[In] Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(g + h\*x)^5,x]

[Out] (b\*p\*r)/(12\*h\*(b\*g - a\*h)\*(g + h\*x)^3) + (d\*q\*r)/(12\*h\*(d\*g - c\*h)\*(g + h\*x)^3) + (b^2\*p\*r)/(8\*h\*(b\*g - a\*h)^2\*(g + h\*x)^2) + (d^2\*q\*r)/(8\*h\*(d\*g - c\*h)^2\*(g + h\*x)^2) + (b^3\*p\*r)/(4\*h\*(b\*g - a\*h)^3\*(g + h\*x)) + (d^3\*q\*r)/(4\*h\*(d\*g - c\*h)^3\*(g + h\*x)) + (b^4\*p\*r\*Log[a + b\*x])/(4\*h\*(b\*g - a\*h)^4) + (d^4\*q\*r\*Log[c + d\*x])/(4\*h\*(d\*g - c\*h)^4) - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)/(4\*h\*(g + h\*x)^4) - (b^4\*p\*r\*Log[g + h\*x])/(4\*h\*(b\*g - a\*h)^4) - (d^4\*q\*r\*Log[g + h\*x])/(4\*h\*(d\*g - c\*h)^4)

**Rule 2495**

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]\*((g\_.) + (h\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]/(h\*(m + 1)), x] + (-Dist[(b\*p\*r)/(h\*(m

+ 1)), Int[(g + h\*x)^(m + 1)/(a + b\*x), x], x] - Dist[(d\*q\*r)/(h\*(m + 1)), Int[(g + h\*x)^(m + 1)/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1]

### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned} \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^5} dx &= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4} + \frac{(bpr) \int \frac{1}{(a+bx)(g+hx)^4} dx}{4h} + \frac{(dqr) \int \frac{1}{(c+dx)(g+hx)^4} dx}{4h} \\ &= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4} + \frac{(bpr) \int \left(\frac{b^4}{(bg-ah)^4(a+bx)} - \frac{h}{(bg-ah)(g+hx)^4} - \frac{bh}{(bg-ah)^2}\right) dx}{4h} \\ &= \frac{bpr}{12h(bg-ah)(g+hx)^3} + \frac{dqr}{12h(dg-ch)(g+hx)^3} + \frac{b^2pr}{8h(bg-ah)^2(g+hx)^2} + \frac{bh}{8h(dg-ch)(g+hx)} \end{aligned}$$

**Mathematica [A]** time = 1.82202, size = 480, normalized size = 1.51

$$\frac{r(g+hx)(2(bg-ah)^3(dg-ch)^3(bdg(p+q)-h(adq+bcq))-(g+hx)(6(g+hx)(-(g+hx)(-\log(g+hx)(6a^2b^2d^4g^2h^2q-4a^3bd^4gh^3q+a^4d^4h^4q-4ab^3d^4g^3hq+b^4(6c^2d^2g^2h^2p$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(g + h\*x)^5,x]

[Out] (-6\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] + (r\*(g + h\*x)\*(2\*(b\*g - a\*h)^3\*(d\*g - c\*h)^3\*(b\*d\*g\*(p + q) - h\*(b\*c\*p + a\*d\*q)) - (g + h\*x)\*((b\*g - a\*h)^2\*(d\*g - c\*h)^2\*(6\*a\*b\*d^2\*g\*h\*q - 3\*a^2\*d^2\*h^2\*q - 3\*b^2\*(-2\*c\*d\*g\*h\*p + c^2\*h^2\*p + d^2\*g^2\*(p + q))) + 6\*(g + h\*x)\*(-(b\*g - a\*h)\*(-(d\*g) + c\*h)\*(3\*a\*b^2\*d^3\*g^2\*h\*q - 3\*a^2\*b\*d^3\*g\*h^2\*q + a^3\*d^3\*h^3\*q - b^3\*(-3\*c\*d^2\*g^2\*h\*p + 3\*c^2\*d\*g\*h^2\*p - c^3\*h^3\*p + d^3\*g^3\*(p + q)))) - (g + h\*x)\*(b^4\*(d\*g - c\*h)^4\*p\*Log[a + b\*x] + d^4\*(b\*g - a\*h)^4\*q\*Log[c + d\*x] - (-4\*a\*b^3\*d^4\*g^3\*h\*q + 6\*a^2\*b^2\*d^4\*g^2\*h^2\*q - 4\*a^3\*b\*d^4\*g\*h^3\*q + a^4\*d^4\*h^4\*q + b^4\*(-4\*c\*d^3\*g^3\*h\*p + 6\*c^2\*d^2\*g^2\*h^2\*p - 4\*c^3\*d\*g\*h^3\*p + c^4\*h^4\*p

+ d^4\*g^4\*(p + q))\*Log[g + h\*x])))))/((b\*g - a\*h)^4\*(d\*g - c\*h)^4))/(24\*h\*(g + h\*x)^4)

**Maple [F]** time = 0.509, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{(hx+g)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(h\*x+g)^5,x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(h\*x+g)^5,x)

**Maxima [B]** time = 1.41247, size = 1048, normalized size = 3.3

$$\left( \left( \frac{6b^3 \log(bx+a)}{b^4g^4-4ab^3g^3h+6a^2b^2g^2h^2-4a^3bg^3h^3+a^4h^4} - \frac{6b^3 \log(hx+g)}{b^4g^4-4ab^3g^3h+6a^2b^2g^2h^2-4a^3bg^3h^3+a^4h^4} + \frac{6b^2h^2x^2}{b^3g^6-3ab^2g^5h+3a^2bg^4h^2-a^3g^3h^3+(b^3g^3h^3-3ab^2g^2h^4+3a^2bg^2h^4-3abg^2h^4+3a^2bg^2h^4)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(h\*x+g)^5,x, algorithm="maxima")

[Out] 1/24\*((6\*b^3\*log(b\*x + a)/(b^4\*g^4 - 4\*a\*b^3\*g^3\*h + 6\*a^2\*b^2\*g^2\*h^2 - 4\*a^3\*b\*g\*h^3 + a^4\*h^4) - 6\*b^3\*log(h\*x + g)/(b^4\*g^4 - 4\*a\*b^3\*g^3\*h + 6\*a^2\*b^2\*g^2\*h^2 - 4\*a^3\*b\*g\*h^3 + a^4\*h^4) + (6\*b^2\*h^2\*x^2 + 11\*b^2\*g^2 - 7\*a\*b\*g\*h + 2\*a^2\*h^2 + 3\*(5\*b^2\*g\*h - a\*b\*h^2)\*x)/(b^3\*g^6 - 3\*a\*b^2\*g^5\*h + 3\*a^2\*b\*g^4\*h^2 - a^3\*g^3\*h^3 + (b^3\*g^3\*h^3 - 3\*a\*b^2\*g^2\*h^4 + 3\*a^2\*b\*g\*h^5 - a^3\*h^6)\*x^3 + 3\*(b^3\*g^4\*h^2 - 3\*a\*b^2\*g^3\*h^3 + 3\*a^2\*b\*g^2\*h^4 - a^3\*g\*h^5)\*x^2 + 3\*(b^3\*g^5\*h - 3\*a\*b^2\*g^4\*h^2 + 3\*a^2\*b\*g^3\*h^3 - a^3\*g^2\*h^4)\*x))\*b\*f\*p + (6\*d^3\*log(d\*x + c)/(d^4\*g^4 - 4\*c\*d^3\*g^3\*h + 6\*c^2\*d^2\*g^2\*h^2 - 4\*c^3\*d\*g\*h^3 + c^4\*h^4) - 6\*d^3\*log(h\*x + g)/(d^4\*g^4 - 4\*c\*d^3\*g^3\*h + 6\*c^2\*d^2\*g^2\*h^2 - 4\*c^3\*d\*g\*h^3 + c^4\*h^4) + (6\*d^2\*h^2\*x^2 + 11\*d^2\*g^2 - 7\*c\*d\*g\*h + 2\*c^2\*h^2 + 3\*(5\*d^2\*g\*h - c\*d\*h^2)\*x)/(d^3\*g^6 - 3\*c\*d^2\*g^5\*h + 3\*c^2\*d\*g^4\*h^2 - c^3\*g^3\*h^3 + (d^3\*g^3\*h^3 - 3\*c\*d^2\*g^2\*h^4 + 3\*c^2\*d\*g\*h^5 - c^3\*h^6)\*x^3 + 3\*(d^3\*g^4\*h^2 - 3\*c\*d^2\*g^3\*h^3 + 3\*c^2\*d\*g^2\*h^4 - c^3\*g\*h^5)\*x^2 + 3\*(d^3\*g^5\*h - 3\*c\*d^2\*g^4\*h^2 + 3\*c^2\*d\*g^3\*h

$$\frac{(c^3 - c^3 g^2 h^4 x) d f q r / (f h) - 1/4 \log((b x + a)^p (d x + c)^q f)^r e}{(h x + g)^4 h}$$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(h\*x+g)^5,x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r)/(h\*x+g)\*\*5,x)

[Out] Timed out

**Giac [B]** time = 2.77682, size = 6884, normalized size = 21.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(h\*x+g)^5,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/4 p r \log(b x + a) / (h^5 x^4 + 4 g h^4 x^3 + 6 g^2 h^3 x^2 + 4 g^3 h^2 x + g^4 h) - 1/4 q r \log(d x + c) / (h^5 x^4 + 4 g h^4 x^3 + 6 g^2 h^3 x^2 + 4 g^3 h^2 x + g^4 h) \\ & - 1/4 (b^4 d^4 g^4 p r - 4 b^4 c d^3 g^3 h p r + 6 b^4 c^2 d^2 g^2 h^2 p r - 4 b^4 c^3 d g h^3 p r + b^4 c^4 h^4 p r + b^4 d^4 g^4 q r - 4 a b^3 d^4 g^3 h q r + 6 a^2 b^2 d^4 g^2 h^2 q r - 4 a^3 b d^4 g h^3 q r + a^4 d^4 h^4 q r) \log(h x + g) / (b^4 d^4 g^8 h - 4 b^4 c d^3 g^7 h^2 - \end{aligned}$$

$$\begin{aligned}
& 4*a*b^3*d^4*g^7*h^2 + 6*b^4*c^2*d^2*g^6*h^3 + 16*a*b^3*c*d^3*g^6*h^3 + 6*a^2*b^2*d^4*g^6*h^3 - 4*b^4*c^3*d*g^5*h^4 - 24*a*b^3*c^2*d^2*g^5*h^4 - 24*a^2*b^2*c*d^3*g^5*h^4 - 4*a^3*b*d^4*g^5*h^4 + b^4*c^4*g^4*h^5 + 16*a*b^3*c^3*d*g^4*h^5 + 36*a^2*b^2*c^2*d^2*g^4*h^5 + 16*a^3*b*c*d^3*g^4*h^5 + a^4*d^4*g^4*h^5 - 4*a*b^3*c^4*g^3*h^6 - 24*a^2*b^2*c^3*d*g^3*h^6 - 24*a^3*b*c^2*d^2*g^3*h^6 - 4*a^4*c*d^3*g^3*h^6 + 6*a^2*b^2*c^4*g^2*h^7 + 16*a^3*b*c^3*d*g^2*h^7 + 6*a^4*c^2*d^2*g^2*h^7 - 4*a^3*b*c^4*g*h^8 - 4*a^4*c^3*d*g*h^8 + a^4*c^4*h^9) + 1/8*(b^4*d^4*g^4*p*r - 4*b^4*c*d^3*g^3*h*p*r + 6*b^4*c^2*d^2*g^2*h^2*p*r - 4*b^4*c^3*d*g*h^3*p*r + b^4*c^4*h^4*p*r + b^4*d^4*g^4*q*r - 4*a*b^3*d^4*g^3*h*q*r + 6*a^2*b^2*d^4*g^2*h^2*q*r - 4*a^3*b*d^4*g*h^3*q*r + a^4*d^4*h^4*q*r)*log(abs(b*d*x^2 + b*c*x + a*d*x + a*c))/(b^4*d^4*g^8*h - 4*b^4*c*d^3*g^7*h^2 - 4*a*b^3*d^4*g^7*h^2 + 6*b^4*c^2*d^2*g^6*h^3 + 16*a*b^3*c*d^3*g^6*h^3 + 6*a^2*b^2*d^4*g^6*h^3 - 4*b^4*c^3*d*g^5*h^4 - 24*a*b^3*c^2*d^2*g^5*h^4 - 24*a^2*b^2*c*d^3*g^5*h^4 - 4*a^3*b*d^4*g^5*h^4 + b^4*c^4*g^4*h^5 + 16*a*b^3*c^3*d*g^4*h^5 + 36*a^2*b^2*c^2*d^2*g^4*h^5 + 16*a^3*b*c*d^3*g^4*h^5 + a^4*d^4*g^4*h^5 - 4*a*b^3*c^4*g^3*h^6 - 24*a^2*b^2*c^3*d*g^3*h^6 - 24*a^3*b*c^2*d^2*g^3*h^6 - 4*a^4*c*d^3*g^3*h^6 + 6*a^2*b^2*c^4*g^2*h^7 + 16*a^3*b*c^3*d*g^2*h^7 + 6*a^4*c^2*d^2*g^2*h^7 - 4*a^3*b*c^4*g*h^8 - 4*a^4*c^3*d*g*h^8 + a^4*c^4*h^9) + 1/24*(6*b^3*d^3*g^3*h^3*p*r*x^3 - 18*b^3*c*d^2*g^2*h^4*p*r*x^3 + 18*b^3*c^2*d*g*h^5*p*r*x^3 - 6*b^3*c^3*h^6*p*r*x^3 + 6*b^3*d^3*g^3*h^3*q*r*x^3 - 18*a*b^2*d^3*g^2*h^4*q*r*x^3 + 18*a^2*b*d^3*g*h^5*q*r*x^3 - 6*a^3*d^3*h^6*q*r*x^3 + 21*b^3*d^3*g^4*h^2*p*r*x^2 - 63*b^3*c*d^2*g^3*h^3*p*r*x^2 - 3*a*b^2*d^3*g^3*h^3*p*r*x^2 + 63*b^3*c^2*d*g^2*h^4*p*r*x^2 + 9*a*b^2*c*d^2*g^2*h^4*p*r*x^2 - 21*b^3*c^3*g*h^5*p*r*x^2 - 9*a*b^2*c^2*d*g*h^5*p*r*x^2 + 3*a*b^2*c^3*h^6*p*r*x^2 + 21*b^3*d^3*g^4*h^2*q*r*x^2 - 3*b^3*c*d^2*g^3*h^3*q*r*x^2 - 63*a*b^2*d^3*g^3*h^3*q*r*x^2 + 9*a*b^2*c*d^2*g^2*h^4*q*r*x^2 + 63*a^2*b*d^3*g^2*h^4*q*r*x^2 - 9*a^2*b*c*d^2*g*h^5*q*r*x^2 - 21*a^3*d^3*g*h^5*q*r*x^2 + 3*a^3*c*d^2*h^6*q*r*x^2 + 26*b^3*d^3*g^5*h*p*r*x - 78*b^3*c*d^2*g^4*h^2*p*r*x - 10*a*b^2*d^3*g^4*h^2*p*r*x + 78*b^3*c^2*d*g^3*h^3*p*r*x + 30*a*b^2*c*d^2*g^3*h^3*p*r*x + 2*a^2*b*d^3*g^3*h^3*p*r*x - 26*b^3*c^3*g^2*h^4*p*r*x - 30*a*b^2*c^2*d*g^2*h^4*p*r*x + 10*a*b^2*c^3*g*h^5*p*r*x + 6*a^2*b*c^2*d*g*h^5*p*r*x - 2*a^2*b*c^3*h^6*p*r*x + 26*b^3*d^3*g^5*h*q*r*x - 10*b^3*c*d^2*g^4*h^2*q*r*x - 78*a*b^2*d^3*g^4*h^2*q*r*x + 2*b^3*c^2*d*g^3*h^3*q*r*x + 30*a*b^2*c*d^2*g^3*h^3*q*r*x + 78*a^2*b*d^3*g^3*h^3*q*r*x - 6*a*b^2*c^2*d*g^2*h^4*q*r*x - 30*a^2*b*c*d^2*g^2*h^4*q*r*x - 26*a^3*d^3*g^2*h^4*q*r*x + 6*a^2*b*c^2*d*g*h^5*q*r*x + 10*a^3*c*d^2*g*h^5*q*r*x - 2*a^3*c^2*d*h^6*q*r*x + 11*b^3*d^3*g^6*p*r - 33*b^3*c*d^2*g^5*h*p*r - 7*a*b^2*d^3*g^5*h*p*r + 33*b^3*c^2*d*g^4*h^2*p*r + 21*a*b^2*c*d^2*g^4*h^2*p*r + 2*a^2*b*d^3*g^4*h^2*p*r - 11*b^3*c^3*g^3*h^3*p*r - 21*a*b^2*c^2*d*g^3*h^3*p*r - 6*a^2*b*c*d^2*g^3*h^3*p*r + 7*a*b^2*c^3*g^2*h^4*p*r + 6*a^2*b*c^2*d*g^2*h^4*p*r - 2*a^2*b*c^3*g*h^5*p*r + 11*b^3*d^3*g^6*q*r - 7*b^3*c*d^2*g^5*h*q*r - 33*a*b^2*d^3*g^5*h*q*r + 2*b^3*c^2*d*g^4*h^2*q*r + 21*a*b^2*c*d^2*g^4*h^2*q*r + 33*a^2*b*d^3*g^4*h^2*q*r - 6*a*b^2*c^2*d*g^3*h^3*q*r - 21*a^2*b*c*d^2*g^3*h^3*q*r - 11*a^3*d^3*g^3*h^3*q*r + 6*a^2*b*c^2*d*g^2*h^4*q*r + 7*a^3*c*d^2*g^2*h^4*q*r - 2*a^3*c^2*d*g*h^5*q*r
\end{aligned}$$

$$\begin{aligned}
& - 6b^3d^3g^6r \log(f) + 18b^3c^2d^2g^5h^r \log(f) + 18ab^2d^3g^5h^r \log(f) - 18b^3c^2d^2g^4h^2r \log(f) - 54ab^2c^2d^2g^4h^2r \log(f) \\
& - 18a^2bd^3g^4h^2r \log(f) + 6b^3c^3g^3h^3r \log(f) + 54ab^2c^2d^2g^3h^3r \log(f) + 54a^2b^2c^2d^2g^3h^3r \log(f) + 6a^3d^3g^3h^3r \log(f) \\
& - 18ab^2c^3g^2h^4r \log(f) - 54a^2b^2c^2d^2g^2h^4r \log(f) - 18a^3c^2d^2g^2h^4r \log(f) + 18a^2b^2c^3g^2h^5r \log(f) + 18a^3c^2d^2g^2h^5r \log(f) \\
& - 6a^3c^3h^6r \log(f) - 6b^3d^3g^6 + 18b^3c^2d^2g^5h + 18ab^2d^3g^5h - 18b^3c^2d^2g^4h^2 - 54ab^2c^2d^2g^4h^2 - 18a^2bd^3g^4h^2 \\
& + 6b^3c^3g^3h^3 + 54ab^2c^2d^2g^3h^3 + 54a^2b^2c^2d^2g^3h^3 + 6a^3d^3g^3h^3 - 18ab^2c^3g^2h^4 - 54a^2b^2c^2d^2g^2h^4 - 18a^3c^2d^2g^2h^4 \\
& + 18a^2b^2c^3g^2h^5 + 18a^3c^2d^2g^2h^5 - 6a^3c^3h^6) / (b^3d^3g^6h^5x^4 - 3b^3c^2d^2g^5h^6x^4 - 3ab^2d^3g^5h^6x^4 + 3b^3c^2d^2g^4h^7x^4 \\
& + 9ab^2c^2d^2g^4h^7x^4 + 3a^2bd^3g^4h^7x^4 - b^3c^3g^3h^8x^4 - 9ab^2c^2d^2g^3h^8x^4 - 9a^2b^2c^2d^2g^3h^8x^4 - a^3d^3g^3h^8x^4 \\
& + 3ab^2c^3g^2h^9x^4 + 9a^2b^2c^2d^2g^2h^9x^4 + 3a^3c^2d^2g^2h^9x^4 - 3a^2b^2c^3g^2h^10x^4 - 3a^3c^2d^2g^2h^10x^4 + a^3c^3h^11x^4 \\
& + 4b^3d^3g^7h^4x^3 - 12b^3c^2d^2g^6h^5x^3 - 12ab^2d^3g^6h^5x^3 + 12b^3c^2d^2g^5h^6x^3 + 36ab^2c^2d^2g^5h^6x^3 + 12a^2bd^3g^5h^6x^3 \\
& - 4b^3c^3g^4h^7x^3 - 36ab^2c^2d^2g^4h^7x^3 - 36a^2b^2c^2d^2g^4h^7x^3 - 4a^3d^3g^4h^7x^3 + 12ab^2c^3g^3h^8x^3 + 36a^2b^2c^2d^2g^3h^8x^3 \\
& + 12a^3c^2d^2g^3h^8x^3 - 12a^2b^2c^3g^2h^9x^3 - 12a^3c^2d^2g^2h^9x^3 + 4a^3c^3g^2h^10x^3 + 6b^3d^3g^8h^3x^2 - 18b^3c^2d^2g^7h^4x^2 \\
& - 18ab^2d^3g^7h^4x^2 + 18b^3c^2d^2g^6h^5x^2 + 54ab^2c^2d^2g^6h^5x^2 + 18a^2bd^3g^6h^5x^2 - 6b^3c^3g^5h^6x^2 - 54ab^2c^2d^2g^5h^6x^2 \\
& - 54a^2b^2c^2d^2g^5h^6x^2 - 6a^3d^3g^5h^6x^2 + 18ab^2c^3g^4h^7x^2 + 54a^2b^2c^2d^2g^4h^7x^2 + 18a^3c^2d^2g^4h^7x^2 - 18a^2b^2c^3g^3h^8x^2 \\
& - 18a^3c^2d^2g^3h^8x^2 + 6a^3c^3g^2h^9x^2 + 4b^3d^3g^9h^2x - 12b^3c^2d^2g^8h^3x - 12ab^2d^3g^8h^3x + 12b^3c^2d^2g^7h^4x \\
& + 36ab^2c^2d^2g^7h^4x + 12a^2bd^3g^7h^4x - 4b^3c^3g^6h^5x - 36ab^2c^2d^2g^6h^5x - 36a^2b^2c^2d^2g^6h^5x - 4a^3d^3g^6h^5x \\
& + 12ab^2c^3g^5h^6x + 36a^2b^2c^2d^2g^5h^6x + 12a^3c^2d^2g^5h^6x - 12a^2b^2c^3g^4h^7x - 12a^3c^2d^2g^4h^7x + 4a^3c^3g^3h^8x \\
& + b^3d^3g^10h - 3b^3c^2d^2g^9h^2 - 3ab^2d^3g^9h^2 + 3b^3c^2d^2g^8h^3 + 9ab^2c^2d^2g^8h^3 + 3a^2bd^3g^8h^3 - b^3c^3g^7h^4 \\
& - 9ab^2c^2d^2g^7h^4 - 9a^2b^2c^2d^2g^7h^4 - a^3d^3g^7h^4 + 3ab^2c^3g^6h^5 + 9a^2b^2c^2d^2g^6h^5 + 3a^3c^2d^2g^6h^5 - 3a^2b^2c^3g^5h^6 \\
& - 3a^3c^2d^2g^5h^6 + a^3c^3g^4h^7) + 1/8(b^5c^2d^4g^4p^r - ab^4d^5g^4p^r - 4b^5c^2d^3g^3h^p^r + 4ab^4c^2d^4g^3h^p^r \\
& + 6b^5c^3d^2g^2h^2p^r - 6ab^4c^2d^3g^2h^2p^r - 4b^5c^4d^2g^2h^2p^r + 4ab^4c^3d^2g^2h^2p^r + b^5c^5h^4p^r - ab^4c^4d^2h^4p^r \\
& - b^5c^4d^4g^4q^r + ab^4d^5g^4q^r + 4ab^4c^2d^4g^3h^q^r - 4a^2b^3d^5g^3h^q^r - 6a^2b^3c^2d^4g^2h^2q^r + 6a^3b^2d^5g^2h^2q^r \\
& + 4a^3b^2c^2d^4g^2h^2q^r + 4a^3b^2c^2d^4g^2h^2q^r - 4a^4b^2d^5g^2h^2q^r - a^4b^2c^2d^4h^4q^r + a^5d^5h^4q^r) * \log(\text{abs}((2bdx + bc + ad - \text{abs}(-bc + ad)
\end{aligned}$$

$$\begin{aligned} &)/(2*b*d*x + b*c + a*d + \text{abs}(-b*c + a*d)))/((b^4*d^4*g^8*h - 4*b^4*c*d^3*g \\ &^7*h^2 - 4*a*b^3*d^4*g^7*h^2 + 6*b^4*c^2*d^2*g^6*h^3 + 16*a*b^3*c*d^3*g^6*h \\ &^3 + 6*a^2*b^2*d^4*g^6*h^3 - 4*b^4*c^3*d*g^5*h^4 - 24*a*b^3*c^2*d^2*g^5*h^4 \\ &- 24*a^2*b^2*c*d^3*g^5*h^4 - 4*a^3*b*d^4*g^5*h^4 + b^4*c^4*g^4*h^5 + 16*a* \\ &b^3*c^3*d*g^4*h^5 + 36*a^2*b^2*c^2*d^2*g^4*h^5 + 16*a^3*b*c*d^3*g^4*h^5 + a \\ &^4*d^4*g^4*h^5 - 4*a*b^3*c^4*g^3*h^6 - 24*a^2*b^2*c^3*d*g^3*h^6 - 24*a^3*b* \\ &c^2*d^2*g^3*h^6 - 4*a^4*c*d^3*g^3*h^6 + 6*a^2*b^2*c^4*g^2*h^7 + 16*a^3*b*c^ \\ &3*d*g^2*h^7 + 6*a^4*c^2*d^2*g^2*h^7 - 4*a^3*b*c^4*g*h^8 - 4*a^4*c^3*d*g*h^8 \\ &+ a^4*c^4*h^9)*\text{abs}(-b*c + a*d)) \end{aligned}$$



$$3.35 \quad \int (g + hx)^3 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$$

**Optimal.** Leaf size=2240

result too large to display

```
[Out] (2*(b*g - a*h)^3*p^2*r^2*x)/b^3 + (5*(b*g - a*h)^3*p*q*r^2*x)/(8*b^3) + (5*
(b*g - a*h)^2*(d*g - c*h)*p*q*r^2*x)/(12*b^2*d) + (5*(b*g - a*h)*(d*g - c*h
)^2*p*q*r^2*x)/(12*b*d^2) + (5*(d*g - c*h)^3*p*q*r^2*x)/(8*d^3) + (2*(d*g -
c*h)^3*q^2*r^2*x)/d^3 + (3*h*(b*g - a*h)^2*p^2*r^2*(a + b*x)^2)/(4*b^4) +
(2*h^2*(b*g - a*h)*p^2*r^2*(a + b*x)^3)/(9*b^4) + (h^3*p^2*r^2*(a + b*x)^4
)/(32*b^4) + (3*h*(d*g - c*h)^2*q^2*r^2*(c + d*x)^2)/(4*d^4) + (2*h^2*(d*g -
c*h)*q^2*r^2*(c + d*x)^3)/(9*d^4) + (h^3*q^2*r^2*(c + d*x)^4)/(32*d^4) + (
3*(b*g - a*h)^2*p*q*r^2*(g + h*x)^2)/(16*b^2*h) + ((b*g - a*h)*(d*g - c*h)*
p*q*r^2*(g + h*x)^2)/(6*b*d*h) + (3*(d*g - c*h)^2*p*q*r^2*(g + h*x)^2)/(16*
d^2*h) + (7*(b*g - a*h)*p*q*r^2*(g + h*x)^3)/(72*b*h) + (7*(d*g - c*h)*p*q*
r^2*(g + h*x)^3)/(72*d*h) + (p*q*r^2*(g + h*x)^4)/(16*h) + ((b*g - a*h)^4*p
*q*r^2*Log[a + b*x])/(8*b^4*h) + ((b*g - a*h)^3*(d*g - c*h)*p*q*r^2*Log[a +
b*x])/(6*b^3*d*h) + ((b*g - a*h)^2*(d*g - c*h)^2*p*q*r^2*Log[a + b*x])/(4*
b^2*d^2*h) - (2*(b*g - a*h)^3*p^2*r^2*(a + b*x)*Log[a + b*x])/b^4 - ((d*g -
c*h)^3*p*q*r^2*(a + b*x)*Log[a + b*x])/(2*b*d^3) - (3*h*(b*g - a*h)^2*p^2*
r^2*(a + b*x)^2*Log[a + b*x])/(2*b^4) - (2*h^2*(b*g - a*h)*p^2*r^2*(a + b*x
)^3*Log[a + b*x])/(3*b^4) - (h^3*p^2*r^2*(a + b*x)^4*Log[a + b*x])/(8*b^4)
- ((d*g - c*h)^2*p*q*r^2*(g + h*x)^2*Log[a + b*x])/(4*d^2*h) - ((d*g - c*h)
*p*q*r^2*(g + h*x)^3*Log[a + b*x])/(6*d*h) - (p*q*r^2*(g + h*x)^4*Log[a + b
*x])/(8*h) - ((b*g - a*h)^4*p^2*r^2*Log[a + b*x]^2)/(4*b^4*h) + ((b*g - a*h
)^2*(d*g - c*h)^2*p*q*r^2*Log[c + d*x])/(4*b^2*d^2*h) + ((b*g - a*h)*(d*g -
c*h)^3*p*q*r^2*Log[c + d*x])/(6*b*d^3*h) + ((d*g - c*h)^4*p*q*r^2*Log[c +
d*x])/(8*d^4*h) - ((b*g - a*h)^3*p*q*r^2*(c + d*x)*Log[c + d*x])/(2*b^3*d)
- (2*(d*g - c*h)^3*q^2*r^2*(c + d*x)*Log[c + d*x])/d^4 - (3*h*(d*g - c*h)^2
*q^2*r^2*(c + d*x)^2*Log[c + d*x])/(2*d^4) - (2*h^2*(d*g - c*h)*q^2*r^2*(c
+ d*x)^3*Log[c + d*x])/(3*d^4) - (h^3*q^2*r^2*(c + d*x)^4*Log[c + d*x])/(8*
d^4) - ((b*g - a*h)^2*p*q*r^2*(g + h*x)^2*Log[c + d*x])/(4*b^2*h) - ((b*g -
a*h)*p*q*r^2*(g + h*x)^3*Log[c + d*x])/(6*b*h) - (p*q*r^2*(g + h*x)^4*Log[
c + d*x])/(8*h) - ((b*g - a*h)^4*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*
Log[c + d*x])/(2*b^4*h) - ((d*g - c*h)^4*q^2*r^2*Log[c + d*x]^2)/(4*d^4*h)
- ((d*g - c*h)^4*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*d^
4*h) + ((b*g - a*h)^3*p*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f
*(a + b*x)^p*(c + d*x)^q]^r)))/(2*b^3) + ((d*g - c*h)^3*q*r*x*(p*r*Log[a +
b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(2*d^3) +
((b*g - a*h)^2*p*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e
*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(4*b^2*h) + ((d*g - c*h)^2*q*r*(g + h*x)^
2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^

```

$$\begin{aligned} & r]))/(4*d^2*h) + ((b*g - a*h)*p*r*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c \\ & + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(6*b*h) + ((d*g - c*h)*q*r \\ & *(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c \\ & + d*x)^q]^r)))/(6*d*h) + (p*r*(g + h*x)^4*(p*r*Log[a + b*x] + q*r*Log[c + \\ & d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(8*h) + (q*r*(g + h*x)^4*(p*r \\ & *Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/( \\ & 8*h) + ((b*g - a*h)^4*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] \\ & - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(2*b^4*h) + ((d*g - c*h)^4*q*r*Lo \\ & g[c + d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + \\ & d*x)^q]^r)))/(2*d^4*h) + ((g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r \\ & ^2)/(4*h) - ((d*g - c*h)^4*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))] \\ & )/(2*d^4*h) - ((b*g - a*h)^4*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] \\ & )/(2*b^4*h) \end{aligned}$$

**Rubi [A]** time = 2.43569, antiderivative size = 2220, normalized size of antiderivative = 0.99, number of steps used = 49, number of rules used = 14, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$ , Rules used = {2498, 2513, 2411, 43, 2334, 12, 2301, 2418, 2389, 2295, 2394, 2393, 2391, 2395}

result too large to display

Antiderivative was successfully verified.

[In] Int[(g + h\*x)^3\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2,x]

[Out]  $(2*(b*g - a*h)^3*p^2*r^2*x)/b^3 + (5*(b*g - a*h)^3*p*q*r^2*x)/(8*b^3) + (5*(b*g - a*h)^2*(d*g - c*h)*p*q*r^2*x)/(12*b^2*d) + (5*(b*g - a*h)*(d*g - c*h)^2*p*q*r^2*x)/(12*b*d^2) + (5*(d*g - c*h)^3*p*q*r^2*x)/(8*d^3) + (2*(d*g - c*h)^3*q^2*r^2*x)/d^3 + (3*h*(b*g - a*h)^2*p^2*r^2*(a + b*x)^2)/(4*b^4) + (2*h^2*(b*g - a*h)*p^2*r^2*(a + b*x)^3)/(9*b^4) + (h^3*p^2*r^2*(a + b*x)^4)/(32*b^4) + (3*h*(d*g - c*h)^2*q^2*r^2*(c + d*x)^2)/(4*d^4) + (2*h^2*(d*g - c*h)*q^2*r^2*(c + d*x)^3)/(9*d^4) + (h^3*q^2*r^2*(c + d*x)^4)/(32*d^4) + (3*(b*g - a*h)^2*p*q*r^2*(g + h*x)^2)/(16*b^2*h) + ((b*g - a*h)*(d*g - c*h)*p*q*r^2*(g + h*x)^2)/(6*b*d*h) + (3*(d*g - c*h)^2*p*q*r^2*(g + h*x)^2)/(16*d^2*h) + (7*(b*g - a*h)*p*q*r^2*(g + h*x)^3)/(72*b*h) + (7*(d*g - c*h)*p*q*r^2*(g + h*x)^3)/(72*d*h) + (p*q*r^2*(g + h*x)^4)/(16*h) + ((b*g - a*h)^4*p*q*r^2*Log[a + b*x])/(8*b^4*h) + ((b*g - a*h)^3*(d*g - c*h)*p*q*r^2*Log[a + b*x])/(6*b^3*d*h) + ((b*g - a*h)^2*(d*g - c*h)^2*p*q*r^2*Log[a + b*x])/(4*b^2*d^2*h) - ((d*g - c*h)^3*p*q*r^2*(a + b*x)*Log[a + b*x])/(2*b*d^3) - ((d*g - c*h)^2*p*q*r^2*(g + h*x)^2*Log[a + b*x])/(4*d^2*h) - ((d*g - c*h)*p*q*r^2*(g + h*x)^3*Log[a + b*x])/(6*d*h) - (p*q*r^2*(g + h*x)^4*Log[a + b*x])/(8*h) + ((b*g - a*h)^4*p^2*r^2*Log[a + b*x]^2)/(4*b^4*h) - (p^2*r^2*Log[a +$

$$\begin{aligned}
& b*x]*((48*h*(b*g - a*h)^3*(a + b*x))/b^4 + (36*h^2*(b*g - a*h)^2*(a + b*x) \\
& ^2)/b^4 + (16*h^3*(b*g - a*h)*(a + b*x)^3)/b^4 + (3*h^4*(a + b*x)^4)/b^4 + \\
& (12*(b*g - a*h)^4*\text{Log}[a + b*x])/b^4)/(24*h) + ((b*g - a*h)^2*(d*g - c*h)^2 \\
& *p*q*r^2*\text{Log}[c + d*x])/(4*b^2*d^2*h) + ((b*g - a*h)*(d*g - c*h)^3*p*q*r^2*\text{L} \\
& \text{og}[c + d*x])/(6*b*d^3*h) + ((d*g - c*h)^4*p*q*r^2*\text{Log}[c + d*x])/(8*d^4*h) - \\
& ((b*g - a*h)^3*p*q*r^2*(c + d*x)*\text{Log}[c + d*x])/(2*b^3*d) - ((b*g - a*h)^2* \\
& p*q*r^2*(g + h*x)^2*\text{Log}[c + d*x])/(4*b^2*h) - ((b*g - a*h)*p*q*r^2*(g + h*x) \\
& )^3*\text{Log}[c + d*x])/(6*b*h) - (p*q*r^2*(g + h*x)^4*\text{Log}[c + d*x])/(8*h) - ((b* \\
& g - a*h)^4*p*q*r^2*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x])/(2*b^4*h \\
& ) + ((d*g - c*h)^4*q^2*r^2*\text{Log}[c + d*x]^2)/(4*d^4*h) - (q^2*r^2*\text{Log}[c + d*x] \\
& ]*((48*h*(d*g - c*h)^3*(c + d*x))/d^4 + (36*h^2*(d*g - c*h)^2*(c + d*x)^2)/ \\
& d^4 + (16*h^3*(d*g - c*h)*(c + d*x)^3)/d^4 + (3*h^4*(c + d*x)^4)/d^4 + (12* \\
& (d*g - c*h)^4*\text{Log}[c + d*x])/d^4)/(24*h) - ((d*g - c*h)^4*p*q*r^2*\text{Log}[a + b \\
& *x]*\text{Log}[(b*(c + d*x))/(b*c - a*d)))/(2*d^4*h) + ((b*g - a*h)^3*p*r*x*(p*r*L \\
& \text{og}[a + b*x] + q*r*\text{Log}[c + d*x] - \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(2* \\
& b^3) + ((d*g - c*h)^3*q*r*x*(p*r*\text{Log}[a + b*x] + q*r*\text{Log}[c + d*x] - \text{Log}[e*(f \\
& *(a + b*x)^p*(c + d*x)^q]^r)))/(2*d^3) + ((b*g - a*h)^2*p*r*(g + h*x)^2*(p* \\
& r*\text{Log}[a + b*x] + q*r*\text{Log}[c + d*x] - \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/ \\
& (4*b^2*h) + ((d*g - c*h)^2*q*r*(g + h*x)^2*(p*r*\text{Log}[a + b*x] + q*r*\text{Log}[c + \\
& d*x] - \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(4*d^2*h) + ((b*g - a*h)*p*r* \\
& (g + h*x)^3*(p*r*\text{Log}[a + b*x] + q*r*\text{Log}[c + d*x] - \text{Log}[e*(f*(a + b*x)^p*(c \\
& + d*x)^q]^r)))/(6*b*h) + ((d*g - c*h)*q*r*(g + h*x)^3*(p*r*\text{Log}[a + b*x] + q \\
& *r*\text{Log}[c + d*x] - \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(6*d*h) + (p*r*(g \\
& + h*x)^4*(p*r*\text{Log}[a + b*x] + q*r*\text{Log}[c + d*x] - \text{Log}[e*(f*(a + b*x)^p*(c + d \\
& *x)^q]^r)))/(8*h) + (q*r*(g + h*x)^4*(p*r*\text{Log}[a + b*x] + q*r*\text{Log}[c + d*x] - \\
& \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(8*h) + ((b*g - a*h)^4*p*r*\text{Log}[a + \\
& b*x]*(p*r*\text{Log}[a + b*x] + q*r*\text{Log}[c + d*x] - \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^ \\
& q]^r)))/(2*b^4*h) + ((d*g - c*h)^4*q*r*\text{Log}[c + d*x]*(p*r*\text{Log}[a + b*x] + q*r \\
& *\text{Log}[c + d*x] - \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(2*d^4*h) + ((g + h* \\
& x)^4*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r^2)/(4*h) - ((d*g - c*h)^4*p*q*r^2 \\
& *PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(2*d^4*h) - ((b*g - a*h)^4*p*q*r \\
& ^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)))/(2*b^4*h)
\end{aligned}$$

### Rule 2498

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
+ 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]

```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFX_), x_Symbol] := Dist[p*r, Int[RFX*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFX*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFX, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFX, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFX, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n
]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] := Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
+ b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n]]^p, RFx, x]},
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

### Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

### Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e^n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^q, x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e^n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

### Rubi steps

$$\begin{aligned}
\int (g + hx)^3 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(g + hx)^4 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4h} - \frac{(bpr) \int \frac{(g+hx)^4 \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{a+bx}}{2h} \\
&= \frac{(g + hx)^4 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4h} - \frac{(bp^2r^2) \int \frac{(g+hx)^4 \log(a+bx)}{a+bx} dx}{2h} \\
&= \frac{(g + hx)^4 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4h} - \frac{(p^2r^2) \text{Subst} \left( \int \frac{\left( \frac{bg-ah}{b} + \frac{hx}{b} \right)^4 \log(a+bx)}{x} dx \right)}{2h} \\
&= -\frac{p^2r^2 \log(a + bx) \left( \frac{48h(bg-ah)^3(a+bx)}{b^4} + \frac{36h^2(bg-ah)^2(a+bx)^2}{b^4} + \frac{16h^3(bg-ah)(a+bx)^3}{b^4} \right)}{24h} \\
&= -\frac{(dg - ch)^2 pqr^2 (g + hx)^2 \log(a + bx)}{4d^2h} - \frac{(dg - ch)pqr^2 (g + hx)^3 \log(a + bx)}{6dh} \\
&= \frac{2(bg - ah)^3 p^2 r^2 x}{b^3} + \frac{(bg - ah)^3 pqr^2 x}{2b^3} + \frac{(dg - ch)^3 pqr^2 x}{2d^3} + \frac{2(dg - ch)^3 q^2}{d^3} \\
&= \frac{2(bg - ah)^3 p^2 r^2 x}{b^3} + \frac{5(bg - ah)^3 pqr^2 x}{8b^3} + \frac{5(bg - ah)^2 (dg - ch)pqr^2 x}{12b^2d} + \frac{5(dg - ch)^3 q^2}{d^3}
\end{aligned}$$

**Mathematica [A]** time = 3.01291, size = 1386, normalized size = 0.62

$$72a \left( -4b^3g^3 + 6ab^2hg^2 - 4a^2bh^2g + a^3h^3 \right) p^2r^2 \log^2(a + bx)d^4 + 12pr \log(a + bx) \left( 12c \left( -4d^3g^3 + 6cd^2hg^2 - 4c^2dh^2g + c^3 \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)^3\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2,x]

[Out] (72\*a\*d^4\*(-4\*b^3\*g^3 + 6\*a\*b^2\*g^2\*h - 4\*a^2\*b\*g\*h^2 + a^3\*h^3)\*p^2\*r^2\*Log[a + b\*x]^2 + 12\*p\*r\*Log[a + b\*x]\*(12\*b^4\*c\*(-4\*d^3\*g^3 + 6\*c\*d^2\*g^2\*h - 4\*c^2\*d\*g\*h^2 + c^3\*h^3)\*q\*r\*Log[c + d\*x] - 12\*(4\*a\*b^3\*d^4\*g^3 - 6\*a^2\*b^2\*d^4\*g^2\*h + 4\*a^3\*b\*d^4\*g\*h^2 - a^4\*d^4\*h^3 + b^4\*c\*(-4\*d^3\*g^3 + 6\*c\*d^2\*g^2\*h - 4\*c^2\*d\*g\*h^2 + c^3\*h^3))\*q\*r\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] + a\*d\*((12\*b^3\*(-4\*d^3\*g^3 + 6\*c\*d^2\*g^2\*h - 4\*c^2\*d\*g\*h^2 + c^3\*h^3)\*q + a^3\*d^3\*h^3\*(25\*p + 3\*q) - 4\*a^2\*b\*d^2\*h^2\*(22\*d\*g\*p + 4\*d\*g\*q - c\*h\*q) + 6\*a\*b^2\*d\*h\*(-4\*c\*d\*g\*h\*q + c^2\*h^2\*q + 6\*d^2\*g^2\*(3\*p + q))))\*r + 12\*d^3\*(4\*b^3\*g^3

$$\begin{aligned}
& - 6ab^2g^2h + 4a^2bgh^2 - a^3h^3) \operatorname{Log}[e^{(f(a+bx)^p(c+dx)^q)^r}] + b(72b^3c(-4d^3g^3 + 6cd^2g^2h - 4c^2dgh^2 + c^3h^3) \\
& )q^2r^2 \operatorname{Log}[c+dx]^2 + 12qr \operatorname{Log}[c+dx] * ((12a^3cd^3h^3p + 6a^2 \\
& *bcd^2h^2(-8d^3g + ch)p + 4ab^2d(12d^3g^3 + 18cd^2g^2h - 6c^2dgh^2 + c^3h^3)p + b^3c(-48d^3g^3(p+q) + 36cd^2g^2h(p+ \\
& 3q) - 8c^2dgh^2(2p+11q) + c^3h^3(3p+25q)))r - 12b^3c(- \\
& 4d^3g^3 + 6cd^2g^2h - 4c^2dgh^2 + c^3h^3) \operatorname{Log}[e^{(f(a+bx)^p(c+dx)^q)^r}] + d(r^2(-60a^3d^3h^3p(5p+3q)x + 6a^2b^2d^2h^2 \\
& *px(-20chq + 16d^2g(11p+8q) + dh(13p+9q)x) + b^3x(-60c^3h^3q(3p+5q) + 6c^2d^2h^2q(16g(8p+11q) + h(9p+13q)x) \\
& - 4cd^2h^2q(p+q)(324g^2 + 60ghx + 7h^2x^2) + d^3(p+q)^2(576 \\
& *g^3 + 216g^2hx + 64gh^2x^2 + 9h^3x^3)) - 4ab^2p(36c^3h^3q + \\
& 6c^2d^2h^2q(-24g + 5hx) - 12cd^2h^2q(-18g^2 + 12ghx + h^2x^2) \\
& ) + d^3(-144g^3q + 324g^2h(p+q)x + 60gh^2(p+q)x^2 + 7h^3(p \\
& + q)x^3)) + 12r(12a^3d^3h^3px - 6a^2b^2d^3h^2p^2x(8g+hx) + \\
& 4ab^2d^3p(-12g^3 + 18g^2hx + 6gh^2x^2 + h^3x^3) - b^3x(-12c^3h^3q + 6c^2d^2h^2q(8g+hx) - 4cd^2h^2q(18g^2 + 6ghx + h^2 \\
& *x^2) + d^3(p+q)(48g^3 + 36g^2hx + 16gh^2x^2 + 3h^3x^3)) \operatorname{Log}[ \\
& e^{(f(a+bx)^p(c+dx)^q)^r}] + 72b^3d^3x(4g^3 + 6g^2hx + 4gh^2 \\
& *x^2 + h^3x^3) \operatorname{Log}[e^{(f(a+bx)^p(c+dx)^q)^r}]^2) - 144(4ab^3d^4 \\
& *g^3 - 6a^2b^2d^4g^2h + 4a^3b^2d^4gh^2 - a^4d^4h^3 + b^4c(-4d^3 \\
& *g^3 + 6cd^2g^2h - 4c^2dgh^2 + c^3h^3))pqr^2 \operatorname{PolyLog}[2, (d(a \\
& + bx))/(-bc) + ad)]/(288b^4d^4)
\end{aligned}$$

**Maple [F]** time = 0.392, size = 0, normalized size = 0.

$$\int (hx + g)^3 \left( \ln \left( e^{(f(bx+a)^p(dx+c)^q)^r} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)^3\*ln(e^(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

[Out] int((h\*x+g)^3\*ln(e^(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

**Maxima [A]** time = 1.45213, size = 2429, normalized size = 1.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x, algorithm="maxima")

[Out]  $\frac{1}{4}(h^3x^4 + 4gh^2x^3 + 6g^2hx^2 + 4g^3x)\log((b*x + a)^p(d*x + c)^qf)^r e^2 + \frac{1}{24}r(12(4ab^3f^3g^3p - 6a^2b^2f^2g^2hp + 4a^3b^2f^2g^2h^2p - a^4f^3h^3p)\log(b*x + a)/b^4 + 12(4cd^3f^3g^3q - 6c^2d^2f^2g^2hq + 4c^3d^2f^2g^2h^2q - c^4f^3h^3q)\log(d*x + c)/d^4 - (3b^3d^3f^3h^3(p + q)x^4 - 4(a^2b^2d^3f^3h^3p - (4d^3f^3g^2h^2(p + q) - cd^2f^2h^3q)b^3)x^3 - 6(4ab^2d^3f^3g^2h^2p - a^2bd^3f^3h^3p - (6d^3f^2g^2h(p + q) - 4cd^2f^2g^2h^2q + c^2d^2f^2h^3q)b^3)x^2 - 12(6a^2b^2d^3f^2g^2hp - 4a^2bd^3f^2g^2h^2p + a^3d^3f^2h^3p - (4d^3f^2g^3(p + q) - 6cd^2f^2g^2hq + 4c^2d^2f^2g^2h^2q - c^3f^2h^3q)b^3)x)/(b^3d^3))\log((b*x + a)^p(d*x + c)^qf)^r e)/f + \frac{1}{288}r^2(12(12a^3cd^3f^2h^3pq - 6(8cd^3f^2g^2h^2pq - c^2d^2f^2h^3pq)a^2b + 4(18cd^3f^2g^2h^2pq - 6c^2d^2f^2g^2h^2pq + c^3d^2f^2h^3pq)a^2b^2 - (48(pq + q^2)cd^3f^2g^3 - 36(pq + 3q^2)c^2d^2f^2g^2h + 8(2pq + 11q^2)c^3d^2f^2g^2h^2 - (3pq + 25q^2)c^4f^2h^3)b^3)\log(d*x + c)/(b^3d^4) - 144(4ab^3d^4f^2g^3pq - 6a^2b^2d^4f^2g^2h^2pq + 4a^3bd^4f^2g^2h^2pq - a^4d^4f^2h^3pq - (4cd^3f^2g^3pq - 6c^2d^2f^2g^2h^2pq + 4c^3d^2f^2g^2h^2pq - c^4f^2h^3pq)b^4)(\log(b*x + a)\log((b*d*x + a*d)/(b*c - a*d) + 1) + \operatorname{dilog}(-(b*d*x + a*d)/(b*c - a*d)))/(b^4d^4) + (9(p^2 + 2pq + q^2)b^4d^4f^2h^3x^4 - 144(4cd^3f^2g^3pq - 6c^2d^2f^2g^2h^2pq + 4c^3d^2f^2g^2h^2pq - c^4f^2h^3pq)b^4)\log(b*x + a)\log(d*x + c) - 72(4cd^3f^2g^3q^2 - 6c^2d^2f^2g^2h^2q^2 + 4c^3d^2f^2g^2h^2q^2 - c^4f^2h^3q^2)b^4\log(d*x + c)^2 - 4(7(p^2 + pq)a^3d^4f^2h^3 - (16(p^2 + 2pq + q^2)d^4f^2g^2h^2 - 7(pq + q^2)cd^3f^2h^3)b^4)x^3 + 6((13p^2 + 9pq)a^2b^2d^4f^2h^3 + 8(cd^3f^2h^3pq - 5(p^2 + pq)d^4f^2g^2h^2)a^2b^3 + (36(p^2 + 2pq + q^2)d^4f^2g^2h - 40(pq + q^2)cd^3f^2g^2h^2 + (9pq + 13q^2)c^2d^2f^2h^3)b^4)x^2 - 72(4ab^3d^4f^2g^3p^2 - 6a^2b^2d^4f^2g^2h^2p^2 + 4a^3bd^4f^2g^2h^2p^2 - a^4d^4f^2h^3p^2)\log(b*x + a)^2 - 12(5(5p^2 + 3pq)a^3bd^4f^2h^3 + 2(5cd^3f^2h^3pq - 4(11p^2 + 8pq)d^4f^2g^2h^2)a^2b^2 - 2(24cd^3f^2g^2h^2pq - 5c^2d^2f^2h^3pq - 54(p^2 + pq)d^4f^2g^2h)a^2b^3 - (48(p^2 + 2pq + q^2)d^4f^2g^3 - 108(pq + q^2)cd^3f^2g^2h + 8(8pq + 11q^2)c^2d^2f^2g^2h^2 - 5(3pq + 5q^2)c^3d^2f^2h^3)b^4)x + 12((25p^2 + 3pq)a^4d^4f^2h^3 + 4(cd^3f^2h^3pq - 2(11p^2 + 2pq)d^4f^2g^2h^2)a^3b - 6(4cd^3f^2g^2h^2pq - c^2d^2f^2h^3pq - 6(3p^2 + pq)d^4f^2g^2h)a^2b^2 + 12(6cd^3f^2g^2h^2pq - 4c^2d^2f^2g^2h^2pq + c^3d^2f^2h^3pq - 4(p^2 + pq)d^4f^2g^3)a^2b^3)\log(b*x + a))/(b^4d^4)/f^2$



**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(h^3x^3 + 3gh^2x^2 + 3g^2hx + g^3\right)\log\left(\left((bx + a)^p(dx + c)^qf\right)^r e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x, algorithm="fricas")

[Out] integral((h^3\*x^3 + 3\*g\*h^2\*x^2 + 3\*g^2\*h\*x + g^3)\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*3\*ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)^3 \log\left(\left((bx + a)^p(dx + c)^qf\right)^r e\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x, algorithm="giac")

[Out] integrate((h\*x + g)^3\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2, x)

### 3.36 $\int (g + hx)^2 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$

**Optimal.** Leaf size=1645

result too large to display

```
[Out] (2*(b*g - a*h)^2*p^2*r^2*x)/b^2 + (8*(b*g - a*h)^2*p*q*r^2*x)/(9*b^2) + (2*(b*g - a*h)*(d*g - c*h)*p*q*r^2*x)/(3*b*d) + (8*(d*g - c*h)^2*p*q*r^2*x)/(9*d^2) + (2*(d*g - c*h)^2*q^2*r^2*x)/d^2 + (h*(b*g - a*h)*p^2*r^2*(a + b*x)^2)/(2*b^3) + (2*h^2*p^2*r^2*(a + b*x)^3)/(27*b^3) + (h*(d*g - c*h)*q^2*r^2*(c + d*x)^2)/(2*d^3) + (2*h^2*q^2*r^2*(c + d*x)^3)/(27*d^3) + (5*(b*g - a*h)*p*q*r^2*(g + h*x)^2)/(18*b*h) + (5*(d*g - c*h)*p*q*r^2*(g + h*x)^2)/(18*d*h) + (4*p*q*r^2*(g + h*x)^3)/(27*h) + (2*(b*g - a*h)^3*p*q*r^2*Log[a + b*x])/((9*b^3*h) + ((b*g - a*h)^2*(d*g - c*h)*p*q*r^2*Log[a + b*x])/(3*b^2*d*h) - (2*(b*g - a*h)^2*p^2*r^2*(a + b*x)*Log[a + b*x])/b^3 - (2*(d*g - c*h)^2*p*q*r^2*(a + b*x)*Log[a + b*x])/(3*b*d^2) - (h*(b*g - a*h)*p^2*r^2*(a + b*x)^2*Log[a + b*x])/b^3 - (2*h^2*p^2*r^2*(a + b*x)^3*Log[a + b*x])/(9*b^3) - ((d*g - c*h)*p*q*r^2*(g + h*x)^2*Log[a + b*x])/(3*d*h) - (2*p*q*r^2*(g + h*x)^3*Log[a + b*x])/(9*h) - ((b*g - a*h)^3*p^2*r^2*Log[a + b*x]^2)/(3*b^3*h) + ((b*g - a*h)*(d*g - c*h)^2*p*q*r^2*Log[c + d*x])/(3*b*d^2*h) + (2*(d*g - c*h)^3*p*q*r^2*Log[c + d*x])/(9*d^3*h) - (2*(b*g - a*h)^2*p*q*r^2*(c + d*x)*Log[c + d*x])/(3*b^2*d) - (2*(d*g - c*h)^2*q^2*r^2*(c + d*x)*Log[c + d*x])/d^3 - (h*(d*g - c*h)*q^2*r^2*(c + d*x)^2*Log[c + d*x])/d^3 - (2*h^2*q^2*r^2*(c + d*x)^3*Log[c + d*x])/(9*d^3) - ((b*g - a*h)*p*q*r^2*(g + h*x)^2*Log[c + d*x])/(3*b*h) - (2*p*q*r^2*(g + h*x)^3*Log[c + d*x])/(9*h) - (2*(b*g - a*h)^3*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b^3*h) - ((d*g - c*h)^3*q^2*r^2*Log[c + d*x]^2)/(3*d^3*h) - (2*(d*g - c*h)^3*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]/(3*d^3*h) + (2*(b*g - a*h)^2*p*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*b^2) + (2*(d*g - c*h)^2*q*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*d^2) + ((b*g - a*h)*p*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*b*h) + ((d*g - c*h)*q*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*d*h) + (2*p*r*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(9*h) + (2*q*r*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(9*h) + (2*(b*g - a*h)^3*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*b^3*h) + (2*(d*g - c*h)^3*q*r*Log[c + d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*d^3*h) + ((g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r^2)/(3*h) - (2*(d*g - c*h)^3*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*d^3*h) - (2*(b*g - a*h)^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]/(3*b^3*h)
```

**Rubi [A]** time = 1.75844, antiderivative size = 1657, normalized size of antiderivative = 1.01, number of steps used = 47, number of rules used = 15, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$ , Rules used = {2498, 2513, 2411, 43, 2334, 12, 14, 2301, 2418, 2389, 2295, 2394, 2393, 2391, 2395}

result too large to display

Antiderivative was successfully verified.

[In] Int[(g + h\*x)^2\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2, x]

[Out]  $(2*(b*g - a*h)^2*p^2*r^2*x)/b^2 + (8*(b*g - a*h)^2*p*q*r^2*x)/(9*b^2) + (2*(b*g - a*h)*(d*g - c*h)*p*q*r^2*x)/(3*b*d) + (8*(d*g - c*h)^2*p*q*r^2*x)/(9*d^2) + (2*(d*g - c*h)^2*q^2*r^2*x)/d^2 + (h*(b*g - a*h)*p^2*r^2*(a + b*x)^2)/(2*b^3) + (2*h^2*p^2*r^2*(a + b*x)^3)/(27*b^3) + (h*(d*g - c*h)*q^2*r^2*(c + d*x)^2)/(2*d^3) + (2*h^2*q^2*r^2*(c + d*x)^3)/(27*d^3) + (5*(b*g - a*h)*p*q*r^2*(g + h*x)^2)/(18*b*h) + (5*(d*g - c*h)*p*q*r^2*(g + h*x)^2)/(18*d*h) + (4*p*q*r^2*(g + h*x)^3)/(27*h) + (2*(b*g - a*h)^3*p*q*r^2*Log[a + b*x])/ (9*b^3*h) + ((b*g - a*h)^2*(d*g - c*h)*p*q*r^2*Log[a + b*x])/ (3*b^2*d*h) - (2*(d*g - c*h)^2*p*q*r^2*(a + b*x)*Log[a + b*x])/ (3*b*d^2) - ((d*g - c*h)*p*q*r^2*(g + h*x)^2*Log[a + b*x])/ (3*d*h) - (2*p*q*r^2*(g + h*x)^3*Log[a + b*x])/ (9*h) + ((b*g - a*h)^3*p^2*r^2*Log[a + b*x]^2)/ (3*b^3*h) - (p^2*r^2*Log[a + b*x]*((18*h*(b*g - a*h)^2*(a + b*x))/b^3 + (9*h^2*(b*g - a*h)*(a + b*x)^2)/b^3 + (2*h^3*(a + b*x)^3)/b^3 + (6*(b*g - a*h)^3*Log[a + b*x])/b^3))/ (9*h) + ((b*g - a*h)*(d*g - c*h)^2*p*q*r^2*Log[c + d*x])/ (3*b*d^2*h) + (2*(d*g - c*h)^3*p*q*r^2*Log[c + d*x])/ (9*d^3*h) - (2*(b*g - a*h)^2*p*q*r^2*(c + d*x)*Log[c + d*x])/ (3*b^2*d) - ((b*g - a*h)*p*q*r^2*(g + h*x)^2*Log[c + d*x])/ (3*b*h) - (2*p*q*r^2*(g + h*x)^3*Log[c + d*x])/ (9*h) - (2*(b*g - a*h)^3*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/ (3*b^3*h) + ((d*g - c*h)^3*q^2*r^2*Log[c + d*x]^2)/ (3*d^3*h) - (q^2*r^2*Log[c + d*x]*((18*h*(d*g - c*h)^2*(c + d*x))/d^3 + (9*h^2*(d*g - c*h)*(c + d*x)^2)/d^3 + (2*h^3*(c + d*x)^3)/d^3 + (6*(d*g - c*h)^3*Log[c + d*x])/d^3))/ (9*h) - (2*(d*g - c*h)^3*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/ (3*d^3*h) + (2*(b*g - a*h)^2*p*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/ (3*b^2) + (2*(d*g - c*h)^2*q*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/ (3*d^2) + ((b*g - a*h)*p*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/ (3*b*h) + ((d*g - c*h)*q*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/ (3*d*h) + (2*p*r*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/ (9*h) + (2*q*r*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/ (9*h) + (2*(b*g - a*h)^3*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a$

$$\begin{aligned} & + b*x)^p*(c + d*x)^q)^r]/(3*b^3*h) + (2*(d*g - c*h)^3*q*r*\text{Log}[c + d*x]*( \\ & p*r*\text{Log}[a + b*x] + q*r*\text{Log}[c + d*x] - \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) \\ & )/(3*d^3*h) + ((g + h*x)^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(3*h) - \\ & (2*(d*g - c*h)^3*p*q*r^2*\text{PolyLog}[2, -((d*(a + b*x))/(b*c - a*d))]/(3*d^3*h) \\ & ) - (2*(b*g - a*h)^3*p*q*r^2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)]/(3*b^3* \\ & h) \end{aligned}$$

### Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*s)
/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
+ 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && NeQ[m, -1]
```

### Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_), x_Symbol] := Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dist
[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0]
&& !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.)] /; IntegersQ[m, n]
```

### Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)
*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] := Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] := Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\* (b\_.)]/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))])\* (b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

### Rubi steps

$$\begin{aligned}
\int (g + hx)^2 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(g + hx)^3 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{3h} - \frac{(2bpr) \int \frac{(g+hx)^3 \log \left( e \left( f(a+bx)^p \right)}{a+bx}}{3h}}{3h} \\
&= \frac{(g + hx)^3 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{3h} - \frac{(2bp^2r^2) \int \frac{(g+hx)^3 \log(a+bx)}{a+bx}}{3h} \\
&= \frac{(g + hx)^3 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{3h} - \frac{(2p^2r^2) \text{Subst} \left( \int \frac{\left( \frac{bg-ah}{b} + \frac{hx}{b} \right)}{x} \right)}{3h} \\
&= -\frac{p^2r^2 \log(a + bx) \left( \frac{18h(bg-ah)^2(a+bx)}{b^3} + \frac{9h^2(bg-ah)(a+bx)^2}{b^3} + \frac{2h^3(a+bx)^3}{b^3} + \frac{6(bg-ah)^2}{b^3} \right)}{9h} \\
&= -\frac{(dg - ch)pqr^2(g + hx)^2 \log(a + bx)}{3dh} - \frac{2pqr^2(g + hx)^3 \log(a + bx)}{9h} - \frac{p}{9h} \\
&= \frac{2(bg - ah)^2 pqr^2 x}{3b^2} + \frac{2(dg - ch)^2 pqr^2 x}{3d^2} - \frac{2(dg - ch)^2 pqr^2 (a + bx) \log(a + bx)}{3bd^2} \\
&= \frac{8(bg - ah)^2 pqr^2 x}{9b^2} + \frac{2(bg - ah)(dg - ch)pqr^2 x}{3bd} + \frac{8(dg - ch)^2 pqr^2 x}{9d^2} + \frac{5}{9} \\
&= \frac{2(bg - ah)^2 p^2 r^2 x}{b^2} + \frac{8(bg - ah)^2 pqr^2 x}{9b^2} + \frac{2(bg - ah)(dg - ch)pqr^2 x}{3bd} + \frac{8}{9}
\end{aligned}$$

**Mathematica [A]** time = 1.89012, size = 899, normalized size = 0.55

$$-18a(3b^2g^2 - 3abhg + a^2h^2)p^2r^2 \log^2(a + bx)d^3 - 6pr \log(a + bx) \left( 6c(3d^2g^2 - 3cdhg + c^2h^2)qr \log(c + dx)b^3 - 6(bc - a^2d) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)^2\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2,x]

[Out] (-18\*a\*d^3\*(3\*b^2\*g^2 - 3\*a\*b\*g\*h + a^2\*h^2)\*p^2\*r^2\*Log[a + b\*x]^2 - 6\*p\*r\*Log[a + b\*x]\*(6\*b^3\*c\*(3\*d^2\*g^2 - 3\*c\*d\*g\*h + c^2\*h^2)\*q\*r\*Log[c + d\*x] - 6\*(b\*c - a\*d)\*(a^2\*d^2\*h^2 + a\*b\*d\*h\*(-3\*d\*g + c\*h) + b^2\*(3\*d^2\*g^2 - 3\*c\*d\*g\*h + c^2\*h^2))\*q\*r\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] + a\*d\*((6\*b^2\*(3\*d^2\*g^2 - 3\*c\*d\*g\*h + c^2\*h^2)\*q + a^2\*d^2\*h^2\*(11\*p + 2\*q) - 3\*a\*b\*d\*h\*(-(c\*h\*q) + 3\*d\*g\*(3\*p + q)))\*r - 6\*d^2\*(3\*b^2\*g^2 - 3\*a\*b\*g\*h + a^2\*h^2)\*Log[e\*(f

$$\begin{aligned} &*(a + b*x)^p*(c + d*x)^q)^r])) + b*(-18*b^2*c*(3*d^2*g^2 - 3*c*d*g*h + c^2* \\ &h^2)*q^2*r^2*\text{Log}[c + d*x]^2 - 6*q*r*\text{Log}[c + d*x]*((6*a^2*c*d^2*h^2*p - 3*a* \\ &b*d*(6*d^2*g^2 + 6*c*d*g*h - c^2*h^2)*p + b^2*c*(18*d^2*g^2*(p + q) - 9*c*d \\ &*g*h*(p + 3*q) + c^2*h^2*(2*p + 11*q)))*r - 6*b^2*c*(3*d^2*g^2 - 3*c*d*g*h \\ &+ c^2*h^2)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + d*(r^2*(6*a^2*d^2*h^2*p* \\ &(11*p + 8*q)*x + b^2*x*(6*c^2*h^2*q*(8*p + 11*q) - 3*c*d*h*q*(p + q)*(54*g \\ &+ 5*h*x) + d^2*(p + q)^2*(108*g^2 + 27*g*h*x + 4*h^2*x^2)) - 3*a*b*p*(-12*c \\ &^2*h^2*q - 12*c*d*h*q*(-3*g + h*x) + d^2*(-36*g^2*q + 54*g*h*(p + q)*x + 5* \\ &h^2*(p + q)*x^2)) - 6*r*(6*a^2*d^2*h^2*p*x + 3*a*b*d^2*p*(6*g^2 - 6*g*h*x \\ &- h^2*x^2) + b^2*x*(6*c^2*h^2*q - 3*c*d*h*q*(6*g + h*x) + d^2*(p + q)*(18*g \\ &^2 + 9*g*h*x + 2*h^2*x^2)))*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*b^2*d \\ &^2*x*(3*g^2 + 3*g*h*x + h^2*x^2)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2) + \\ &36*(b*c - a*d)*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3* \\ &c*d*g*h + c^2*h^2))*p*q*r^2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]/(54*b \\ &^3*d^3) \end{aligned}$$

**Maple [F]** time = 0.385, size = 0, normalized size = 0.

$$\int (hx + g)^2 \left( \ln \left( e \left( f(bx + a)^p (dx + c)^q \right)^r \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)^2\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

[Out] int((h\*x+g)^2\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

**Maxima [A]** time = 1.37896, size = 1516, normalized size = 0.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x, algorithm="maxima")

[Out] 1/3\*(h^2\*x^3 + 3\*g\*h\*x^2 + 3\*g^2\*x)\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2 + 1/9\*r\*(6\*(3\*a\*b^2\*f\*g^2\*p - 3\*a^2\*b\*f\*g\*h\*p + a^3\*f\*h^2\*p)\*log(b\*x + a)/b^3 + 6\*(3\*c\*d^2\*f\*g^2\*q - 3\*c^2\*d\*f\*g\*h\*q + c^3\*f\*h^2\*q)\*log(d\*x + c)/d^3 - (2\*b^2\*d^2\*f\*h^2\*(p + q)\*x^3 - 3\*(a\*b\*d^2\*f\*h^2\*p - (3\*d^2\*f\*g\*h\*(p + q) -



$$\begin{aligned}
& c*d*f*h^2*q)*b^2)*x^2 - 6*(3*a*b*d^2*f*g*h*p - a^2*d^2*f*h^2*p - (3*d^2*f* \\
& g^2*(p + q) - 3*c*d*f*g*h*q + c^2*f*h^2*q)*b^2)*x)/(b^2*d^2))*\log(((b*x + a \\
& )^p*(d*x + c)^q*f)^r*e)/f - 1/54*r^2*(6*(6*a^2*c*d^2*f^2*h^2*p*q - 3*(6*c*d \\
& ^2*f^2*g*h*p*q - c^2*d*f^2*h^2*p*q)*a*b + (18*(p*q + q^2)*c*d^2*f^2*g^2 - 9 \\
& *(p*q + 3*q^2)*c^2*d*f^2*g*h + (2*p*q + 11*q^2)*c^3*f^2*h^2)*b^2)*\log(d*x + \\
& c)/(b^2*d^3) + 36*(3*a*b^2*d^3*f^2*g^2*p*q - 3*a^2*b*d^3*f^2*g*h*p*q + a^3 \\
& *d^3*f^2*h^2*p*q - (3*c*d^2*f^2*g^2*p*q - 3*c^2*d*f^2*g*h*p*q + c^3*f^2*h^2 \\
& *p*q)*b^3)*(\log(b*x + a)*\log((b*d*x + a*d)/(b*c - a*d) + 1) + \operatorname{dilog}(-(b*d*x \\
& + a*d)/(b*c - a*d)))/(b^3*d^3) - (4*(p^2 + 2*p*q + q^2)*b^3*d^3*f^2*h^2*x^ \\
& 3 - 36*(3*c*d^2*f^2*g^2*p*q - 3*c^2*d*f^2*g*h*p*q + c^3*f^2*h^2*p*q)*b^3*\log \\
& (b*x + a)*\log(d*x + c) - 18*(3*c*d^2*f^2*g^2*q^2 - 3*c^2*d*f^2*g*h*q^2 + c \\
& ^3*f^2*h^2*q^2)*b^3*\log(d*x + c)^2 - 3*(5*(p^2 + p*q)*a*b^2*d^3*f^2*h^2 - ( \\
& 9*(p^2 + 2*p*q + q^2)*d^3*f^2*g*h - 5*(p*q + q^2)*c*d^2*f^2*h^2)*b^3)*x^2 - \\
& 18*(3*a*b^2*d^3*f^2*g^2*p^2 - 3*a^2*b*d^3*f^2*g*h*p^2 + a^3*d^3*f^2*h^2*p^ \\
& 2)*\log(b*x + a)^2 + 6*((11*p^2 + 8*p*q)*a^2*b*d^3*f^2*h^2 + 3*(2*c*d^2*f^2* \\
& h^2*p*q - 9*(p^2 + p*q)*d^3*f^2*g*h)*a*b^2 + (18*(p^2 + 2*p*q + q^2)*d^3*f^ \\
& 2*g^2 - 27*(p*q + q^2)*c*d^2*f^2*g*h + (8*p*q + 11*q^2)*c^2*d*f^2*h^2)*b^3) \\
& *x - 6*((11*p^2 + 2*p*q)*a^3*d^3*f^2*h^2 + 3*(c*d^2*f^2*h^2*p*q - 3*(3*p^2 \\
& + p*q)*d^3*f^2*g*h)*a^2*b - 6*(3*c*d^2*f^2*g*h*p*q - c^2*d*f^2*h^2*p*q - 3* \\
& (p^2 + p*q)*d^3*f^2*g^2)*a*b^2)*\log(b*x + a))/(b^3*d^3))/f^2
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(h^2x^2 + 2ghx + g^2\right)\log\left(\left((bx + a)^p(dx + c)^qf\right)^r e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x, algorithm="fricas")

[Out] integral((h^2\*x^2 + 2\*g\*h\*x + g^2)\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)^2 \log\left(\left((bx + a)^p (dx + c)^q f\right)^r e\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((h*x + g)^2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

$$3.37 \quad \int (g + hx) \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$$

**Optimal.** Leaf size=1063

result too large to display

```
[Out] (3*(b*g - a*h)*p*q*r^2*x)/(2*b) + (3*(d*g - c*h)*p*q*r^2*x)/(2*d) + (p*q*r^
2*(g + h*x)^2)/(2*h) + (p^2*r^2*(4*b*g - 3*a*h + b*h*x)^2)/(4*b^2*h) + (q^2
*r^2*(4*d*g - 3*c*h + d*h*x)^2)/(4*d^2*h) + ((b*g - a*h)^2*p*q*r^2*Log[a +
b*x])/(2*b^2*h) - (2*(b*g - a*h)*p^2*r^2*(a + b*x)*Log[a + b*x])/b^2 - ((d*
g - c*h)*p*q*r^2*(a + b*x)*Log[a + b*x])/(b*d) - (h*p^2*r^2*(a + b*x)^2*Log
[a + b*x])/(2*b^2) - (p*q*r^2*(g + h*x)^2*Log[a + b*x])/(2*h) - ((b*g - a*h
)^2*p^2*r^2*Log[a + b*x]^2)/(2*b^2*h) + ((d*g - c*h)^2*p*q*r^2*Log[c + d*x]
)/(2*d^2*h) - ((b*g - a*h)*p*q*r^2*(c + d*x)*Log[c + d*x])/(b*d) - (2*(d*g
- c*h)*q^2*r^2*(c + d*x)*Log[c + d*x])/d^2 - (h*q^2*r^2*(c + d*x)^2*Log[c +
d*x])/(2*d^2) - (p*q*r^2*(g + h*x)^2*Log[c + d*x])/(2*h) - ((b*g - a*h)^2*
p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b^2*h) - ((d*g - c
*h)^2*q^2*r^2*Log[c + d*x]^2)/(2*d^2*h) - ((d*g - c*h)^2*p*q*r^2*Log[a + b*
x]*Log[(b*(c + d*x))/(b*c - a*d)))/(d^2*h) + ((b*g - a*h)*p*r*x*(p*r*Log[a
+ b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/b + ((d*
g - c*h)*q*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p
*(c + d*x)^q]^r))/d + (p*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x]
- Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/(2*h) + (q*r*(g + h*x)^2*(p*r*Log
[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/(2*h)
+ ((b*g - a*h)^2*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - L
og[e*(f*(a + b*x)^p*(c + d*x)^q]^r))/(b^2*h) + ((d*g - c*h)^2*q*r*Log[c +
d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r))/(d^2*h) + ((g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r^2)/(2*h
) - ((d*g - c*h)^2*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(d^2*h
) - ((b*g - a*h)^2*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]/(b^2*h)
```

**Rubi [A]** time = 1.16285, antiderivative size = 1097, normalized size of antiderivative = 1.03, number of steps used = 39, number of rules used = 15, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$ , Rules used = {2498, 2513, 2411, 43, 2334, 12, 14, 2301, 2418, 2389, 2295, 2394, 2393, 2391, 2395}

$$\frac{p^2 r^2 \log^2(a + bx)(bg - ah)^2}{2b^2 h} + \frac{p q r^2 \log(a + bx)(bg - ah)^2}{2b^2 h} - \frac{p q r^2 \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c + dx)(bg - ah)^2}{b^2 h} + \frac{p r \log(a + bx)}{b^2 h}$$

Antiderivative was successfully verified.

[In] Int[(g + h\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2,x]

[Out] (3\*(b\*g - a\*h)\*p\*q\*r^2\*x)/(2\*b) + (3\*(d\*g - c\*h)\*p\*q\*r^2\*x)/(2\*d) + (p\*q\*r^2\*(g + h\*x)^2)/(2\*h) + (p^2\*r^2\*(4\*b\*g - 3\*a\*h + b\*h\*x)^2)/(4\*b^2\*h) + (q^2\*r^2\*(4\*d\*g - 3\*c\*h + d\*h\*x)^2)/(4\*d^2\*h) + ((b\*g - a\*h)^2\*p\*q\*r^2\*Log[a + b\*x])/(2\*b^2\*h) - ((d\*g - c\*h)\*p\*q\*r^2\*(a + b\*x)\*Log[a + b\*x])/(b\*d) - (p\*q\*r^2\*(g + h\*x)^2\*Log[a + b\*x])/(2\*h) + ((b\*g - a\*h)^2\*p^2\*r^2\*Log[a + b\*x]^2)/(2\*b^2\*h) - (p^2\*r^2\*Log[a + b\*x]\*((4\*h\*(b\*g - a\*h)\*(a + b\*x))/b^2 + (h^2\*(a + b\*x)^2)/b^2 + (2\*(b\*g - a\*h)^2\*Log[a + b\*x])/b^2))/(2\*h) + ((d\*g - c\*h)^2\*p\*q\*r^2\*Log[c + d\*x])/(2\*d^2\*h) - ((b\*g - a\*h)\*p\*q\*r^2\*(c + d\*x)\*Log[c + d\*x])/(b\*d) - (p\*q\*r^2\*(g + h\*x)^2\*Log[c + d\*x])/(2\*h) - ((b\*g - a\*h)^2\*p\*q\*r^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x])/(b^2\*h) + ((d\*g - c\*h)^2\*q^2\*r^2\*Log[c + d\*x]^2)/(2\*d^2\*h) - (q^2\*r^2\*Log[c + d\*x]\*((4\*h\*(d\*g - c\*h)\*(c + d\*x))/d^2 + (h^2\*(c + d\*x)^2)/d^2 + (2\*(d\*g - c\*h)^2\*Log[c + d\*x])/d^2))/(2\*h) - ((d\*g - c\*h)^2\*p\*q\*r^2\*Log[a + b\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(d^2\*h) + ((b\*g - a\*h)\*p\*r\*x\*(p\*r\*Log[a + b\*x] + q\*r\*Log[c + d\*x] - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r))/b + ((d\*g - c\*h)\*q\*r\*x\*(p\*r\*Log[a + b\*x] + q\*r\*Log[c + d\*x] - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r))/d + (p\*r\*(g + h\*x)^2\*(p\*r\*Log[a + b\*x] + q\*r\*Log[c + d\*x] - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)))/(2\*h) + (q\*r\*(g + h\*x)^2\*(p\*r\*Log[a + b\*x] + q\*r\*Log[c + d\*x] - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)))/(2\*h) + ((b\*g - a\*h)^2\*p\*r\*Log[a + b\*x]\*(p\*r\*Log[a + b\*x] + q\*r\*Log[c + d\*x] - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)))/(b^2\*h) + ((d\*g - c\*h)^2\*q\*r\*Log[c + d\*x]\*(p\*r\*Log[a + b\*x] + q\*r\*Log[c + d\*x] - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)))/(d^2\*h) + ((g + h\*x)^2\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2)/(2\*h) - ((d\*g - c\*h)^2\*p\*q\*r^2\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(d^2\*h) - ((b\*g - a\*h)^2\*p\*q\*r^2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])/(b^2\*h)

### Rule 2498

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]^(s\_.)\*((g\_.) + (h\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(h\*(m + 1)), x] + (-Dist[(b\*p\*r\*s)/(h\*(m + 1)), Int[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(a + b\*x), x], x] - Dist[(d\*q\*r\*s)/(h\*(m + 1)), Int[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IntQ[s, 0] && NeQ[m, -1]

### Rule 2513

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]\*(RFx\_.), x\_Symbol] := Dist[p\*r, Int[RFx\*Log[a + b\*x], x], x] + (Dist[q\*r, Int[RFx\*Log[c + d\*x], x], x] - Dist[p\*r\*Log[a + b\*x] + q\*r\*Log[c + d

```
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x] /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n
]
```

### Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] := Dist[1/e, Subst[Int
[(g*x)/e]^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
+ b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1
] && EqQ[m, -1])
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol]
:> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol]
:> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a,
b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x]
/; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)),
x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x]
- Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x]
&& NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol]
:= Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2,
-(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_))^(q_.),
x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])/(g*(q + 1)), x]
- Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x]
&& NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int (g + hx) \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(g + hx)^2 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2h} - \frac{(bpr) \int \frac{(g+hx)^2 \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{a+bx} dx}{h} \\
&= \frac{(g + hx)^2 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2h} - \frac{(bp^2r^2) \int \frac{(g+hx)^2 \log(a+bx)}{a+bx} dx}{h} \\
&= \frac{(g + hx)^2 \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{2h} - \frac{(p^2r^2) \text{Subst} \left( \int \frac{\left( \frac{bg-ah}{b} + \frac{hx}{b} \right)^2 \log(a+bx)}{x} dx \right)}{h} \\
&= -\frac{p^2r^2 \log(a + bx) \left( \frac{4h(bg-ah)(a+bx)}{b^2} + \frac{h^2(a+bx)^2}{b^2} + \frac{2(bg-ah)^2 \log(a+bx)}{b^2} \right)}{2h} - \frac{p^2r^2 \int \frac{\left( \frac{bg-ah}{b} + \frac{hx}{b} \right)^2 \log(a+bx)}{x} dx}{h} \\
&= -\frac{pqr^2(g + hx)^2 \log(a + bx)}{2h} - \frac{p^2r^2 \log(a + bx) \left( \frac{4h(bg-ah)(a+bx)}{b^2} + \frac{h^2(a+bx)^2}{b^2} \right)}{2h} \\
&= \frac{(bg - ah)pqr^2x}{b} + \frac{(dg - ch)pqr^2x}{d} - \frac{(dg - ch)pqr^2(a + bx) \log(a + bx)}{bd} \\
&= \frac{3(bg - ah)pqr^2x}{2b} + \frac{3(dg - ch)pqr^2x}{2d} + \frac{pqr^2(g + hx)^2}{2h} + \frac{p^2r^2(4bg - 3ah)}{4b^2h} \\
&= \frac{3(bg - ah)pqr^2x}{2b} + \frac{3(dg - ch)pqr^2x}{2d} + \frac{pqr^2(g + hx)^2}{2h} + \frac{p^2r^2(4bg - 3ah)}{4b^2h}
\end{aligned}$$

**Mathematica [A]** time = 1.12735, size = 480, normalized size = 0.45

$$-4pqr^2(bc - ad)(adh + bch - 2bdg) \text{PolyLog} \left( 2, \frac{d(a+bx)}{ad-bc} \right) + 2pr \log(a + bx) \left( ad \left( (4bdg - 2adh) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2, x]

[Out] (2\*a\*d^2\*(-2\*b\*g + a\*h)\*p^2\*r^2\*Log[a + b\*x]^2 + 2\*p\*r\*Log[a + b\*x]\*(2\*b^2\*c\*(-2\*d\*g + c\*h)\*q\*r\*Log[c + d\*x] - 2\*(b\*c - a\*d)\*(-2\*b\*d\*g + b\*c\*h + a\*d\*h)\*q\*r\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] + a\*d\*(2\*b\*(-2\*d\*g + c\*h)\*q\*r + a\*d\*h\*(3\*p + q)\*r + (4\*b\*d\*g - 2\*a\*d\*h)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)) + b\*(2\*b\*c\*(-2\*d\*g + c\*h)\*q^2\*r^2\*Log[c + d\*x]^2 + 2\*q\*r\*Log[c + d\*x]\*(2\*a\*d\*

$$(2*d*g + c*h)*p*r + b*c*(-4*d*g*(p + q) + c*h*(p + 3*q))*r - 2*b*c*(-2*d*g + c*h)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + d*(r^2*(-2*a*p*(-4*d*g*q + 2*c*h*q + 3*d*h*(p + q)*x) + b*(p + q)*x*(-6*c*h*q + d*(p + q)*(8*g + h*x))) - 2*r*(2*a*d*p*(2*g - h*x) + b*x*(-2*c*h*q + d*(p + q)*(4*g + h*x)))*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*d*x*(2*g + h*x)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2) - 4*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h)*p*q*r^2*\text{Poly}[\text{Log}[2, (d*(a + b*x))/(-b*c + a*d)]]/(4*b^2*d^2)$$

**Maple [F]** time = 0.176, size = 0, normalized size = 0.

$$\int (hx + g) \left( \ln \left( e \left( f(bx + a)^p (dx + c)^q \right)^r \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

[Out] int((h\*x+g)\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

**Maxima [A]** time = 1.37604, size = 841, normalized size = 0.79

$$\frac{1}{2} (hx^2 + 2gx) \log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2 + \frac{r \left( \frac{2(2abfgp - a^2fhp) \log(bx+a)}{b^2} + \frac{2(2cdfgq - c^2fhq) \log(dx+c)}{d^2} - \frac{bdfh(p+q)x^2 - 2(adfhp - (2abfgp - a^2fhp))}{bd} \right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x, algorithm="maxima")

[Out] 1/2\*(h\*x^2 + 2\*g\*x)\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2 + 1/2\*r\*(2\*(2\*a\*b\*f\*g\*p - a^2\*f\*h\*p)\*log(b\*x + a)/b^2 + 2\*(2\*c\*d\*f\*g\*q - c^2\*f\*h\*q)\*log(d\*x + c)/d^2 - (b\*d\*f\*h\*(p + q)\*x^2 - 2\*(a\*d\*f\*h\*p - (2\*d\*f\*g\*(p + q) - c\*f\*h\*q)\*b)\*x)/(b\*d))\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)/f + 1/4\*r^2\*(2\*(2\*a\*c\*d\*f^2\*h\*p\*q - (4\*(p\*q + q^2)\*c\*d\*f^2\*g - (p\*q + 3\*q^2)\*c^2\*f^2\*h)\*b)\*log(d\*x + c)/(b\*d^2) - 4\*(2\*a\*b\*d^2\*f^2\*g\*p\*q - a^2\*d^2\*f^2\*h\*p\*q - (2\*c\*d\*f^2\*g\*p\*q - c^2\*f^2\*h\*p\*q)\*b^2)\*(log(b\*x + a)\*log((b\*d\*x + a\*d)/(b\*c - a\*d) + 1) + dilog(-(b\*d\*x + a\*d)/(b\*c - a\*d)))/(b^2\*d^2) + ((p^2 + 2\*p\*q + q^2)\*b^2\*d^2\*f^2\*h\*x^2 - 4\*(2\*c\*d\*f^2\*g\*p\*q - c^2\*f^2\*h\*p\*q)\*b^2\*log(b\*x + a)\*log(d\*x + c) - 2\*(2\*c\*d\*f^2\*g\*q^2 - c^2\*f^2\*h\*q^2)\*b^2\*log(d\*x + c)^2 - 2\*(2\*a\*b\*d



$$\frac{\begin{aligned} &^2 f^2 g p^2 - a^2 d^2 f^2 h p^2 \log(bx + a)^2 - 2(3(p^2 + pq) a b d^2 \\ & f^2 h - (4(p^2 + 2pq + q^2) d^2 f^2 g - 3(pq + q^2) c d f^2 h) b^2) x \\ & + 2((3p^2 + pq) a^2 d^2 f^2 h + 2(c d f^2 h p q - 2(p^2 + pq) d^2 f^2 \\ & g) a b) \log(bx + a) / (b^2 d^2) \end{aligned}}{f^2}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(hx + g\right) \log\left(\left((bx + a)^p (dx + c)^q f\right)^r e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x, algorithm="fricas")

[Out] integral((h\*x + g)\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g) \log\left(\left((bx + a)^p (dx + c)^q f\right)^r e\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x, algorithm="giac")

[Out] integrate((h\*x + g)\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2, x)

### 3.38 $\int \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx$

**Optimal.** Leaf size=269

$$\frac{2pqr^2(bc - ad)\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bd} + \frac{(a + bx)\log^2\left(e\left(f(a + bx)^p(c + dx)^q\right)^r\right)}{b} - \frac{2r(p + q)(a + bx)\log\left(e\left(f(a + bx)^p(c + dx)^q\right)^r\right)}{b}$$

[Out]  $2*(p + q)^2*r^2*x - (2*(b*c - a*d)*q*(p + q)*r^2*\text{Log}[c + d*x])/(b*d) - (2*(b*c - a*d)*p*q*r^2*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x])/(b*d) - ((b*c - a*d)*q^2*r^2*\text{Log}[c + d*x]^2)/(b*d) - (2*(p + q)*r*(a + b*x)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b + (2*(b*c - a*d)*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b + ((a + b*x)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/b - (2*(b*c - a*d)*p*q*r^2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/b$

**Rubi [A]** time = 0.153009, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$ , Rules used = {2487, 2494, 2394, 2393, 2391, 2390, 2301, 31, 8}

$$\frac{2pqr^2(bc - ad)\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bd} + \frac{(a + bx)\log^2\left(e\left(f(a + bx)^p(c + dx)^q\right)^r\right)}{b} - \frac{2r(p + q)(a + bx)\log\left(e\left(f(a + bx)^p(c + dx)^q\right)^r\right)}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2, x]$

[Out]  $2*(p + q)^2*r^2*x - (2*(b*c - a*d)*q*(p + q)*r^2*\text{Log}[c + d*x])/(b*d) - (2*(b*c - a*d)*p*q*r^2*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x])/(b*d) - ((b*c - a*d)*q^2*r^2*\text{Log}[c + d*x]^2)/(b*d) - (2*(p + q)*r*(a + b*x)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b + (2*(b*c - a*d)*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b + ((a + b*x)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/b - (2*(b*c - a*d)*p*q*r^2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/b$

#### Rule 2487

$\text{Int}[\text{Log}[(e._)*((f._)*((a._) + (b._)*(x._))^(p._))*((c._) + (d._)*(x._))^(q._)]^(r._)]^(s._), x\_Symbol] \rightarrow \text{Simp}[(a + b*x)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/b, x] + (\text{Dist}[(q*r*s*(b*c - a*d))/b, \text{Int}[\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - \text{Dist}[r*s*(p + q), \text{Int}[\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x), x], x]$

$b*x)^p*(c + d*x)^q)^r)^{s-1}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, p, q, r, s\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[p + q, 0] \ \&\& \ \text{IGtQ}[s, 0] \ \&\& \ \text{LtQ}[s, 4]$

#### Rule 2494

$\text{Int}[\text{Log}[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^{(p_.)*((c_.) + (d_.)*(x_.))^{(q_.)})^{(r_.)}]/((g_.) + (h_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[g + h*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/h, x] + (-\text{Dist}[(b*p*r)/h, \text{Int}[\text{Log}[g + h*x]/(a + b*x), x], x] - \text{Dist}[(d*q*r)/h, \text{Int}[\text{Log}[g + h*x]/(c + d*x), x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, p, q, r\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

#### Rule 2394

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^{(n_.)}]*(b_.)]/((f_.) + (g_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n]))/g, x] - \text{Dist}[(b*e*n)/g, \text{Int}[\text{Log}[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0]$

#### Rule 2393

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))]*(b_.)]/((f_.) + (g_.)*(x_.)), x\_Symbol] \rightarrow \text{Dist}[1/g, \text{Subst}[\text{Int}[(a + b*\text{Log}[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{EqQ}[g + c*(e*f - d*g), 0]$

#### Rule 2391

$\text{Int}[\text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^{(n_.)}]/(x_.), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$

#### Rule 2390

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^{(n_.)}]*(b_.))^{(p_.)*((f_.) + (g_.)*(x_.))^{(q_.)}, x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(f*x)/d]^q*(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n, p, q\}, x] \ \&\& \ \text{EqQ}[e*f - d*g, 0]$

#### Rule 2301

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.))^{(n_.)}]*(b_.)]/(x_.), x\_Symbol] \rightarrow \text{Simp}[(a + b*\text{Log}[c*x^n])^2/(2*b*n), x] /; \text{FreeQ}[\{a, b, c, n\}, x]$

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^( -1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned}
 \int \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) dx &= \frac{(a + bx) \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{b} + \frac{(2(bc - ad)qr) \int \frac{\log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{c + dx} dx}{b} \\
 &= -\frac{2(p + q)r(a + bx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{b} + \frac{2(bc - ad)qr \log(c + dx) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{bd} \\
 &= 2(p + q)^2 r^2 x - \frac{2(bc - ad)q(p + q)r^2 \log(c + dx)}{bd} - \frac{2(bc - ad)pqr^2 \log \left( -\frac{d(a + bx)}{bc - ad} \right) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{bd} \\
 &= 2(p + q)^2 r^2 x - \frac{2(bc - ad)q(p + q)r^2 \log(c + dx)}{bd} - \frac{2(bc - ad)pqr^2 \log \left( -\frac{d(a + bx)}{bc - ad} \right) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{bd} \\
 &= 2(p + q)^2 r^2 x - \frac{2(bc - ad)q(p + q)r^2 \log(c + dx)}{bd} - \frac{2(bc - ad)pqr^2 \log \left( -\frac{d(a + bx)}{bc - ad} \right) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{bd}
 \end{aligned}$$

**Mathematica [A]** time = 0.215741, size = 437, normalized size = 1.62

$$\frac{2pqr^2(bc - ad)\text{PolyLog} \left( 2, \frac{d(a + bx)}{ad - bc} \right) + bdx \log^2 \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) - 2adpr \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right) - 2bdprx}{1}$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2,x]

[Out] (p^2\*r^2\*(2\*b\*x - 2\*(a + b\*x)\*Log[a + b\*x] + (a + b\*x)\*Log[a + b\*x]^2))/b + (2\*a\*d\*p\*q\*r^2 + 4\*b\*d\*p\*q\*r^2\*x + 2\*b\*d\*q^2\*r^2\*x - d\*p^2\*r^2\*(2\*a + b\*x)\*Log[a + b\*x]^2 - 2\*b\*c\*p\*q\*r^2\*Log[c + d\*x] + 2\*a\*d\*p\*q\*r^2\*Log[c + d\*x] - 2\*b\*c\*q^2\*r^2\*Log[c + d\*x] - b\*c\*q^2\*r^2\*Log[c + d\*x]^2 - 2\*a\*d\*p\*r\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] - 2\*b\*d\*p\*r\*x\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] - 2\*b\*d\*q\*r\*x\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] + 2\*b\*c\*q\*r\*Log[c + d\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] + b\*d\*x\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]

$c + d*x)^q)^r]^2 + 2*p*r*Log[a + b*x]*(-(b*c*q*r*Log[c + d*x]) + (b*c - a*d)*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + d*(a*(p - q)*r + b*p*r*x + a*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) + 2*(b*c - a*d)*p*q*r^2*PolyLog[2, (d*(a + b*x))/(-b*c + a*d)]/(b*d)$

**Maple [F]** time = 0.131, size = 0, normalized size = 0.

$$\int \left( \ln \left( e \left( f (bx + a)^p (dx + c)^q \right)^r \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x)

**Maxima [A]** time = 1.36595, size = 402, normalized size = 1.49

$$x \log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2 - \frac{2 \left( f(p + q)x - \frac{afp \log(bx+a)}{b} - \frac{cfq \log(dx+c)}{d} \right) r \log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)}{f} - \frac{\left( \frac{2(pq+q^2)c}{d} \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2,x, algorithm="maxima")

[Out]  $x*\log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 - 2*(f*(p + q)*x - a*f*p*\log(b*x + a)/b - c*f*q*\log(d*x + c)/d)*r*\log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f - (2*(p*q + q^2)*c*f^2*\log(d*x + c)/d - 2*(b*c*f^2*p*q - a*d*f^2*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*d) + (a*d*f^2*p^2*\log(b*x + a)^2 + 2*b*c*f^2*p*q*\log(b*x + a)*\log(d*x + c) + b*c*f^2*q^2*\log(d*x + c)^2 - 2*(p^2 + 2*p*q + q^2)*b*d*f^2*x + 2*(p^2 + p*q)*a*d*f^2*\log(b*x + a))/(b*d))*r^2/f^2$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

$$3.39 \quad \int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx} dx$$

Optimal. Leaf size=1471

result too large to display

```
[Out] (p*q*r^2*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[((b*g - a*h)*(c + d*x))/((d*
g - c*h)*(a + b*x))]^2)/h + (p^2*r^2*Log[a + b*x]^2*Log[g + h*x])/h + (2*p*
q*r^2*Log[a + b*x]*Log[c + d*x]*Log[g + h*x])/h + (q^2*r^2*Log[c + d*x]^2*L
og[g + h*x])/h - (2*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)*L
og[g + h*x])/h - (2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)*L
og[g + h*x])/h + (Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)^2*Log[g + h*x])/h -
(p^2*r^2*Log[a + b*x]^2*Log[(b*(g + h*x))/(b*g - a*h)]/h - (2*p*q*r^2*Log[
a + b*x]*Log[-((h*(c + d*x))/(d*g - c*h))]*Log[(b*(g + h*x))/(b*g - a*h)]/
h + (p*q*r^2*Log[-((h*(c + d*x))/(d*g - c*h))]^2*Log[(b*(g + h*x))/(b*g - a
*h)]/h - (2*p*q*r^2*Log[-((h*(c + d*x))/(d*g - c*h))]*Log[((b*g - a*h)*(c
+ d*x))/((d*g - c*h)*(a + b*x))]*Log[(b*(g + h*x))/(b*g - a*h)]/h + (p*q*r
^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]^2*Log[(b*(g + h*x))
/(b*g - a*h)]/h + (2*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]
*Log[(b*(g + h*x))/(b*g - a*h)]/h - (2*p*q*r^2*Log[a + b*x]*Log[c + d*x]*L
og[(d*(g + h*x))/(d*g - c*h)]/h - (q^2*r^2*Log[c + d*x]^2*Log[(d*(g + h*x)
)/(d*g - c*h)]/h + (2*p*q*r^2*Log[a + b*x]*Log[-((h*(c + d*x))/(d*g - c*h)
)]*Log[(d*(g + h*x))/(d*g - c*h)]/h - (p*q*r^2*Log[-((h*(c + d*x))/(d*g -
c*h))]^2*Log[(d*(g + h*x))/(d*g - c*h)]/h + (2*p*q*r^2*Log[-((h*(c + d*x))
/(d*g - c*h))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Log[(d*
(g + h*x))/(d*g - c*h)]/h + (2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]*Log[(d*(g + h*x))/(d*g - c*h)]/h - (p*q*r^2*Log[((b*g - a*h)*(c
+ d*x))/((d*g - c*h)*(a + b*x))]^2*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c
*h)*(a + b*x)))]/h - (2*p*r*(q*r*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r])*PolyLog[2, -((h*(a + b*
x))/(b*g - a*h))]/h + (2*q*r*(p*r*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)
*(a + b*x))] + Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r])*PolyLog[2, -((h*(c + d
*x))/(d*g - c*h))]/h + (2*p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)
*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))]/h - (2*p*q*r^2*Log[((
b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*PolyLog[2, ((b*g - a*h)*(c +
d*x))/((d*g - c*h)*(a + b*x))]/h - (2*p^2*r^2*PolyLog[3, -((h*(a + b*x))/
(b*g - a*h))]/h - (2*p*q*r^2*PolyLog[3, -((h*(a + b*x))/(b*g - a*h))]/h -
(2*p*q*r^2*PolyLog[3, -((h*(c + d*x))/(d*g - c*h))]/h - (2*q^2*r^2*PolyLo
g[3, -((h*(c + d*x))/(d*g - c*h))]/h - (2*p*q*r^2*PolyLog[3, (b*(c + d*x))
/(d*(a + b*x))]/h + (2*p*q*r^2*PolyLog[3, ((b*g - a*h)*(c + d*x))/((d*g -
c*h)*(a + b*x))]/h
```

**Rubi [A]** time = 1.93148, antiderivative size = 2096, normalized size of antiderivative = 1.42, number of steps used = 29, number of rules used = 14, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$ , Rules used = {2497, 2500, 2394, 2393, 2391, 2433, 2375, 2317, 2374, 6589, 2440, 2437, 2435, 2315}

result too large to display

Antiderivative was successfully verified.

[In] Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2/(g + h\*x), x]

[Out] 
$$\begin{aligned} & -((\text{Log}[(a + b*x)^{(p*r)}]^2 * \text{Log}[g + h*x])/h) - (2*p*q*r^2 * \text{Log}[-((d*(a + b*x)) / (b*c - a*d))] * \text{Log}[c + d*x] * \text{Log}[g + h*x])/h - (2*p*q*r^2 * \text{Log}[a + b*x] * \text{Log}[(b*(c + d*x)) / (b*c - a*d)] * \text{Log}[g + h*x])/h + (2*q*r*(p*r * \text{Log}[a + b*x] - \text{Log}[(a + b*x)^{(p*r)}]) * \text{Log}[-((h*(c + d*x)) / (d*g - c*h))] * \text{Log}[g + h*x])/h + (2*p*r * \text{Log}[-((h*(a + b*x)) / (b*g - a*h))] * (q*r * \text{Log}[c + d*x] - \text{Log}[(c + d*x)^{(q*r)}]) * \text{Log}[g + h*x])/h - (\text{Log}[(c + d*x)^{(q*r)}]^2 * \text{Log}[g + h*x])/h + (2*p*r * \text{Log}[-((h*(a + b*x)) / (b*g - a*h))] * (\text{Log}[(a + b*x)^{(p*r)}] + \text{Log}[(c + d*x)^{(q*r)}] - \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r) * \text{Log}[g + h*x])/h + (2*q*r * \text{Log}[-((h*(c + d*x)) / (d*g - c*h))] * (\text{Log}[(a + b*x)^{(p*r)}] + \text{Log}[(c + d*x)^{(q*r)}] - \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r) * \text{Log}[g + h*x])/h + (\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)^2 * \text{Log}[g + h*x])/h + (\text{Log}[(a + b*x)^{(p*r)}]^2 * \text{Log}[(b*(g + h*x)) / (b*g - a*h)])/h + (\text{Log}[(c + d*x)^{(q*r)}]^2 * \text{Log}[(d*(g + h*x)) / (d*g - c*h)])/h - (p*q*r^2 * (\text{Log}[(b*(c + d*x)) / (b*c - a*d)] + \text{Log}[(b*g - a*h) / (b*(g + h*x))]) - \text{Log}[(b*g - a*h)*(c + d*x)] / ((b*c - a*d)*(g + h*x))) * \text{Log}[-(((b*c - a*d)*(g + h*x)) / ((d*g - c*h)*(a + b*x)))^2] / h + (p*q*r^2 * (\text{Log}[(b*(c + d*x)) / (b*c - a*d)] - \text{Log}[-((h*(c + d*x)) / (d*g - c*h))]) * (\text{Log}[a + b*x] + \text{Log}[-(((b*c - a*d)*(g + h*x)) / ((d*g - c*h)*(a + b*x)))^2] / h - (p*q*r^2 * (\text{Log}[-((d*(a + b*x)) / (b*c - a*d))] + \text{Log}[(d*g - c*h) / (d*(g + h*x))] - \text{Log}[-(((d*g - c*h)*(a + b*x)) / ((b*c - a*d)*(g + h*x))]) * \text{Log}[(b*c - a*d)*(g + h*x)] / ((b*g - a*h)*(c + d*x)))^2) / h + (p*q*r^2 * (\text{Log}[-((d*(a + b*x)) / (b*c - a*d))] - \text{Log}[-((h*(a + b*x)) / (b*g - a*h))]) * (\text{Log}[c + d*x] + \text{Log}[(b*c - a*d)*(g + h*x)] / ((b*g - a*h)*(c + d*x)))^2) / h - (2*p*q*r^2 * (\text{Log}[g + h*x] - \text{Log}[-(((b*c - a*d)*(g + h*x)) / ((d*g - c*h)*(a + b*x))]) * \text{PolyLog}[2, -((d*(a + b*x)) / (b*c - a*d))]) / h + (2*p*r * \text{Log}[(a + b*x)^{(p*r)}] * \text{PolyLog}[2, -((h*(a + b*x)) / (b*g - a*h))]) / h - (2*p*q*r^2 * (\text{Log}[g + h*x] - \text{Log}[(b*c - a*d)*(g + h*x)] / ((b*g - a*h)*(c + d*x)))) * \text{PolyLog}[2, (b*(c + d*x)) / (b*c - a*d)] / h + (2*q*r * \text{Log}[(c + d*x)^{(q*r)}] * \text{PolyLog}[2, -((h*(c + d*x)) / (d*g - c*h))]) / h + (2*p*q*r^2 * \text{Log}[-(((b*c - a*d)*(g + h*x)) / ((d*g - c*h)*(a + b*x))]) * \text{PolyLog}[2, (h*(a + b*x)) / (b*(g + h*x))]) / h - (2*p*q*r^2 * \text{Log}[-(((b*c - a*d)*(g + h*x)) / ((d*g - c*h)*(a + b*x))]) * \text{PolyLog}[2, -(((d*g - c*h)*(a + b*x)) / ((b*c - a*d)*(g + h*x))]) / h + (2*p*q*r^2 * \text{Log}[(b*c - a*d)*(g + h*x)] / ((b*g - a*h)*(c + d*x))) * \text{PolyLog}[2, (h*(c + d*x)) / (d*(g + h*x))]) / h - (2*p*q*r^2 * \text{Log}[(b*c - a*d)*(g + h*x)] / ((b*g - a*h)*(c + d*x))) * \text{PolyLog}[2, ((b*g - a*h)*(c + d*x)) / (b*c - a \end{aligned}$$



```

*d)*(g + h*x)))/h + (2*p*r*(q*r*Log[c + d*x] - Log[(c + d*x)^(q*r)])*PolyL
og[2, (b*(g + h*x))/(b*g - a*h)]/h + (2*p*r*(Log[(a + b*x)^(p*r)] + Log[(c
+ d*x)^(q*r)] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r])*PolyLog[2, (b*(g + h
*x))/(b*g - a*h)]/h - (2*p*q*r^2*(Log[c + d*x] + Log[((b*c - a*d)*(g + h*x
)))/((b*g - a*h)*(c + d*x))])*PolyLog[2, (b*(g + h*x))/(b*g - a*h)]/h + (2*
q*r*(p*r*Log[a + b*x] - Log[(a + b*x)^(p*r)])*PolyLog[2, (d*(g + h*x))/(d*g
- c*h)]/h + (2*q*r*(Log[(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e*(
f*(a + b*x)^p*(c + d*x)^q]^r])*PolyLog[2, (d*(g + h*x))/(d*g - c*h)]/h - (
2*p*q*r^2*(Log[a + b*x] + Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b
*x))]))*PolyLog[2, (d*(g + h*x))/(d*g - c*h)]/h + (2*p*q*r^2*PolyLog[3, -(
(d*(a + b*x))/(b*c - a*d))]/h - (2*p^2*r^2*PolyLog[3, -((h*(a + b*x))/(b*g
- a*h))]/h + (2*p*q*r^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]/h - (2*q^2
*r^2*PolyLog[3, -((h*(c + d*x))/(d*g - c*h))]/h + (2*p*q*r^2*PolyLog[3, (h
*(a + b*x))/(b*(g + h*x))]/h - (2*p*q*r^2*PolyLog[3, -(((d*g - c*h)*(a + b
*x))/((b*c - a*d)*(g + h*x)))]/h + (2*p*q*r^2*PolyLog[3, (h*(c + d*x))/(d*
(g + h*x))]/h - (2*p*q*r^2*PolyLog[3, ((b*g - a*h)*(c + d*x))/((b*c - a*d)
*(g + h*x))]/h + (2*p*q*r^2*PolyLog[3, (b*(g + h*x))/(b*g - a*h)]/h + (2*
p*q*r^2*PolyLog[3, (d*(g + h*x))/(d*g - c*h)]/h

```

### Rule 2497

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^2/((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(Log[g + h*x]*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q]^r]^2)/h, x] + (-Dist[(2*b*p*r)/h, Int[(Log[g + h*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)]/(a + b*x), x], x] - Dist[(2*d*q*r)/h,
Int[(Log[g + h*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)]/(c + d*x), x], x])
/; FreeQ[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]

```

### Rule 2500

```

Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*(s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))/((j_.) + (k
_.)*(x_)), x_Symbol] := Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n]]/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n))]/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n))]/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]

```

### Rule 2394

```

Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x
_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]

```

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))])\*(b\_.)^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_)^(r\_.)), x\_Symbol] := Dist[1/e, Subst[Int[((k\*x)/d)^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

Rule 2375

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))^(r\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)^(p\_.))/(x\_), x\_Symbol] := Simp[(Log[d\*(e + f\*x^m)^r]\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*n\*(p + 1)), x] - Dist[(f\*m\*r)/(b\*n\*(p + 1)), Int[(x^(m - 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(e + f\*x^m), x], x] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d\*e, 1]

Rule 2317

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)^(p\_.))/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[1 + (e\*x)/d]\*(a + b\*Log[c\*x^n])^p)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[1 + (e\*x)/d]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))])\*(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)^(p\_.))/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol]
:> Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n)]*(f +
g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m)], x], x, k + l*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m, n}, x]
&& IntegerQ[r]
```

Rule 2437

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m_.)])/(x_), x_Symbol]
:> Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x, x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x]
&& NeQ[e*i - d*j, 0] && NeQ[i + j*x, h*(i + j*x)^m]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol]
:> Simp[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)] - Log[-((b*c - a*d)*x]/(a*(c + d*x))))] + Log[(b*c - a*d)/(b*(c + d*x))])*Log[(a*(c + d*x))/(c*(a + b*x))]^2/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x)]))^2/2, x] + Simp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x)])]*PolyLog[2, 1 + (b*x)/a], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x)])]*PolyLog[2, 1 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a + b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))], x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol]
:> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{g+hx} dx &= \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) \log(g+hx)}{h} - \frac{(2bpr) \int \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) \log(g+hx)}{a+bx} dx}{h} \\
&= \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) \log(g+hx)}{h} - \frac{(2bpr) \int \frac{\log((a+bx)^{pr}) \log(g+hx)}{a+bx} dx}{h} - \frac{2pr \log \left( -\frac{h(a+bx)}{bg-ah} \right) \left( \log((a+bx)^{pr}) + \log((c+dx)^{qr}) - \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) \right)}{h} \\
&= -\frac{\log^2((a+bx)^{pr}) \log(g+hx)}{h} - \frac{\log^2((c+dx)^{qr}) \log(g+hx)}{h} + \frac{2pr \log \left( -\frac{h(a+bx)}{bg-ah} \right)}{h} \\
&= -\frac{\log^2((a+bx)^{pr}) \log(g+hx)}{h} - \frac{2pqr^2 \log \left( -\frac{d(a+bx)}{bc-ad} \right) \log(c+dx) \log(g+hx)}{h} - \frac{2pqr^2 \log \left( -\frac{d(a+bx)}{bc-ad} \right) \log(c+dx) \log(g+hx)}{h} - \frac{2pqr^2 \log \left( -\frac{d(a+bx)}{bc-ad} \right) \log(c+dx) \log(g+hx)}{h} - \frac{2pqr^2 \log \left( -\frac{d(a+bx)}{bc-ad} \right) \log(c+dx) \log(g+hx)}{h}
\end{aligned}$$

**Mathematica [A]** time = 0.280696, size = 1370, normalized size = 0.93

result too large to display

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2/(g + h\*x),x]

[Out] (p\*q\*r^2\*Log[(-b\*c) + a\*d]/(d\*(a + b\*x)))\*Log[((b\*g - a\*h)\*(c + d\*x))/((d\*g - c\*h)\*(a + b\*x))]^2 + p^2\*r^2\*Log[a + b\*x]^2\*Log[g + h\*x] + 2\*p\*q\*r^2\*Log[g[a + b\*x]\*Log[c + d\*x]\*Log[g + h\*x] + q^2\*r^2\*Log[c + d\*x]^2\*Log[g + h\*x] - 2\*p\*r\*Log[a + b\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]\*Log[g + h\*x] - 2\*q\*r\*Log[c + d\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]\*Log[g + h\*x] + Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2\*Log[g + h\*x] - p^2\*r^2\*Log[a + b\*x]^2\*Log[(b\*(g + h\*x))/(b\*g - a\*h)] - 2\*p\*q\*r^2\*Log[a + b\*x]\*Log[(h\*(c + d\*x))/(-(d\*g) + c\*h)]\*Log[(b\*(g + h\*x))/(b\*g - a\*h)] + p\*q\*r^2\*Log[(h\*(c + d\*x))/(-(d\*g) + c\*h)]

$$\begin{aligned}
& g) + c*h)]^2 * \text{Log}[(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2 * \text{Log}[(h*(c + d*x))/(- \\
& -(d*g) + c*h)] * \text{Log}[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] * \text{Log}[(b* \\
& (g + h*x))/(b*g - a*h)] + p*q*r^2 * \text{Log}[((b*g - a*h)*(c + d*x))/((d*g - c*h)* \\
& (a + b*x))]^2 * \text{Log}[(b*(g + h*x))/(b*g - a*h)] + 2*p*r * \text{Log}[a + b*x] * \text{Log}[e*(f* \\
& (a + b*x)^p*(c + d*x)^q)^r] * \text{Log}[(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2 * \text{Log}[ \\
& a + b*x] * \text{Log}[c + d*x] * \text{Log}[(d*(g + h*x))/(d*g - c*h)] - q^2*r^2 * \text{Log}[c + d*x] \\
& ^2 * \text{Log}[(d*(g + h*x))/(d*g - c*h)] + 2*p*q*r^2 * \text{Log}[a + b*x] * \text{Log}[(h*(c + d*x) \\
& )/(-(d*g) + c*h)] * \text{Log}[(d*(g + h*x))/(d*g - c*h)] - p*q*r^2 * \text{Log}[(h*(c + d*x) \\
& )/(-(d*g) + c*h)]^2 * \text{Log}[(d*(g + h*x))/(d*g - c*h)] + 2*p*q*r^2 * \text{Log}[(h*(c + \\
& d*x))/(-(d*g) + c*h)] * \text{Log}[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] * \\
& \text{Log}[(d*(g + h*x))/(d*g - c*h)] + 2*q*r * \text{Log}[c + d*x] * \text{Log}[e*(f*(a + b*x)^p*(c \\
& + d*x)^q)^r] * \text{Log}[(d*(g + h*x))/(d*g - c*h)] - p*q*r^2 * \text{Log}[((b*g - a*h)*(c \\
& + d*x))/((d*g - c*h)*(a + b*x))]^2 * \text{Log}[( -(b*c) + a*d)*(g + h*x))/((d*g - c \\
& *h)*(a + b*x))] + 2*p*r * (-(q*r * \text{Log}[(b*(g - a*h)*(c + d*x))/((d*g - c*h)*(a \\
& + b*x)])) + \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) * \text{PolyLog}[2, (h*(a + b*x))/ \\
& (-(b*g) + a*h)] + 2*q*r * (p*r * \text{Log}[(b*(g - a*h)*(c + d*x))/((d*g - c*h)*(a + \\
& b*x)])) + \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) * \text{PolyLog}[2, (h*(c + d*x))/(-( \\
& d*g) + c*h)] + 2*p*q*r^2 * \text{Log}[(b*(g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x) \\
& )] * \text{PolyLog}[2, (b*(c + d*x))/(d*(a + b*x))] - 2*p*q*r^2 * \text{Log}[(b*(g - a*h)*(c \\
& + d*x))/((d*g - c*h)*(a + b*x))] * \text{PolyLog}[2, ((b*g - a*h)*(c + d*x))/((d*g - \\
& c*h)*(a + b*x))] - 2*p^2*r^2 * \text{PolyLog}[3, (h*(a + b*x))/(-(b*g) + a*h)] - 2* \\
& p*q*r^2 * \text{PolyLog}[3, (h*(a + b*x))/(-(b*g) + a*h)] - 2*p*q*r^2 * \text{PolyLog}[3, (h* \\
& (c + d*x))/(-(d*g) + c*h)] - 2*q^2*r^2 * \text{PolyLog}[3, (h*(c + d*x))/(-(d*g) + c \\
& *h)] - 2*p*q*r^2 * \text{PolyLog}[3, (b*(c + d*x))/(d*(a + b*x))] + 2*p*q*r^2 * \text{PolyLo \\
& g}[3, ((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]/h
\end{aligned}$$

**Maple [F]** time = 0.578, size = 0, normalized size = 0.

$$\int \frac{\left( \ln \left( e \left( f(bx + a)^p (dx + c)^q \right)^r \right) \right)^2}{hx + g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(h\*x+g),x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(h\*x+g),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{hx+g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(h\*x+g),x, algorithm="maxima")

[Out] integrate(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2/(h\*x + g), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{hx+g}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(h\*x+g),x, algorithm="fricas")

[Out] integral(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2/(h\*x + g), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r)\*\*2/(h\*x+g),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{hx+g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

$$3.40 \quad \int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2} dx$$

**Optimal.** Leaf size=832

$$\frac{2bpq \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c+dx)r^2}{h(bg-ah)} + \frac{2dpq \log(a+bx) \log\left(\frac{b(c+dx)}{bc-ad}\right) r^2}{h(dg-ch)} - \frac{2dpq \log(a+bx) \log\left(\frac{b(g+hx)}{bg-ah}\right) r^2}{h(dg-ch)} - \frac{2bpq \log(c+dx)r^2}{h(bg-ah)}$$

```
[Out] (2*b*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(h*(b*g - a*h))
+ (2*d*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(h*(d*g - c*h))
- (2*b*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(h*(b*g - a*h))
- (2*d*q*r*Log[c + d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(h*(d*g - c*h))
- Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2/(h*(g + h*x)) + (2*b*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))*Log[g + h*x])/(h*(b*g - a*h))
+ (2*d*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))*Log[g + h*x])/(h*(d*g - c*h))
- (2*d*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(h*(d*g - c*h))
- (2*b*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(h*(b*g - a*h))
- (2*b*p^2*r^2*Log[a + b*x]*Log[1 + (b*g - a*h)/(h*(a + b*x))])/(h*(b*g - a*h))
- (2*d*q^2*r^2*Log[c + d*x]*Log[1 + (d*g - c*h)/(h*(c + d*x))])/(h*(d*g - c*h))
+ (2*b*p^2*r^2*PolyLog[2, -((b*g - a*h)/(h*(a + b*x)))])/(h*(b*g - a*h))
+ (2*d*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(h*(d*g - c*h))
- (2*d*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])/(h*(d*g - c*h))
+ (2*d*q^2*r^2*PolyLog[2, -((d*g - c*h)/(h*(c + d*x)))])/(h*(d*g - c*h))
+ (2*b*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(h*(b*g - a*h))
- (2*b*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(h*(b*g - a*h))
)
```

**Rubi [A]** time = 0.93174, antiderivative size = 878, normalized size of antiderivative = 1.06, number of steps used = 35, number of rules used = 12, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$ , Rules used = {2498, 2513, 2411, 2344, 2301, 2317, 2391, 2418, 2394, 2393, 36, 31}

$$\frac{bp^2 \log^2(a+bx)r^2}{h(bg-ah)} + \frac{dq^2 \log^2(c+dx)r^2}{h(dg-ch)} + \frac{2bpq \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c+dx)r^2}{h(bg-ah)} + \frac{2dpq \log(a+bx) \log\left(\frac{b(c+dx)}{bc-ad}\right) r^2}{h(dg-ch)} - \frac{2bp^2 \log^2(a+bx)r^2}{h(bg-ah)}$$

Antiderivative was successfully verified.

[In] Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2/(g + h\*x)^2,x]



```
[Out] (b*p^2*r^2*Log[a + b*x]^2)/(h*(b*g - a*h)) + (2*b*p*q*r^2*Log[-((d*(a + b*x)))/(b*c - a*d)]*Log[c + d*x])/(h*(b*g - a*h)) + (d*q^2*r^2*Log[c + d*x]^2)/(h*(d*g - c*h)) + (2*d*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(h*(d*g - c*h)) - (2*b*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(h*(b*g - a*h)) - (2*d*q*r*Log[c + d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(h*(d*g - c*h)) - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^2/(h*(g + h*x)) + (2*b*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))*Log[g + h*x])/(h*(b*g - a*h)) + (2*d*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))*Log[g + h*x])/(h*(d*g - c*h)) - (2*b*p^2*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(h*(b*g - a*h)) - (2*d*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(h*(d*g - c*h)) - (2*b*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(h*(b*g - a*h)) - (2*d*q^2*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(h*(d*g - c*h)) + (2*d*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(h*(d*g - c*h)) - (2*b*p^2*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])/(h*(b*g - a*h)) - (2*d*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])/(h*(d*g - c*h)) + (2*b*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(h*(b*g - a*h)) - (2*b*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(h*(b*g - a*h)) - (2*d*q^2*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(h*(d*g - c*h))
```

### Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(c + d*x), x], x)) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IntegerQ[s, 0] && NeQ[m, -1]
```

### Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]*(RFx_), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dist[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFx, x], x]) /; FreeQ[{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0] && !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.)] /; IntegerQ[m, n]
```

### Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] := Dist[1/e, Subst[Int[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

#### Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_Symbol] := Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e, Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))/((f_.) + (g_.)*(x_))), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(g+hx)^2} dx &= -\frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{h(g+hx)} + \frac{(2bpr) \int \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)(g+hx)} dx}{h} + \frac{(2dqr) \int \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)(g+hx)} dx}{h} \\
&= -\frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{h(g+hx)} + \frac{(2bp^2r^2) \int \frac{\log(a+bx)}{(a+bx)(g+hx)} dx}{h} + \frac{(2bpqr^2) \int \frac{\log(c+dx)}{(a+bx)(g+hx)} dx}{h} \\
&= -\frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{h(g+hx)} + \frac{(2p^2r^2) \text{Subst} \left( \int \frac{\log(x)}{x \left( \frac{bg-ah}{b} + \frac{hx}{b} \right)} dx, x, a+bx \right)}{h} + \\
&= -\frac{2bpr \log(a+bx) \left( pr \log(a+bx) + qr \log(c+dx) - \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) \right)}{h(bg-ah)} \\
&= \frac{bp^2r^2 \log^2(a+bx)}{h(bg-ah)} + \frac{2bpqr^2 \log \left( -\frac{d(a+bx)}{bc-ad} \right) \log(c+dx)}{h(bg-ah)} + \frac{dq^2r^2 \log^2(c+dx)}{h(dg-ch)} + \frac{2bpqr^2 \log \left( -\frac{d(a+bx)}{bc-ad} \right) \log(c+dx)}{h(bg-ah)} \\
&= \frac{bp^2r^2 \log^2(a+bx)}{h(bg-ah)} + \frac{2bpqr^2 \log \left( -\frac{d(a+bx)}{bc-ad} \right) \log(c+dx)}{h(bg-ah)} + \frac{dq^2r^2 \log^2(c+dx)}{h(dg-ch)} + \frac{2bpqr^2 \log \left( -\frac{d(a+bx)}{bc-ad} \right) \log(c+dx)}{h(bg-ah)} \\
&= \frac{bp^2r^2 \log^2(a+bx)}{h(bg-ah)} + \frac{2bpqr^2 \log \left( -\frac{d(a+bx)}{bc-ad} \right) \log(c+dx)}{h(bg-ah)} + \frac{dq^2r^2 \log^2(c+dx)}{h(dg-ch)} + \frac{2bpqr^2 \log \left( -\frac{d(a+bx)}{bc-ad} \right) \log(c+dx)}{h(bg-ah)}
\end{aligned}$$

**Mathematica [B]** time = 2.67099, size = 2930, normalized size = 3.52

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2/(g + h\*x)^2,x]

[Out]  $(-(b*d*g^2*p^2*r^2*\text{Log}[a + b*x]^2) + b*c*g*h*p^2*r^2*\text{Log}[a + b*x]^2 - b*d*g*h*p^2*r^2*x*\text{Log}[a + b*x]^2 + b*c*h^2*p^2*r^2*x*\text{Log}[a + b*x]^2 - 2*b*d*g^2*p*q*r^2*\text{Log}[a + b*x]*\text{Log}[c + d*x] + 2*a*d*g*h*p*q*r^2*\text{Log}[a + b*x]*\text{Log}[c + d*x] - 2*b*d*g*h*p*q*r^2*x*\text{Log}[a + b*x]*\text{Log}[c + d*x] + 2*a*d*h^2*p*q*r^2*x*\text{Log}[a + b*x]*\text{Log}[c + d*x] - b*d*g^2*q^2*r^2*\text{Log}[c + d*x]^2 + a*d*g*h*q^2*r^2*\text{Log}[c + d*x]^2 - b*d*g*h*q^2*r^2*x*\text{Log}[c + d*x]^2 + a*d*h^2*q^2*r^2*x*\text{Log}[c + d*x]^2 + 2*b*c*g*h*p*q*r^2*\text{Log}[a + b*x]*\text{Log}[(h*(c + d*x))/(-(d*g) + c*h)] - 2*a*d*g*h*p*q*r^2*\text{Log}[a + b*x]*\text{Log}[(h*(c + d*x))/(-(d*g) + c*h)] + 2*$

$$\begin{aligned}
& b*c*h^2*p*q*r^2*x*\text{Log}[a + b*x]*\text{Log}[(h*(c + d*x))/(-d*g) + c*h] - 2*a*d*h^2*p*q*r^2*x*\text{Log}[a + b*x]*\text{Log}[(h*(c + d*x))/(-d*g) + c*h] - b*c*g*h*p*q*r^2*\text{Log}[(h*(c + d*x))/(-d*g) + c*h]^2 + a*d*g*h*p*q*r^2*\text{Log}[(h*(c + d*x))/(-d*g) + c*h]^2 - b*c*h^2*p*q*r^2*x*\text{Log}[(h*(c + d*x))/(-d*g) + c*h]^2 + a*d*h^2*p*q*r^2*x*\text{Log}[(h*(c + d*x))/(-d*g) + c*h]^2 + 2*b*c*g*h*p*q*r^2*\text{Log}[(-b*c) + a*d]/(d*(a + b*x))*\text{Log}[(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*a*d*g*h*p*q*r^2*\text{Log}[(-b*c) + a*d]/(d*(a + b*x))*\text{Log}[(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*\text{Log}[(-b*c) + a*d]/(d*(a + b*x))*\text{Log}[(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*a*d*h^2*p*q*r^2*x*\text{Log}[(-b*c) + a*d]/(d*(a + b*x))*\text{Log}[(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*b*c*g*h*p*q*r^2*\text{Log}[(h*(c + d*x))/(-d*g) + c*h]*\text{Log}[(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*a*d*g*h*p*q*r^2*\text{Log}[(h*(c + d*x))/(-d*g) + c*h]*\text{Log}[(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*\text{Log}[(h*(c + d*x))/(-d*g) + c*h]*\text{Log}[(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*a*d*h^2*p*q*r^2*x*\text{Log}[(h*(c + d*x))/(-d*g) + c*h]*\text{Log}[(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*\text{Log}[(h*(c + d*x))/(-d*g) + c*h]*\text{Log}[(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*a*d*g*h*p*q*r^2*\text{Log}[(h*(c + d*x))/(-d*g) + c*h]*\text{Log}[(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]^2 + a*d*g*h*p*q*r^2*\text{Log}[(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]^2 - b*c*h^2*p*q*r^2*x*\text{Log}[(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]^2 + a*d*h^2*p*q*r^2*x*\text{Log}[(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]^2 + 2*b*d*g^2*p*r*\text{Log}[a + b*x]*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r - 2*b*c*g*h*p*r*\text{Log}[a + b*x]*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r + 2*b*d*g*h*p*r*x*\text{Log}[a + b*x]*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r - 2*b*c*h^2*p*r*x*\text{Log}[a + b*x]*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r + 2*b*d*g^2*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r - 2*a*d*g*h*q*r*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r + 2*b*d*g*h*q*r*x*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r - 2*a*d*h^2*q*r*x*\text{Log}[c + d*x]*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r - b*d*g^2*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r^2 + b*c*g*h*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r^2 + a*d*g*h*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r^2 - a*c*h^2*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r^2 - 2*b*d*g^2*p*q*r^2*\text{Log}[a + b*x]*\text{Log}[(b*(g + h*x))/(b*g - a*h)] + 2*a*d*g*h*p*q*r^2*\text{Log}[a + b*x]*\text{Log}[(b*(g + h*x))/(b*g - a*h)] - 2*b*d*g*h*p*q*r^2*x*\text{Log}[a + b*x]*\text{Log}[(b*(g + h*x))/(b*g - a*h)] + 2*a*d*h^2*p*q*r^2*x*\text{Log}[a + b*x]*\text{Log}[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g^2*p*q*r^2*\text{Log}[(h*(c + d*x))/(-d*g) + c*h]*\text{Log}[(b*(g + h*x))/(b*g - a*h)] - 2*b*c*g*h*p*q*r^2*\text{Log}[(h*(c + d*x))/(-d*g) + c*h]*\text{Log}[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g*h*p*q*r^2*x*\text{Log}[(h*(c + d*x))/(-d*g) + c*h]*\text{Log}[(b*(g + h*x))/(b*g - a*h)] - 2*b*c*h^2*p*q*r^2*x*\text{Log}[(h*(c + d*x))/(-d*g) + c*h]*\text{Log}[(b*(g + h*x))/(b*g - a*h)] - 2*b*d*g^2*p*r*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r*\text{Log}[(b*(g + h*x))/(b*g - a*h)] + 2*b*c*g*h*p*r*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r*\text{Log}[(b*(g + h*x))/(b*g - a*h)] - 2*b*d*g*h*p*r*x*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r*\text{Log}[(b*(g + h*x))/(b*g - a*h)] + 2*b*c*h^2*p*r*x*\text{Log}[e*(f*(a + b*x))^p*(c + d*x)^q]^r*\text{Log}[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g^2*p*q*r^2*\text{Log}[a + b*x]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] - 2*a*d*g*h*p*q*r^2*\text{Log}[a + b*x]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] + 2*b*d*g*h*p*q*r^2*x*\text{Log}[a + b*x]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] + 2*b*d*g*h*p*q*r^2*x*\text{Log}[a + b*x]*\text{Log}[(d*(g + h*x))/(d*g - c*h)]
\end{aligned}$$

$$\begin{aligned}
& - c*h)] - 2*a*d*h^2*p*q*r^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g - c*h)] \\
& - 2*b*d*g^2*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g + h*x))/(d* \\
& g - c*h)] + 2*b*c*g*h*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g + \\
& h*x))/(d*g - c*h)] - 2*b*d*g*h*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)] \\
& *Log[(d*(g + h*x))/(d*g - c*h)] + 2*b*c*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-( \\
& d*g) + c*h)]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*b*d*g^2*q*r*Log[e*(f*(a + b \\
& *x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*a*d*g*h*q*r*Log[e* \\
& (f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*b*d*g*h*q \\
& *r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] + \\
& 2*a*d*h^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g \\
& - c*h)] + 2*p*(b*c*h*p + a*d*h*q - b*d*g*(p + q))*r^2*(g + h*x)*PolyLog[2, \\
& (h*(a + b*x))/(-(b*g) + a*h)] + 2*q*(b*c*h*p + a*d*h*q - b*d*g*(p + q))*r^ \\
& 2*(g + h*x)*PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)] + 2*b*c*g*h*p*q*r^2*Po \\
& lyLog[2, (b*(c + d*x))/(d*(a + b*x))] - 2*a*d*g*h*p*q*r^2*PolyLog[2, (b*(c \\
& + d*x))/(d*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*PolyLog[2, (b*(c + d*x))/(d*(a \\
& + b*x))] - 2*a*d*h^2*p*q*r^2*x*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))]/(h \\
& *(-(b*g) + a*h)*(-(d*g) + c*h)*(g + h*x))
\end{aligned}$$

**Maple [F]** time = 0.498, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)\right)^2}{(hx+g)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(h\*x+g)^2,x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(h\*x+g)^2,x)

**Maxima [A]** time = 1.57674, size = 1006, normalized size = 1.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(h\*x+g)^2,x, algorithm="maxima")

```
[Out] 2*(b*f*p*log(b*x + a)/(b*g - a*h) + d*f*q*log(d*x + c)/(d*g - c*h) - (a*d*f
*h*q - (d*f*g*(p + q) - c*f*h*p)*b)*log(h*x + g)/((d*g*h - c*h^2)*a - (d*g^
2 - c*g*h)*b))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(f*h) - (2*(b*c*f^2*h
*p*q - a*d*f^2*h*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + di
log(-(b*d*x + a*d)/(b*c - a*d)))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b) +
2*(a*d*f^2*h*p*q + (c*f^2*h*p^2 - (p^2 + p*q)*d*f^2*g)*b)*(log(b*x + a)*log
((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))/((d*g*
h - c*h^2)*a - (d*g^2 - c*g*h)*b) + 2*(a*d*f^2*h*q^2 + (c*f^2*h*p*q - (p*q
+ q^2)*d*f^2*g)*b)*(log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilog
(-(d*h*x + c*h)/(d*g - c*h)))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b) - ((d
*f^2*g*p^2 - c*f^2*h*p^2)*b*log(b*x + a)^2 + 2*(b*d*f^2*g*p*q - a*d*f^2*h*p
*q)*log(b*x + a)*log(d*x + c) + (b*d*f^2*g*q^2 - a*d*f^2*h*q^2)*log(d*x + c
)^2 + 2*((a*d*f^2*h*p*q + (c*f^2*h*p^2 - (p^2 + p*q)*d*f^2*g)*b)*log(b*x +
a) + (a*d*f^2*h*q^2 + (c*f^2*h*p*q - (p*q + q^2)*d*f^2*g)*b)*log(d*x + c))*
log(h*x + g))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*r^2/(f^2*h) - log(((
b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((h*x + g)*h)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2}{h^2 x^2 + 2ghx + g^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^2*x^2 + 2*g*h*x + g^2),
x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**2,x)
```

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx + a)^p(dx + c)^q f\right)^r e\right)^2}{(hx + g)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(h\*x+g)^2,x, algorithm="giac")

[Out] integrate(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2/(h\*x + g)^2, x)



$$3.41 \quad \int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^3} dx$$

Optimal. Leaf size=1304

result too large to display

```
[Out] -((b*d*p*q*r^2*Log[a + b*x])/(h*(b*g - a*h)*(d*g - c*h))) + (d*p*q*r^2*Log[a + b*x])/(h*(d*g - c*h)*(g + h*x)) - (b*p^2*r^2*(a + b*x)*Log[a + b*x])/((b*g - a*h)^2*(g + h*x)) - (b*d*p*q*r^2*Log[c + d*x])/(h*(b*g - a*h)*(d*g - c*h)) + (b*p*q*r^2*Log[c + d*x])/(h*(b*g - a*h)*(g + h*x)) - (d*q^2*r^2*(c + d*x)*Log[c + d*x])/((d*g - c*h)^2*(g + h*x)) + (b^2*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(h*(b*g - a*h)^2) + (d^2*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(h*(d*g - c*h)^2) - (b*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(h*(b*g - a*h)*(g + h*x)) - (d*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(h*(d*g - c*h)*(g + h*x)) - (b^2*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(h*(b*g - a*h)^2) - (d^2*q*r*Log[c + d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(h*(d*g - c*h)^2) - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)^2/(2*h*(g + h*x)^2) + (b^2*p^2*r^2*Log[g + h*x])/(h*(b*g - a*h)^2) + (2*b*d*p*q*r^2*Log[g + h*x])/(h*(b*g - a*h)*(d*g - c*h)) + (d^2*q^2*r^2*Log[g + h*x])/(h*(d*g - c*h)^2) + (b^2*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))*Log[g + h*x])/(h*(b*g - a*h)^2) + (d^2*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))*Log[g + h*x])/(h*(d*g - c*h)^2) - (d^2*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(h*(d*g - c*h)^2) - (b^2*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(h*(b*g - a*h)^2) - (b^2*p^2*r^2*Log[a + b*x]*Log[1 + (b*g - a*h)/(h*(a + b*x))])/(h*(b*g - a*h)^2) - (d^2*q^2*r^2*Log[c + d*x]*Log[1 + (d*g - c*h)/(h*(c + d*x))])/(h*(d*g - c*h)^2) + (b^2*p^2*r^2*PolyLog[2, -((b*g - a*h)/(h*(a + b*x)))])/(h*(b*g - a*h)^2) + (d^2*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(h*(d*g - c*h)^2) - (d^2*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])/(h*(d*g - c*h)^2) + (d^2*q^2*r^2*PolyLog[2, -((d*g - c*h)/(h*(c + d*x)))])/(h*(d*g - c*h)^2) + (b^2*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(h*(b*g - a*h)^2) - (b^2*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(h*(b*g - a*h)^2)
```

---

**Rubi [A]** time = 1.40757, antiderivative size = 1362, normalized size of antiderivative = 1.04, number of steps used = 47, number of rules used = 16, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$ , Rules used = {2498, 2513, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2418, 2394,

2393, 2395, 36, 44}

$$\frac{p^2 r^2 \log^2(a + bx) b^2}{2h(bg - ah)^2} + \frac{pqr^2 \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c + dx) b^2}{h(bg - ah)^2} - \frac{pr \log(a + bx) \left( pr \log(a + bx) + qr \log(c + dx) - \log\left(e\left(f(a + bx)\right)\right)\right)}{h(bg - ah)^2}$$

Antiderivative was successfully verified.

[In] Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2/(g + h\*x)^3,x]

[Out] -((b\*d\*p\*q\*r^2\*Log[a + b\*x])/(h\*(b\*g - a\*h)\*(d\*g - c\*h))) + (d\*p\*q\*r^2\*Log[a + b\*x])/(h\*(d\*g - c\*h)\*(g + h\*x)) - (b\*p^2\*r^2\*(a + b\*x)\*Log[a + b\*x])/((b\*g - a\*h)^2\*(g + h\*x)) + (b^2\*p^2\*r^2\*Log[a + b\*x]^2)/(2\*h\*(b\*g - a\*h)^2) - (b\*d\*p\*q\*r^2\*Log[c + d\*x])/(h\*(b\*g - a\*h)\*(d\*g - c\*h)) + (b\*p\*q\*r^2\*Log[c + d\*x])/(h\*(b\*g - a\*h)\*(g + h\*x)) - (d\*q^2\*r^2\*(c + d\*x)\*Log[c + d\*x])/((d\*g - c\*h)^2\*(g + h\*x)) + (b^2\*p\*q\*r^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x])/(h\*(b\*g - a\*h)^2) + (d^2\*q^2\*r^2\*Log[c + d\*x]^2)/(2\*h\*(d\*g - c\*h)^2) + (d^2\*p\*q\*r^2\*Log[a + b\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(h\*(d\*g - c\*h)^2) - (b\*p\*r\*(p\*r\*Log[a + b\*x] + q\*r\*Log[c + d\*x] - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)))/(h\*(b\*g - a\*h)\*(g + h\*x)) - (d\*q\*r\*(p\*r\*Log[a + b\*x] + q\*r\*Log[c + d\*x] - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)))/(h\*(d\*g - c\*h)\*(g + h\*x)) - (b^2\*p\*r\*Log[a + b\*x]\*(p\*r\*Log[a + b\*x] + q\*r\*Log[c + d\*x] - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)))/(h\*(b\*g - a\*h)^2) - (d^2\*q\*r\*Log[c + d\*x]\*(p\*r\*Log[a + b\*x] + q\*r\*Log[c + d\*x] - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)))/(h\*(d\*g - c\*h)^2) - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^2/(2\*h\*(g + h\*x)^2) + (b^2\*p^2\*r^2\*Log[g + h\*x])/(h\*(b\*g - a\*h)^2) + (2\*b\*d\*p\*q\*r^2\*Log[g + h\*x])/(h\*(b\*g - a\*h)\*(d\*g - c\*h)) + (d^2\*q^2\*r^2\*Log[g + h\*x])/(h\*(d\*g - c\*h)^2) + (b^2\*p\*r\*(p\*r\*Log[a + b\*x] + q\*r\*Log[c + d\*x] - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r))\*Log[g + h\*x])/(h\*(b\*g - a\*h)^2) + (d^2\*q\*r\*(p\*r\*Log[a + b\*x] + q\*r\*Log[c + d\*x] - Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r))\*Log[g + h\*x])/(h\*(d\*g - c\*h)^2) - (b^2\*p^2\*r^2\*Log[a + b\*x]\*Log[(b\*(g + h\*x))/(b\*g - a\*h)])/(h\*(b\*g - a\*h)^2) - (d^2\*p\*q\*r^2\*Log[a + b\*x]\*Log[(b\*(g + h\*x))/(b\*g - a\*h)])/(h\*(d\*g - c\*h)^2) - (b^2\*p\*q\*r^2\*Log[c + d\*x]\*Log[(d\*(g + h\*x))/(d\*g - c\*h)])/(h\*(b\*g - a\*h)^2) - (d^2\*q^2\*r^2\*Log[c + d\*x]\*Log[(d\*(g + h\*x))/(d\*g - c\*h)])/(h\*(d\*g - c\*h)^2) + (d^2\*p\*q\*r^2\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(h\*(d\*g - c\*h)^2) - (b^2\*p^2\*r^2\*PolyLog[2, -((h\*(a + b\*x))/(b\*g - a\*h))])/(h\*(b\*g - a\*h)^2) - (d^2\*p\*q\*r^2\*PolyLog[2, -((h\*(a + b\*x))/(b\*g - a\*h))])/(h\*(d\*g - c\*h)^2) + (b^2\*p\*q\*r^2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])/(h\*(b\*g - a\*h)^2) - (b^2\*p\*q\*r^2\*PolyLog[2, -((h\*(c + d\*x))/(d\*g - c\*h))])/(h\*(b\*g - a\*h)^2) - (d^2\*q^2\*r^2\*PolyLog[2, -((h\*(c + d\*x))/(d\*g - c\*h))])/(h\*(d\*g - c\*h)^2)

### Rule 2498

Int[Log[(e\_.)\*((f\_.)\*(a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]^(s\_.)\*((g\_.) + (h\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(g + h\*x)^(m +

```

1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && NeQ[m, -1]

```

### Rule 2513

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]*(RFX_), x_Symbol] := Dist[p*r, Int[RFX*Log[a + b*x], x], x] + (Dist[q*r, Int[RFX*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFX, x], x]) /; FreeQ[{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFX, x] && NeQ[b*c - a*d, 0] && !MatchQ[RFX, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]]

```

### Rule 2411

```

Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] := Dist[1/e, Subst[Int[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]

```

### Rule 2347

```

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + (e_.)*(x_))^(q_.))/(x_), x_Symbol] := Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]

```

### Rule 2344

```

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_.)*((d_.) + (e_.)*(x_))), x_Symbol] := Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

```

### Rule 2301

```

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]

```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
:= Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
  Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b,
  c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2,
  -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x_Symbol]
:= Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b*n)/d,
  Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x]
&& EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol]
:= With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
  Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
  RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)),
x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g,
x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /;
FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol]
:= Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /;
FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^ (q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(g+hx)^3} dx &= -\frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{2h(g+hx)^2} + \frac{(bpr) \int \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)(g+hx)^2} dx}{h} + \frac{(dqr) \int \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)(g+hx)^2} dx}{h} \\
&= -\frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{2h(g+hx)^2} + \frac{(bp^2r^2) \int \frac{\log(a+bx)}{(a+bx)(g+hx)^2} dx}{h} + \frac{(bpqr^2) \int \frac{\log(c+dx)}{(a+bx)(g+hx)^2} dx}{h} \\
&= -\frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{2h(g+hx)^2} + \frac{(p^2r^2) \text{Subst} \left( \int \frac{\log(x)}{x \left( \frac{bg-ah}{b} + \frac{hx}{b} \right)^2} dx, x, a+bx \right)}{h} + \frac{(dqr) \int \frac{\log(c+dx)}{(a+bx)(g+hx)^2} dx}{h} \\
&= -\frac{bpr \left( pr \log(a+bx) + qr \log(c+dx) - \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) \right)}{h(bg-ah)(g+hx)} - \frac{dqr \left( pr \log(a+bx) + qr \log(c+dx) - \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) \right)}{h(bg-ah)(g+hx)} \\
&= \frac{dpqr^2 \log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx) \log(a+bx)}{(bg-ah)^2(g+hx)} + \frac{bpqr^2 \log(c+dx)}{h(bg-ah)(g+hx)} - \frac{dq^2r^2(c+dx) \log(c+dx)}{(dg-ch)(g+hx)} \\
&= \frac{dpqr^2 \log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx) \log(a+bx)}{(bg-ah)^2(g+hx)} + \frac{b^2p^2r^2 \log^2(a+bx)}{2h(bg-ah)^2} + \frac{bpqr^2 \log(c+dx)}{h(bg-ah)(g+hx)} - \frac{dq^2r^2(c+dx) \log(c+dx)}{(dg-ch)(g+hx)} \\
&= -\frac{bdpqr^2 \log(a+bx)}{h(bg-ah)(dg-ch)} + \frac{dpqr^2 \log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx) \log(a+bx)}{(bg-ah)^2(g+hx)} + \frac{b^2p^2r^2 \log^2(a+bx)}{2h(bg-ah)^2} + \frac{bpqr^2 \log(c+dx)}{h(bg-ah)(g+hx)} - \frac{dq^2r^2(c+dx) \log(c+dx)}{(dg-ch)(g+hx)}
\end{aligned}$$

**Mathematica [B]** time = 6.32159, size = 15976, normalized size = 12.25

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2/(g + h\*x)^3,x]

[Out] Result too large to show

**Maple [F]** time = 0.5, size = 0, normalized size = 0.

$$\int \frac{\left( \ln \left( e \left( f(bx+a)^p (dx+c)^q \right)^r \right) \right)^2}{(hx+g)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x)$

[Out]  $\text{int}(\ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x)$

**Maxima [A]** time = 2.44984, size = 2507, normalized size = 1.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, \text{algorithm}=\text{"maxima"})$

[Out]  $(b^2*f*p*\log(b*x + a)/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) + d^2*f*q*\log(d*x + c)/(d^2*g^2 - 2*c*d*g*h + c^2*h^2) + (2*a*b*d^2*f*g*h*q - a^2*d^2*f*h^2*q - (d^2*f*g^2*(p + q) - 2*c*d*f*g*h*p + c^2*f*h^2*p)*b^2)*\log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + (a*d*f*h*q - (d*f*g*(p + q) - c*f*h*p)*b)/((d*g^2*h - c*g*h^2)*a - (d*g^3 - c*g^2*h)*b + ((d*g*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*r*\log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(f*h) + 1/2*(2*(2*a*b*d^2*f^2*g*h*p*q - a^2*d^2*f^2*h^2*p*q - (2*c*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/((d*g*h^2 - c*h^3)*a^2 - 2*(d*g^2*h - c*g*h^2)*a*b + (d*g^3 - c*g^2*h)*b^2) - 2*(2*a*b*d^2*f^2*g*h*p*q - a^2*d^2*f^2*h^2*p*q + (2*c*d*f^2*g*h*p^2 - c^2*f^2*h^2*p^2 - (p^2 + p*q)*d^2*f^2*g^2)*b^2)*(log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))/((d*g*h^2 - c*h^3)*a^2 - 2*(d*g^2*h - c*g*h^2)*a*b + (d*g^3 - c*g^2*h)*b^2) + 2*(2*a*b*d^2*f^2*g*h*q^2 - a^2*d^2*f^2*h^2*q^2 + (2*c*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q - (p*q + q^2)*d^2*f^2*g^2)*b^2)*(log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilog(-(d*h*x + c*h)/(d*g - c*h)))/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) - 2*(a*d^2*f^2*h*q^2 + (c*d*f^2*h*p*q - (p*q + q^2)*d^2*f^2*g)*b)*log(d*x + c)/((d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3)*a - (d^2*g^3 - 2*c*d*g^2*h + c^2*g*h^2)*b) + 2*(a^2*d^2*f^2*h^2*q^2 + 2*(c*d*f^2*h^2*p*q - (p*q + q^2)*d^2*f^2*g*h)*a*b + (c^2*f^2*h^2*p^2 + (p^2 + 2*p*q + q^2)*d^2*f^2*g^2 - 2*(p^2 + p*q)*c*d*f^2*g*h)*b^2)*log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + ((d^3*f^2*g^3*p^2 - 3*c*d^2*f^2*g^2*h*p^2 + 3*c^2*d*f^2*g*h^2*p^2 - c^3*f^2*h^3*p^2)*b^2*log(b*x + a)^2 - 2*(b^2*d^2*f^2*g^2*p$

$$\begin{aligned}
& *q - 2*a*b*d^2*f^2*g*h*p*q + a^2*d^2*f^2*h^2*p*q) * \log(b*x + a) * \log(d*x + c) \\
& - (b^2*d^2*f^2*g^2*q^2 - 2*a*b*d^2*f^2*g*h*q^2 + a^2*d^2*f^2*h^2*q^2) * \log( \\
& d*x + c)^2 + 2*((d^2*f^2*g*h*p*q - c*d*f^2*h^2*p*q)*a*b - (c^2*f^2*h^2*p^2 \\
& + (p^2 + p*q)*d^2*f^2*g^2 - (2*p^2 + p*q)*c*d*f^2*g*h)*b^2) * \log(b*x + a) - \\
& 2*((2*a*b*d^2*f^2*g*h*p*q - a^2*d^2*f^2*h^2*p*q + (2*c*d*f^2*g*h*p^2 - c^2* \\
& f^2*h^2*p^2 - (p^2 + p*q)*d^2*f^2*g^2)*b^2) * \log(b*x + a) + (2*a*b*d^2*f^2*g \\
& *h*q^2 - a^2*d^2*f^2*h^2*q^2 + (2*c*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q - (p*q \\
& + q^2)*d^2*f^2*g^2)*b^2) * \log(d*x + c)) * \log(h*x + g) / ((d^2*g^2*h^2 - 2*c*d* \\
& g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2 \\
& *g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2)) * r^2 / (f^2*h) - 1/2 * \log(((b*x + a)^p \\
& (d*x + c)^q * f)^r * e)^2 / ((h*x + g)^2 * h)
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2}{h^3 x^3 + 3gh^2 x^2 + 3g^2 hx + g^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(h\*x+g)^3,x, algorithm="fricas")

[Out] integral(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2/(h^3\*x^3 + 3\*g\*h^2\*x^2 + 3\*g^2\*h\*x + g^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r)\*\*2/(h\*x+g)\*\*3,x)

[Out] Timed out



**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.42 \quad \int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^4} dx$$

Optimal. Leaf size=1957

result too large to display

```
[Out] -(b^2*p^2*r^2)/(3*h*(b*g - a*h)^2*(g + h*x)) - (2*b*d*p*q*r^2)/(3*h*(b*g - a*h)*(d*g - c*h)*(g + h*x)) - (d^2*q^2*r^2)/(3*h*(d*g - c*h)^2*(g + h*x)) - (b^3*p^2*r^2*Log[a + b*x])/(3*h*(b*g - a*h)^3) - (2*b*d^2*p*q*r^2*Log[a + b*x])/(3*h*(b*g - a*h)*(d*g - c*h)^2) - (b^2*d*p*q*r^2*Log[a + b*x])/(3*h*(b*g - a*h)^2*(d*g - c*h)) + (b*p^2*r^2*Log[a + b*x])/(3*h*(b*g - a*h)*(g + h*x)^2) + (d*p*q*r^2*Log[a + b*x])/(3*h*(d*g - c*h)*(g + h*x)^2) + (2*d^2*p*q*r^2*Log[a + b*x])/(3*h*(d*g - c*h)^2*(g + h*x)) - (2*b^2*p^2*r^2*(a + b*x)*Log[a + b*x])/(3*(b*g - a*h)^3*(g + h*x)) - (b*d^2*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)*(d*g - c*h)^2) - (2*b^2*d*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)^2*(d*g - c*h)) - (d^3*q^2*r^2*Log[c + d*x])/(3*h*(d*g - c*h)^3) + (b*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)*(g + h*x)^2) + (d*q^2*r^2*Log[c + d*x])/(3*h*(d*g - c*h)*(g + h*x)^2) + (2*b^2*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)^2*(g + h*x)) - (2*d^2*q^2*r^2*(c + d*x)*Log[c + d*x])/(3*(d*g - c*h)^3*(g + h*x)) + (2*b^3*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*h*(b*g - a*h)^3) + (2*d^3*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*h*(d*g - c*h)^3) - (b*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*h*(b*g - a*h)*(g + h*x)^2) - (d*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*h*(d*g - c*h)*(g + h*x)^2) - (2*b^2*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*h*(b*g - a*h)^2*(g + h*x)) - (2*d^2*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*h*(d*g - c*h)^2*(g + h*x)) - (2*b^3*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*h*(b*g - a*h)^3) - (2*d^3*q*r*Log[c + d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*h*(d*g - c*h)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)^2/(3*h*(g + h*x)^3) + (b^3*p^2*r^2*Log[g + h*x])/(h*(b*g - a*h)^3) + (b*d^2*p*q*r^2*Log[g + h*x])/(h*(b*g - a*h)*(d*g - c*h)^2) + (b^2*d*p*q*r^2*Log[g + h*x])/(h*(b*g - a*h)^2*(d*g - c*h)) + (d^3*q^2*r^2*Log[g + h*x])/(h*(d*g - c*h)^3) + (2*b^3*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))*Log[g + h*x])/(3*h*(b*g - a*h)^3) + (2*d^3*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r))*Log[g + h*x])/(3*h*(d*g - c*h)^3) - (2*d^3*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(3*h*(d*g - c*h)^3) - (2*b^3*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(3*h*(b*g - a*h)^3) - (2*b^3*p^2*r^2*Log[a + b*x]*Log[1 + (b*g - a*h)/(h*(a + b*x))])/(3*h*(b*g - a*h)^3) - (2*d^3*q^2*r^2*Log[c + d*x]*Log[1 + (d*g - c*h)/(h*(c + d*x))])/(3*h*(d*g - c*h)^3) + (2*b^3*p^2*r^2*PolyLog[2, -(b*g
```

$$- a*h)/(h*(a + b*x)))]/(3*h*(b*g - a*h)^3) + (2*d^3*p*q*r^2*PolyLog[2, -(d*(a + b*x))/(b*c - a*d)])/(3*h*(d*g - c*h)^3) - (2*d^3*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h)))]/(3*h*(d*g - c*h)^3) + (2*d^3*q^2*r^2*PolyLog[2, -((d*g - c*h)/(h*(c + d*x)))])/(3*h*(d*g - c*h)^3) + (2*b^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*h*(b*g - a*h)^3) - (2*b^3*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h)))]/(3*h*(b*g - a*h)^3)$$

**Rubi [A]** time = 2.10192, antiderivative size = 2013, normalized size of antiderivative = 1.03, number of steps used = 61, number of rules used = 17, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$ , Rules used = {2498, 2513, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44, 2418, 2394, 2393, 2395, 36}

result too large to display

Antiderivative was successfully verified.

[In] Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2/(g + h\*x)^4,x]

[Out]  $-(b^2*p^2*r^2)/(3*h*(b*g - a*h)^2*(g + h*x)) - (2*b*d*p*q*r^2)/(3*h*(b*g - a*h)*(d*g - c*h)*(g + h*x)) - (d^2*q^2*r^2)/(3*h*(d*g - c*h)^2*(g + h*x)) - (b^3*p^2*r^2*Log[a + b*x])/(3*h*(b*g - a*h)^3) - (2*b*d^2*p*q*r^2*Log[a + b*x])/(3*h*(b*g - a*h)*(d*g - c*h)^2) - (b^2*d*p*q*r^2*Log[a + b*x])/(3*h*(b*g - a*h)^2*(d*g - c*h)) + (b*p^2*r^2*Log[a + b*x])/(3*h*(b*g - a*h)*(g + h*x)^2) + (d*p*q*r^2*Log[a + b*x])/(3*h*(d*g - c*h)*(g + h*x)^2) + (2*d^2*p*q*r^2*Log[a + b*x])/(3*h*(d*g - c*h)^2*(g + h*x)) - (2*b^2*p^2*r^2*(a + b*x)*Log[a + b*x])/(3*(b*g - a*h)^3*(g + h*x)) + (b^3*p^2*r^2*Log[a + b*x]^2)/(3*h*(b*g - a*h)^3) - (b*d^2*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)*(d*g - c*h)^2) - (2*b^2*d*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)^2*(d*g - c*h)) - (d^3*q^2*r^2*Log[c + d*x])/(3*h*(d*g - c*h)^3) + (b*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)*(g + h*x)^2) + (d*q^2*r^2*Log[c + d*x])/(3*h*(d*g - c*h)*(g + h*x)^2) + (2*b^2*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)^2*(g + h*x)) - (2*d^2*q^2*r^2*(c + d*x)*Log[c + d*x])/(3*(d*g - c*h)^3*(g + h*x)) + (2*b^3*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*h*(b*g - a*h)^3) + (d^3*q^2*r^2*Log[c + d*x]^2)/(3*h*(d*g - c*h)^3) + (2*d^3*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*h*(d*g - c*h)^3) - (b*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*h*(b*g - a*h)*(g + h*x)^2) - (d*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*h*(d*g - c*h)*(g + h*x)^2) - (2*b^2*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*h*(b*g - a*h)^2*(g + h*x)) - (2*d^2*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)))/(3*h*(d*g - c*h)^2*(g + h*x)) - (2*b^3*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] -$

$$\begin{aligned} & \text{Log}[e^{*(f*(a + b*x)^p*(c + d*x)^q)^r}]/(3*h*(b*g - a*h)^3) - (2*d^3*q*r*Lo \\ & g[c + d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - \text{Log}[e^{*(f*(a + b*x)^p*(c + \\ & d*x)^q)^r}])/(3*h*(d*g - c*h)^3) - \text{Log}[e^{*(f*(a + b*x)^p*(c + d*x)^q)^r}]^2/ \\ & (3*h*(g + h*x)^3) + (b^3*p^2*r^2*Log[g + h*x])/(h*(b*g - a*h)^3) + (b*d^2*p \\ & *q*r^2*Log[g + h*x])/(h*(b*g - a*h)*(d*g - c*h)^2) + (b^2*d*p*q*r^2*Log[g + \\ & h*x])/(h*(b*g - a*h)^2*(d*g - c*h)) + (d^3*q^2*r^2*Log[g + h*x])/(h*(d*g - \\ & c*h)^3) + (2*b^3*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - \text{Log}[e^{*(f*(a + \\ & b*x)^p*(c + d*x)^q)^r}])*Log[g + h*x]/(3*h*(b*g - a*h)^3) + (2*d^3*q*r*(p*r \\ & *Log[a + b*x] + q*r*Log[c + d*x] - \text{Log}[e^{*(f*(a + b*x)^p*(c + d*x)^q)^r}])*Lo \\ & g[g + h*x]/(3*h*(d*g - c*h)^3) - (2*b^3*p^2*r^2*Log[a + b*x]*Log[(b*(g + h \\ & *x))/(b*g - a*h)])/(3*h*(b*g - a*h)^3) - (2*d^3*p*q*r^2*Log[a + b*x]*Log[(b \\ & *(g + h*x))/(b*g - a*h)])/(3*h*(d*g - c*h)^3) - (2*b^3*p*q*r^2*Log[c + d*x] \\ & *Log[(d*(g + h*x))/(d*g - c*h)])/(3*h*(b*g - a*h)^3) - (2*d^3*q^2*r^2*Log[c \\ & + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(3*h*(d*g - c*h)^3) + (2*d^3*p*q*r^ \\ & 2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))]/(3*h*(d*g - c*h)^3) - (2*b^3*p^ \\ & 2*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))]/(3*h*(b*g - a*h)^3) - (2*d^ \\ & 3*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))]/(3*h*(d*g - c*h)^3) + ( \\ & 2*b^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]/(3*h*(b*g - a*h)^3) - \\ & (2*b^3*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))]/(3*h*(b*g - a*h)^3 \\ & ) - (2*d^3*q^2*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))]/(3*h*(d*g - c* \\ & h)^3) \end{aligned}$$

### Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*s)
/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
+ 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

### Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_), x_Symbol] := Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.)] /; IntegersQ[m, n
]
```

### Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] := Dist[1/e, Subst[Int[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

#### Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)/ (x_), x_Symbol] := Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

#### Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_Symbol] := Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e, Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x_Symbol] := Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_) + Log[(c_)*(x_)^(n_)]*(b_))^(p_)*((d_) + (e_)*(x_))^(q_),
x_Symbol] := Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] && !IGtQ[q, 0]) || (EqQ[p, 2] &&
NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2418

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_))^(p_)*(RFx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_))/((f_) + (g_)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_))/((f_) + (g_)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
(e*f - d*g), 0]
```

Rule 2395

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_))*((f_) + (g_)*(x_
```

```

))^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]

```

### Rule 36

```

Int[1/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(g+hx)^4} dx &= -\frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{3h(g+hx)^3} + \frac{(2bpr) \int \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)(g+hx)^3} dx}{3h} + \frac{(2dqr) \int \frac{\log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{(a+bx)(g+hx)^3} dx}{3h} \\
&= -\frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{3h(g+hx)^3} + \frac{(2bp^2r^2) \int \frac{\log(a+bx)}{(a+bx)(g+hx)^3} dx}{3h} + \frac{(2bpqr^2) \int \frac{\log(c+dx)}{(a+bx)(g+hx)^3} dx}{3h} \\
&= -\frac{\log^2 \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right)}{3h(g+hx)^3} + \frac{(2p^2r^2) \text{Subst} \left( \int \frac{\log(x)}{x \left( \frac{bg-ah}{b} + \frac{hx}{b} \right)^3} dx, x, a+bx \right)}{3h} \\
&= -\frac{bpr \left( pr \log(a+bx) + qr \log(c+dx) - \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) \right)}{3h(bg-ah)(g+hx)^2} - \frac{dqr \left( pr \log(a+bx) + qr \log(c+dx) - \log \left( e \left( f(a+bx)^p (c+dx)^q \right)^r \right) \right)}{3h(bg-ah)(g+hx)^2} \\
&= \frac{bp^2r^2 \log(a+bx)}{3h(bg-ah)(g+hx)^2} + \frac{dpqr^2 \log(a+bx)}{3h(dg-ch)(g+hx)^2} + \frac{2d^2pqr^2 \log(a+bx)}{3h(dg-ch)^2(g+hx)} + \frac{bpqr^2 \log(c+dx)}{3h(bg-ah)(g+hx)^2} \\
&= \frac{bp^2r^2 \log(a+bx)}{3h(bg-ah)(g+hx)^2} + \frac{dpqr^2 \log(a+bx)}{3h(dg-ch)(g+hx)^2} + \frac{2d^2pqr^2 \log(a+bx)}{3h(dg-ch)^2(g+hx)} - \frac{2b^2p^2r^2 \log(c+dx)}{3h(bg-ah)(g+hx)^2} \\
&= -\frac{b^2p^2r^2}{3h(bg-ah)^2(g+hx)} - \frac{2bdpqr^2}{3h(bg-ah)(dg-ch)(g+hx)} - \frac{d^2q^2r^2}{3h(dg-ch)^2(g+hx)} \\
&= -\frac{b^2p^2r^2}{3h(bg-ah)^2(g+hx)} - \frac{2bdpqr^2}{3h(bg-ah)(dg-ch)(g+hx)} - \frac{d^2q^2r^2}{3h(dg-ch)^2(g+hx)}
\end{aligned}$$

**Mathematica [B]** time = 6.54072, size = 47110, normalized size = 24.07

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^2/(g + h\*x)^4,x]

[Out] Result too large to show

**Maple [F]** time = 0.52, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)\right)^2}{(hx+g)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(h\*x+g)^4,x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(h\*x+g)^4,x)

**Maxima [B]** time = 5.21219, size = 6388, normalized size = 3.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(h\*x+g)^4,x, algorithm="maxima")

[Out]  $\frac{1}{3}(2b^3f^p\log(bx+a)/(b^3g^3 - 3a*b^2g^2h + 3a^2b*gh^2 - a^3h^3) + 2d^3f^q\log(dx+c)/(d^3g^3 - 3c*d^2g^2h + 3c^2d*gh^2 - c^3h^3) - 2(3a*b^2d^3f^p g^2h^q - 3a^2b*d^3f^p g^2h^q + a^3d^3f^p h^3q - (d^3f^p g^3(p+q) - 3c*d^2f^p g^2h^p + 3c^2d*f^p g^2h^2p - c^3f^p h^3p)*b^3)\log(hx+g)/((d^3g^3h^3 - 3c*d^2g^2h^4 + 3c^2d*gh^5 - c^3h^6)a^3 - 3(d^3g^4h^2 - 3c*d^2g^3h^3 + 3c^2d*g^2h^4 - c^3g^2h^5)a^2b + 3(d^3g^5h - 3c*d^2g^4h^2 + 3c^2d*g^3h^3 - c^3g^2h^4)a*b^2 - (d^3g^6 - 3c*d^2g^5h + 3c^2d*g^4h^2 - c^3g^3h^3)*b^3) + ((3d^3$



$$\begin{aligned}
& 2*f*g*h^2*q - c*d*f*h^3*q)*a^2 - (d^2*f*g^2*h*(p + 6*q) - 2*c*d*f*g*h^2*(p \\
& + q) + c^2*f*h^3*p)*a*b - (c*d*f*g^2*h*(6*p + q) - 3*d^2*f*g^3*(p + q) - 3* \\
& c^2*f*g*h^2*p)*b^2 - 2*(2*a*b*d^2*f*g*h^2*q - a^2*d^2*f*h^3*q - (d^2*f*g^2* \\
& h*(p + q) - 2*c*d*f*g*h^2*p + c^2*f*h^3*p)*b^2)*x)/((d^2*g^4*h^2 - 2*c*d*g^ \\
& 3*h^3 + c^2*g^2*h^4)*a^2 - 2*(d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*a*b \\
& + (d^2*g^6 - 2*c*d*g^5*h + c^2*g^4*h^2)*b^2 + ((d^2*g^2*h^4 - 2*c*d*g*h^5 + \\
& c^2*h^6)*a^2 - 2*(d^2*g^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a*b + (d^2*g^4* \\
& h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*b^2)*x^2 + 2*((d^2*g^3*h^3 - 2*c*d*g^2*h \\
& ^4 + c^2*g*h^5)*a^2 - 2*(d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a*b + ( \\
& d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*b^2)*x)*r*\log((b*x + a)^p*(d*x + \\
& c)^q*f)^r*e)/(f*h) + 1/3*(2*(3*a*b^2*d^3*f^2*g^2*h*p*q - 3*a^2*b*d^3*f^2*g \\
& *h^2*p*q + a^3*d^3*f^2*h^3*p*q - (3*c*d^2*f^2*g^2*h*p*q - 3*c^2*d*f^2*g*h^2 \\
& *p*q + c^3*f^2*h^3*p*q)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + \\
& 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3 \\
& *c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^ \\
& 2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 \\
& - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^ \\
& 3*h^3)*b^3) - 2*(3*a*b^2*d^3*f^2*g^2*h*p*q - 3*a^2*b*d^3*f^2*g*h^2*p*q + a^ \\
& 3*d^3*f^2*h^3*p*q + (3*c*d^2*f^2*g^2*h*p^2 - 3*c^2*d*f^2*g*h^2*p^2 + c^3*f^ \\
& 2*h^3*p^2 - (p^2 + p*q)*d^3*f^2*g^3)*b^3)*(log(b*x + a)*log((b*h*x + a*h)/( \\
& b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))/((d^3*g^3*h^3 - 3*c*d^ \\
& 2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 \\
& + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3* \\
& c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4 \\
& *h^2 - c^3*g^3*h^3)*b^3) - 2*(3*a*b^2*d^3*f^2*g^2*h*q^2 - 3*a^2*b*d^3*f^2*g \\
& *h^2*q^2 + a^3*d^3*f^2*h^3*q^2 + (3*c*d^2*f^2*g^2*h*p*q - 3*c^2*d*f^2*g*h^2 \\
& *p*q + c^3*f^2*h^3*p*q - (p*q + q^2)*d^3*f^2*g^3)*b^3)*(log(d*x + c)*log((d \\
& *h*x + c*h)/(d*g - c*h) + 1) + dilog(-(d*h*x + c*h)/(d*g - c*h)))/((d^3*g^3 \\
& *h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3* \\
& c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2 \\
& *g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h \\
& + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) - (3*a^2*d^3*f^2*h^2*q^2 + (c*d^2*f^2 \\
& *h^2*p*q - (p*q + 6*q^2)*d^3*f^2*g*h)*a*b - (5*c*d^2*f^2*g*h*p*q - 2*c^2*d* \\
& f^2*h^2*p*q - 3*(p*q + q^2)*d^3*f^2*g^2)*b^2)*log(d*x + c)/((d^3*g^3*h^2 - \\
& 3*c*d^2*g^2*h^3 + 3*c^2*d*g*h^4 - c^3*h^5)*a^2 - 2*(d^3*g^4*h - 3*c*d^2*g^3 \\
& *h^2 + 3*c^2*d*g^2*h^3 - c^3*g*h^4)*a*b + (d^3*g^5 - 3*c*d^2*g^4*h + 3*c^2* \\
& d*g^3*h^2 - c^3*g^2*h^3)*b^2) + 3*(a^3*d^3*f^2*h^3*q^2 + (c*d^2*f^2*h^3*p*q \\
& - (p*q + 3*q^2)*d^3*f^2*g*h^2)*a^2*b - (4*c*d^2*f^2*g*h^2*p*q - c^2*d*f^2* \\
& h^3*p*q - 3*(p*q + q^2)*d^3*f^2*g^2*h)*a*b^2 + (c^3*f^2*h^3*p^2 - (p^2 + 2* \\
& p*q + q^2)*d^3*f^2*g^3 + 3*(p^2 + p*q)*c*d^2*f^2*g^2*h - (3*p^2 + p*q)*c^2* \\
& d*f^2*g*h^2)*b^3)*log(h*x + g)/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g* \\
& h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c \\
& ^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^ \\
& 2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^ \\
& 3) - ((d^3*f^2*g*h^3*q^2 - c*d^2*f^2*h^4*q^2)*a^3 - (2*c^2*d*f^2*h^4*p*q +
\end{aligned}$$

$$\begin{aligned}
& (2pq + 3q^2)d^3f^2g^2h^2 - (4pq + 3q^2)c^2d^2f^2g^3h^3)a^2b - \\
& (c^3f^2h^4p^2 - (p^2 + 4pq + 3q^2)d^3f^2g^3h + (3p^2 + 8pq + 3q^2)c^2d^2f^2g^2h^2 - (3p^2 + 4pq)c^2d^2f^2g^3h^3)a^2b^2 + (c^3f^2g^3h^3p^2 - (p^2 + 2pq + q^2)d^3f^2g^4 + (3p^2 + 4pq + q^2)c^2d^2f^2g^3h - (3p^2 + 2pq)c^2d^2f^2g^2h^2)b^3 - ((d^3f^2g^3h^3p^2 - 3c^2d^2f^2g^2h^2p^2 + 3c^2d^2f^2g^3h^3p^2 - c^3f^2h^4p^2)b^3x + (d^3f^2g^4p^2 - 3c^2d^2f^2g^3h^3p^2 + 3c^2d^2f^2g^2h^2p^2 - c^3f^2g^3h^3p^2)b^3)\log(bx + a)^2 - 2(b^3d^3f^2g^4pq - 3a^2b^2d^3f^2g^3h^3pq + 3a^2bd^3f^2g^2h^2pq - a^3d^3f^2g^3h^3pq + (b^3d^3f^2g^3h^3pq - 3a^2b^2d^3f^2g^2h^2pq + 3a^2bd^3f^2g^3h^3pq - a^3d^3f^2h^4pq)x)\log(bx + a)\log(dx + c) - (b^3d^3f^2g^4q^2 - 3a^2b^2d^3f^2g^3h^3q^2 + 3a^2bd^3f^2g^2h^2q^2 - a^3d^3f^2g^3h^3q^2 + (b^3d^3f^2g^3h^3q^2 - 3a^2b^2d^3f^2g^2h^2q^2 + 3a^2bd^3f^2g^3h^3q^2 - a^3d^3f^2h^4q^2)x)\log(dx + c)^2 - (2(d^3f^2g^2h^2pq - c^2d^2f^2g^3h^3pq)a^2b - (5d^3f^2g^3h^3pq - 6c^2d^2f^2g^2h^2pq + c^2d^2f^2g^3h^3pq)a^2b^2 - (3c^3f^2g^3h^3p^2 - 3(p^2 + pq)d^3f^2g^4 + (9p^2 + 4pq)c^2d^2f^2g^3h - (9p^2 + pq)c^2d^2f^2g^2h^2)b^3 + (2(d^3f^2g^3h^3pq - c^2d^2f^2h^4pq)a^2b - (5d^3f^2g^2h^2pq - 6c^2d^2f^2g^3h^3pq + c^2d^2f^2h^4pq)a^2b^2 - (3c^3f^2h^4p^2 - 3(p^2 + pq)d^3f^2g^3h + (9p^2 + 4pq)c^2d^2f^2g^2h^2 - (9p^2 + pq)c^2d^2f^2g^3h^3)b^3)x)\log(bx + a) - 2((3a^2b^2d^3f^2g^3h^3pq - 3a^2bd^3f^2g^2h^2pq + a^3d^3f^2g^3h^3pq + (3c^2d^2f^2g^3h^3p^2 - 3c^2d^2f^2g^2h^2p^2 + c^3f^2g^3h^3p^2 - (p^2 + pq)d^3f^2g^4)b^3 + (3a^2b^2d^3f^2g^2h^2pq - 3a^2bd^3f^2g^3h^3pq + a^3d^3f^2h^4pq + (3c^2d^2f^2g^2h^2p^2 - 3c^2d^2f^2g^3h^3p^2 + c^3f^2h^4p^2 - (p^2 + pq)d^3f^2g^3h^3)b^3)x)\log(bx + a) + (3a^2b^2d^3f^2g^3h^3q^2 - 3a^2bd^3f^2g^2h^2q^2 + a^3d^3f^2g^3h^3q^2 + (3c^2d^2f^2g^3h^3pq - 3c^2d^2f^2g^2h^2pq + c^3f^2g^3h^3pq - (pq + q^2)d^3f^2g^4)b^3 + (3a^2b^2d^3f^2g^2h^2q^2 - 3a^2bd^3f^2g^3h^3q^2 + a^3d^3f^2h^4q^2 + (3c^2d^2f^2g^2h^2pq - 3c^2d^2f^2g^3h^3pq + c^3f^2h^4pq - (pq + q^2)d^3f^2g^3h^3)b^3)x)\log(dx + c))\log(hx + g))/((d^3g^4h^3 - 3c^2d^2g^3h^4 + 3c^2d^2g^2h^5 - c^3g^2h^6)a^3 - 3(d^3g^5h^2 - 3c^2d^2g^4h^3 + 3c^2d^2g^3h^4 - c^3g^2h^5)a^2b + 3(d^3g^6h - 3c^2d^2g^5h^2 + 3c^2d^2g^4h^3 - c^3g^3h^4)a^2b^2 - (d^3g^7 - 3c^2d^2g^6h + 3c^2d^2g^5h^2 - c^3g^4h^3)b^3 + ((d^3g^3h^4 - 3c^2d^2g^2h^5 + 3c^2d^2g^3h^6 - c^3h^7)a^3 - 3(d^3g^4h^3 - 3c^2d^2g^3h^4 + 3c^2d^2g^2h^5 - c^3g^3h^6)a^2b + 3(d^3g^5h^2 - 3c^2d^2g^4h^3 + 3c^2d^2g^3h^4 - c^3g^2h^5)a^2b^2 - (d^3g^6h - 3c^2d^2g^5h^2 + 3c^2d^2g^4h^3 - c^3g^3h^4)b^3)x))r^2/(f^2h) - 1/3\log((bx + a)^p(dx + c)^qf)^2e)^2/((hx + g)^3h)
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2}{h^4 x^4 + 4gh^3 x^3 + 6g^2 h^2 x^2 + 4g^3 hx + g^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(h\*x+g)^4,x, algorithm="fricas")

[Out] integral(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2/(h^4\*x^4 + 4\*g\*h^3\*x^3 + 6\*g^2\*h^2\*x^2 + 4\*g^3\*h\*x + g^4), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r)\*\*2/(h\*x+g)\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)^2}{(hx + g)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)^2/(h\*x+g)^4,x, algorithm="giac")

[Out] integrate(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)^2/(h\*x + g)^4, x)

$$3.43 \quad \int \frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

**Optimal.** Leaf size=42

$$\frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^{n+1}}{bc(n+1)}$$

[Out] -((a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^(1 + n)/(b\*c\*(1 + n)))

**Rubi [A]** time = 0.0802486, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$ , Rules used = {2512, 2302, 30}

$$\frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

[Out] -((a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^(1 + n)/(b\*c\*(1 + n)))

#### Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

#### Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
x]
```

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned} \int \frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1 - c^2x^2} dx &= -\frac{\text{Subst}\left(\int \frac{(a+b \log(x))^n}{x} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\ &= -\frac{\text{Subst}\left(\int x^n dx, x, a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc} \\ &= -\frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^{1+n}}{bc(1+n)} \end{aligned}$$

**Mathematica [A]** time = 0.0290376, size = 42, normalized size = 1.

$$-\frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

[Out] -((a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^(1 + n)/(b\*c\*(1 + n)))

**Maple [F]** time = 0.485, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2 + 1} \left( a + b \ln \left( \sqrt{-cx + 1} \frac{1}{\sqrt{cx + 1}} \right) \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^n/(-c^2\*x^2+1), x)

[Out] int((a+b\*ln((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^n/(-c^2\*x^2+1), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^n}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^n/(-c^2\*x^2+1),x, algorithm="maxima")

[Out] -integrate((b\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)^n/(c^2\*x^2 - 1), x)

**Fricas [A]** time = 2.19345, size = 138, normalized size = 3.29

$$\frac{\left(b \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)\left(b \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^n}{bcn + bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^n/(-c^2\*x^2+1),x, algorithm="fricas")

[Out] -(b\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)\*(b\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)^n/(b\*c\*n + b\*c)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln((-c\*x+1)\*\*(1/2)/(c\*x+1)\*\*(1/2)))\*\*n/(-c\*\*2\*x\*\*2+1),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^n}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

$$3.44 \quad \int \frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2} dx$$

**Optimal.** Leaf size=37

$$-\frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc}$$

[Out]  $-(a + b \cdot \text{Log}[\text{Sqrt}[1 - c \cdot x]/\text{Sqrt}[1 + c \cdot x]])^4/(4 \cdot b \cdot c)$

**Rubi [A]** time = 0.0611698, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$ , Rules used = {2512, 2302, 30}

$$-\frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b \cdot \text{Log}[\text{Sqrt}[1 - c \cdot x]/\text{Sqrt}[1 + c \cdot x]])^3/(1 - c^2 \cdot x^2), x]$

[Out]  $-(a + b \cdot \text{Log}[\text{Sqrt}[1 - c \cdot x]/\text{Sqrt}[1 + c \cdot x]])^4/(4 \cdot b \cdot c)$

#### Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] := Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

#### Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] := Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
x]
```

#### Rule 30



```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1 - c^2x^2} dx &= -\frac{\text{Subst}\left(\int \frac{(a+b \log(x))^3}{x} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\ &= -\frac{\text{Subst}\left(\int x^3 dx, x, a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc} \\ &= -\frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} \end{aligned}$$

**Mathematica [A]** time = 0.0089422, size = 37, normalized size = 1.

$$-\frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

```
[Out] -(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(4*b*c)
```

**Maple [F]** time = 0.411, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2 + 1} \left( a + b \ln\left(\sqrt{-cx + 1} \frac{1}{\sqrt{cx + 1}}\right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1), x)
```

```
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1), x)
```

**Maxima [B]** time = 1.35167, size = 710, normalized size = 19.19

$$\frac{1}{2} b^3 \left( \frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c} \right) \log \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^3 + \frac{3}{2} ab^2 \left( \frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c} \right) \log \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^2 + \frac{3}{2} a^2 b \left( \frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c} \right) \log \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3/(-c^2\*x^2+1),x, algorithm="maxima")

[Out] 1/2\*b^3\*(log(c\*x + 1)/c - log(c\*x - 1)/c)\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^3 + 3/2\*a\*b^2\*(log(c\*x + 1)/c - log(c\*x - 1)/c)\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^2 + 3/2\*a^2\*b\*(log(c\*x + 1)/c - log(c\*x - 1)/c)\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + 1/64\*(24\*(log(c\*x + 1)^2 - 2\*log(c\*x + 1)\*log(c\*x - 1) + log(c\*x - 1)^2)\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^2/c + 8\*(log(c\*x + 1)^3 - 3\*log(c\*x + 1)^2\*log(c\*x - 1) + 3\*log(c\*x + 1)\*log(c\*x - 1)^2 - log(c\*x - 1)^3)\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1))/c + (log(c\*x + 1)^4 - 4\*log(c\*x + 1)^3\*log(c\*x - 1) + 6\*log(c\*x + 1)^2\*log(c\*x - 1)^2 - 4\*log(c\*x + 1)\*log(c\*x - 1)^3 + log(c\*x - 1)^4)/c)\*b^3 + 1/8\*a\*b^2\*(6\*(log(c\*x + 1)^2 - 2\*log(c\*x + 1)\*log(c\*x - 1) + log(c\*x - 1)^2)\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1))/c + (log(c\*x + 1)^3 - 3\*log(c\*x + 1)^2\*log(c\*x - 1) + 3\*log(c\*x + 1)\*log(c\*x - 1)^2 - log(c\*x - 1)^3)/c) + 1/2\*a^3\*(log(c\*x + 1)/c - log(c\*x - 1)/c) + 3/8\*(log(c\*x + 1)^2 - 2\*log(c\*x + 1)\*log(c\*x - 1) + log(c\*x - 1)^2)\*a^2\*b/c

**Fricas [B]** time = 2.01189, size = 248, normalized size = 6.7

$$\frac{b^3 \log \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^4 + 4ab^2 \log \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^3 + 6a^2b \log \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^2 + 4a^3 \log \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3/(-c^2\*x^2+1),x, algorithm="fricas")

[Out] -1/4\*(b^3\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^4 + 4\*a\*b^2\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^3 + 6\*a^2\*b\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^2 + 4\*a^3\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1)))/c

**Sympy [A]** time = 47.0202, size = 65, normalized size = 1.76

$$\begin{cases} \frac{a^3 \operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2 \sqrt{-\frac{1}{c^2}}} & \text{for } b = 0 \\ a^3 x & \text{for } c = 0 \\ -\frac{\left(a + b \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^4}{4bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a**3*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*
*3*x, Eq(c, 0)), (-a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**4/(4*b*c), Tr
ue))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^3}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x)
```

$$3.45 \quad \int \frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2} dx$$

**Optimal.** Leaf size=37

$$-\frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}$$

[Out]  $-(a + b \cdot \text{Log}[\text{Sqrt}[1 - c \cdot x]/\text{Sqrt}[1 + c \cdot x]])^3/(3 \cdot b \cdot c)$

**Rubi [A]** time = 0.0628152, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$ , Rules used = {2512, 2302, 30}

$$-\frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b \cdot \text{Log}[\text{Sqrt}[1 - c \cdot x]/\text{Sqrt}[1 + c \cdot x]])^2/(1 - c^2 \cdot x^2), x]$

[Out]  $-(a + b \cdot \text{Log}[\text{Sqrt}[1 - c \cdot x]/\text{Sqrt}[1 + c \cdot x]])^3/(3 \cdot b \cdot c)$

#### Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

#### Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
x]
```

#### Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1 - c^2x^2} dx &= -\frac{\text{Subst}\left(\int \frac{(a+b \log(x))^2}{x} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\ &= -\frac{\text{Subst}\left(\int x^2 dx, x, a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc} \\ &= -\frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} \end{aligned}$$

**Mathematica [A]** time = 0.0100674, size = 37, normalized size = 1.

$$-\frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

```
[Out] -(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(3*b*c)
```

**Maple [F]** time = 0.342, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2 + 1} \left( a + b \ln \left( \sqrt{-cx + 1} \frac{1}{\sqrt{cx + 1}} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1), x)
```

```
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1), x)
```

**Maxima [B]** time = 1.27657, size = 362, normalized size = 9.78

$$\frac{1}{2} b^2 \left( \frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c} \right) \log \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^2 + ab \left( \frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c} \right) \log \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + \frac{1}{24} b^2 \left( \frac{6(\log(cx+1))^2 - 2\log(cx+1)\log(cx-1) + \log^2(cx-1)}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2/(-c^2\*x^2+1),x, algorithm="maxima")

[Out] 1/2\*b^2\*(log(c\*x + 1)/c - log(c\*x - 1)/c)\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^2 + a\*b\*(log(c\*x + 1)/c - log(c\*x - 1)/c)\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + 1/24\*b^2\*(6\*(log(c\*x + 1))^2 - 2\*log(c\*x + 1)\*log(c\*x - 1) + log(c\*x - 1)^2)\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1))/c + (log(c\*x + 1)^3 - 3\*log(c\*x + 1)^2\*log(c\*x - 1) + 3\*log(c\*x + 1)\*log(c\*x - 1)^2 - log(c\*x - 1)^3)/c + 1/2\*a^2\*(log(c\*x + 1)/c - log(c\*x - 1)/c) + 1/4\*(log(c\*x + 1)^2 - 2\*log(c\*x + 1)\*log(c\*x - 1) + log(c\*x - 1)^2)\*a\*b/c

**Fricas [B]** time = 2.12897, size = 184, normalized size = 4.97

$$\frac{b^2 \log \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^3 + 3ab \log \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^2 + 3a^2 \log \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2/(-c^2\*x^2+1),x, algorithm="fricas")

[Out] -1/3\*(b^2\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^3 + 3\*a\*b\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^2 + 3\*a^2\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1)))/c

**Sympy [A]** time = 23.3409, size = 65, normalized size = 1.76

$$\begin{cases} \frac{a^2 \operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2 \sqrt{-\frac{1}{c^2}}} & \text{for } b = 0 \\ a^2 x & \text{for } c = 0 \\ -\frac{\left(a + b \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^3}{3bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a**2*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*
*2*x, Eq(c, 0)), (-a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**3/(3*b*c), Tr
ue))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^2}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x)
```

$$3.46 \quad \int \frac{a+b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2} dx$$

**Optimal.** Leaf size=37

$$-\frac{\left(a+b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}$$

[Out]  $-(a + b*\text{Log}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[1 + c*x]])^2/(2*b*c)$

**Rubi [A]** time = 0.0379821, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2512, 2301}

$$-\frac{\left(a+b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Log}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[1 + c*x]])/(1 - c^2*x^2), x]$

[Out]  $-(a + b*\text{Log}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[1 + c*x]])^2/(2*b*c)$

#### Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rubi steps



$$\int \frac{a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2} dx = -\frac{\text{Subst}\left(\int \frac{a+b \log(x)}{x} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}$$

**Mathematica [A]** time = 0.0063933, size = 37, normalized size = 1.

$$\frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])/(1 - c^2\*x^2), x]

[Out] -(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2/(2\*b\*c)

**Maple [F]** time = 0.364, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2 + 1} \left( a + b \ln\left(\sqrt{-cx + 1} \frac{1}{\sqrt{cx + 1}}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))/(-c^2\*x^2+1), x)

[Out] int((a+b\*ln((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))/(-c^2\*x^2+1), x)

**Maxima [B]** time = 1.20672, size = 142, normalized size = 3.84

$$\frac{1}{2} b \left( \frac{\log(cx + 1)}{c} - \frac{\log(cx - 1)}{c} \right) \log\left(\frac{\sqrt{-cx + 1}}{\sqrt{cx + 1}}\right) + \frac{1}{2} a \left( \frac{\log(cx + 1)}{c} - \frac{\log(cx - 1)}{c} \right) + \frac{(\log(cx + 1))^2 - 2 \log(cx + 1) \log(cx - 1)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) +
1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(log(c*x + 1)^2 - 2*log(c*x
+ 1)*log(c*x - 1) + log(c*x - 1)^2)*b/c
```

**Fricas [A]** time = 1.99544, size = 119, normalized size = 3.22

$$\frac{b \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 2a \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*log(sqrt(-c*x + 1)/sqrt(c
*x + 1)))/c
```

**Sympy [A]** time = 30.8092, size = 61, normalized size = 1.65

$$\left\{ \begin{array}{ll} \frac{a \operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2 \sqrt{-\frac{1}{c^2}}} & \text{for } b = 0 \\ ax & \text{for } c = 0 \\ -\frac{\left(a+b \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}{2bc} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*x,
Eq(c, 0)), (-a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))**2/(2*b*c), True))
```

---

**Giac [B]** time = 1.26335, size = 116, normalized size = 3.14

$$-\frac{b \log(cx+1)^2}{8c} + \frac{b \log(cx-1)^2}{8c} + \frac{1}{4} \left( \frac{b \log(cx+1)}{c} - \frac{b \log(cx-1)}{c} \right) \log(-cx+1) + \frac{a \log(cx+1)}{2c} - \frac{a \log(cx-1)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))/(-c^2\*x^2+1),x, algorithm="giac")

[Out] -1/8\*b\*log(c\*x + 1)^2/c + 1/8\*b\*log(c\*x - 1)^2/c + 1/4\*(b\*log(c\*x + 1)/c - b\*log(c\*x - 1)/c)\*log(-c\*x + 1) + 1/2\*a\*log(c\*x + 1)/c - 1/2\*a\*log(c\*x - 1)/c

$$3.47 \quad \int \frac{1}{(1-c^2x^2)\left(a+b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$

**Optimal.** Leaf size=34

$$-\frac{\log\left(a+b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

[Out] -(Log[a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]]]/(b\*c))

**Rubi [A]** time = 0.0677073, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$ , Rules used = {2512, 2302, 29}

$$-\frac{\log\left(a+b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - c^2\*x^2)\*(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])),x]

[Out] -(Log[a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]]]/(b\*c))

#### Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

#### Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
x]
```

#### Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+b\log(x))} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\ &= -\frac{\text{Subst}\left(\int \frac{1}{x} dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc} \\ &= -\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc} \end{aligned}$$

**Mathematica [A]** time = 0.0360779, size = 34, normalized size = 1.

$$-\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 - c^2\*x^2)\*(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])),x]

[Out] -(Log[a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]]]/(b\*c))

**Maple [F]** time = 0.351, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1} \left( a + b \ln \left( \sqrt{-cx+1} \frac{1}{\sqrt{cx+1}} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-c^2\*x^2+1)/(a+b\*ln((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x)

[Out] int(1/(-c^2\*x^2+1)/(a+b\*ln((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x)

**Maxima [A]** time = 1.48186, size = 49, normalized size = 1.44

$$-\frac{\log\left(-\frac{b\log(cx+1)-b\log(-cx+1)-2a}{2b}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorithm="maxima")
```

```
[Out] -log(-1/2*(b*log(c*x + 1) - b*log(-c*x + 1) - 2*a)/b)/(b*c)
```

**Fricas [A]** time = 2.00456, size = 72, normalized size = 2.12

$$\frac{\log\left(b \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorithm="fricas")
```

```
[Out] -log(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(b*c)
```

**Sympy [A]** time = 164.757, size = 53, normalized size = 1.56

$$\begin{cases} \frac{x}{a} & \text{for } b = 0 \wedge c = 0 \\ \frac{-\frac{\log\left(x-\frac{1}{c}\right)}{2c} + \frac{\log\left(x+\frac{1}{c}\right)}{2c}}{a} & \text{for } b = 0 \\ \frac{x}{a} & \text{for } c = 0 \\ -\frac{\log\left(\frac{a}{b} + \frac{\log(-cx+1)}{2} - \frac{\log(cx+1)}{2}\right)}{bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] Piecewise((x/a, Eq(b, 0) & Eq(c, 0)), ((-log(x - 1/c)/(2*c) + log(x + 1/c)/(2*c))/a, Eq(b, 0)), (x/a, Eq(c, 0)), (-log(a/b + log(-c*x + 1)/2 - log(c*x + 1)/2)/(b*c), True))
```

**Giac [A]** time = 1.16082, size = 42, normalized size = 1.24

$$\frac{\log(-b \log(cx + 1) + b \log(-cx + 1) + 2a)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorithm="giac")
```

```
[Out] -log(-b*log(c*x + 1) + b*log(-c*x + 1) + 2*a)/(b*c)
```

$$3.48 \quad \int \frac{1}{(1-c^2x^2)\left(a+b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2} dx$$

**Optimal.** Leaf size=34

$$\frac{1}{bc\left(a+b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}$$

[Out] 1/(b\*c\*(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]]))

**Rubi [A]** time = 0.0654521, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$ , Rules used = {2512, 2302, 30}

$$\frac{1}{bc\left(a+b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - c^2\*x^2)\*(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2),x]

[Out] 1/(b\*c\*(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]]))

#### Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

#### Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
x]
```

#### Rule 30



Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(1 - c^2 x^2) \left( a + b \log \left( \frac{\sqrt{1 - cx}}{\sqrt{1 + cx}} \right) \right)^2} dx &= - \frac{\text{Subst} \left( \int \frac{1}{x(a + b \log(x))^2} dx, x, \frac{\sqrt{1 - cx}}{\sqrt{1 + cx}} \right)}{c} \\ &= - \frac{\text{Subst} \left( \int \frac{1}{x^2} dx, x, a + b \log \left( \frac{\sqrt{1 - cx}}{\sqrt{1 + cx}} \right) \right)}{bc} \\ &= \frac{1}{bc \left( a + b \log \left( \frac{\sqrt{1 - cx}}{\sqrt{1 + cx}} \right) \right)} \end{aligned}$$

**Mathematica [A]** time = 0.0104415, size = 34, normalized size = 1.

$$\frac{1}{bc \left( a + b \log \left( \frac{\sqrt{1 - cx}}{\sqrt{cx + 1}} \right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 - c^2\*x^2)\*(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2),x]

[Out] 1/(b\*c\*(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]]))

**Maple [F]** time = 0.356, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2 x^2 + 1} \left( a + b \ln \left( \sqrt{-cx + 1} \frac{1}{\sqrt{cx + 1}} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-c^2\*x^2+1)/(a+b\*ln((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x)

[Out] int(1/(-c^2\*x^2+1)/(a+b\*ln((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x)

---

**Maxima [A]** time = 1.58297, size = 46, normalized size = 1.35

$$\frac{2}{b^2c \log(cx + 1) - b^2c \log(-cx + 1) - 2abc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*log((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x, algorithm="maxima")

[Out] -2/(b^2\*c\*log(c\*x + 1) - b^2\*c\*log(-c\*x + 1) - 2\*a\*b\*c)

---

**Fricas [A]** time = 1.99131, size = 72, normalized size = 2.12

$$\frac{1}{b^2c \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + abc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*log((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x, algorithm="fricas")

[Out] 1/(b^2\*c\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a\*b\*c)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c\*\*2\*x\*\*2+1)/(a+b\*ln((-c\*x+1)\*\*(1/2)/(c\*x+1)\*\*(1/2)))\*\*2,x)

[Out] Timed out

---

**Giac [A]** time = 1.25946, size = 46, normalized size = 1.35

$$\frac{2}{b^2c \log(cx + 1) - b^2c \log(-cx + 1) - 2abc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*log((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x, algorithm="giac")

[Out] -2/(b^2\*c\*log(c\*x + 1) - b^2\*c\*log(-c\*x + 1) - 2\*a\*b\*c)

$$3.49 \quad \int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3} dx$$

**Optimal.** Leaf size=37

$$\frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}$$

[Out] 1/(2\*b\*c\*(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2)

**Rubi [A]** time = 0.0651834, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$ , Rules used = {2512, 2302, 30}

$$\frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - c^2\*x^2)\*(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^3),x]

[Out] 1/(2\*b\*c\*(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2)

#### Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

#### Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
x]
```

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(1 - c^2x^2) \left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+b \log(x))^3} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\ &= -\frac{\text{Subst}\left(\int \frac{1}{x^3} dx, x, a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc} \\ &= \frac{1}{2bc \left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2} \end{aligned}$$

**Mathematica [A]** time = 0.0105605, size = 37, normalized size = 1.

$$\frac{1}{2bc \left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 - c^2\*x^2)\*(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^3),x]

[Out] 1/(2\*b\*c\*(a + b\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2)

**Maple [F]** time = 0.349, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1} \left(a + b \ln\left(\sqrt{-cx+1} \frac{1}{\sqrt{cx+1}}\right)\right)^{-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-c^2\*x^2+1)/(a+b\*ln((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3,x)

[Out] int(1/(-c^2\*x^2+1)/(a+b\*ln((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3,x)

---

**Maxima [B]** time = 1.60891, size = 108, normalized size = 2.92

$$\frac{2}{b^3c \log(cx+1)^2 + b^3c \log(-cx+1)^2 - 4ab^2c \log(cx+1) + 4a^2bc - 2(b^3c \log(cx+1) - 2ab^2c) \log(-cx+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*log((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3,x, algorithm="maxima")

[Out] 2/(b^3\*c\*log(c\*x + 1)^2 + b^3\*c\*log(-c\*x + 1)^2 - 4\*a\*b^2\*c\*log(c\*x + 1) + 4\*a^2\*b\*c - 2\*(b^3\*c\*log(c\*x + 1) - 2\*a\*b^2\*c)\*log(-c\*x + 1))

---

**Fricas [A]** time = 1.84696, size = 142, normalized size = 3.84

$$\frac{1}{2 \left( b^3c \log \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^2 + 2ab^2c \log \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a^2bc \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*log((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3,x, algorithm="fricas")

[Out] 1/2/(b^3\*c\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^2 + 2\*a\*b^2\*c\*log(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a^2\*b\*c)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c\*\*2\*x\*\*2+1)/(a+b\*ln((-c\*x+1)\*\*(1/2)/(c\*x+1)\*\*(1/2)))\*\*3,x)

[Out] Timed out

---

**Giac [B]** time = 1.30372, size = 115, normalized size = 3.11

$$2$$

$$b^3c \log(cx+1)^2 - 2b^3c \log(cx+1) \log(-cx+1) + b^3c \log(-cx+1)^2 - 4ab^2c \log(cx+1) + 4ab^2c \log(-cx+1) + 4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*log((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3,x, algorithm="giac")

[Out] 2/(b^3\*c\*log(c\*x + 1)^2 - 2\*b^3\*c\*log(c\*x + 1)\*log(-c\*x + 1) + b^3\*c\*log(-c\*x + 1)^2 - 4\*a\*b^2\*c\*log(c\*x + 1) + 4\*a\*b^2\*c\*log(-c\*x + 1) + 4\*a^2\*b\*c)

$$3.50 \quad \int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

**Optimal.** Leaf size=30

$$-\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

[Out] -Log[Sqrt[1 - a\*x]/Sqrt[1 + a\*x]]^2/(2\*a)

**Rubi [A]** time = 0.0232066, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {2505}

$$-\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Log[Sqrt[1 - a\*x]/Sqrt[1 + a\*x]]/(1 - a^2\*x^2), x]

[Out] -Log[Sqrt[1 - a\*x]/Sqrt[1 + a\*x]]^2/(2\*a)

### Rule 2505

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{h = Simplify[u*(a + b*x)*(c + d*x)]},
Simp[(h*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^(s + 1))/(p*r*(s + 1)*(b*c -
a*d)), x] /; FreeQ[h, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && N
eQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[s, -1]
```

### Rubi steps

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx = -\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$



**Mathematica [A]** time = 0.0085736, size = 30, normalized size = 1.

$$-\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[Log[Sqrt[1 - a\*x]/Sqrt[1 + a\*x]]/(1 - a^2\*x^2), x]

[Out] -Log[Sqrt[1 - a\*x]/Sqrt[1 + a\*x]]^2/(2\*a)

**Maple [F]** time = 0.389, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2 + 1} \ln\left(\sqrt{-ax + 1} \frac{1}{\sqrt{ax + 1}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln((-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1), x)

[Out] int(ln((-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1), x)

**Maxima [B]** time = 1.9242, size = 112, normalized size = 3.73

$$\frac{1}{2} \left( \frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a} \right) \log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right) + \frac{\log(ax-1)^2}{8a} + \frac{\log(ax+1)^2 - 2 \log(ax+1) \log(ax-1)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1), x, algorithm="maxima")

[Out] 1/2\*(log(a\*x + 1)/a - log(a\*x - 1)/a)\*log(sqrt(-a\*x + 1)/sqrt(a\*x + 1)) + 1/8\*log(a\*x - 1)^2/a + 1/8\*(log(a\*x + 1)^2 - 2\*log(a\*x + 1)\*log(a\*x - 1))/a

**Fricas [A]** time = 1.94668, size = 59, normalized size = 1.97

$$\frac{\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1),x, algorithm="fricas")

[Out] -1/2\*log(sqrt(-a\*x + 1)/sqrt(a\*x + 1))^2/a

**Sympy [B]** time = 13.5192, size = 65, normalized size = 2.17

$$\frac{\operatorname{atan}^2\left(\frac{x}{\sqrt{-\frac{1}{a^2}}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)\operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{a^2}}}\right)}{a^2\sqrt{-\frac{1}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln((-a\*x+1)\*\*(1/2)/(a\*x+1)\*\*(1/2))/(-a\*\*2\*x\*\*2+1),x)

[Out] -atan(x/sqrt(-1/a\*\*2))\*2/(2\*a) - log(sqrt(-a\*x + 1)/sqrt(a\*x + 1))\*atan(x/sqrt(-1/a\*\*2))/(a\*\*2\*sqrt(-1/a\*\*2))

**Giac [B]** time = 1.29754, size = 78, normalized size = 2.6

$$\frac{1}{4}\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\log(-ax+1) - \frac{\log(ax+1)^2}{8a} + \frac{\log(ax-1)^2}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1),x, algorithm="giac")

[Out] 1/4\*(log(a\*x + 1)/a - log(a\*x - 1)/a)\*log(-a\*x + 1) - 1/8\*log(a\*x + 1)^2/a + 1/8\*log(a\*x - 1)^2/a

$$3.51 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t \log(i(g+hx)^n)\right)^2}{gk+hkx} dx$$

**Optimal.** Leaf size=410

$$\frac{pr \operatorname{PolyLog}\left(2, \frac{b(g+hx)}{bg-ah}\right)\left(t \log(i(g+hx)^n) + s\right)^2}{hk} + \frac{2np \operatorname{PolyLog}\left(3, \frac{b(g+hx)}{bg-ah}\right)\left(t \log(i(g+hx)^n) + s\right)}{hk} - \frac{2n^2 p r^2 \operatorname{PolyLog}\left(4, \frac{b(g+hx)}{bg-ah}\right)\left(t \log(i(g+hx)^n) + s\right)^2}{hk}$$

[Out]  $-(p*r*\operatorname{Log}[-((h*(a+b*x))/(b*g-a*h))]*(s+t*\operatorname{Log}[i*(g+h*x)^n])^3)/(3*h*k*n*t) - (q*r*\operatorname{Log}[-((h*(c+d*x))/(d*g-c*h))]*(s+t*\operatorname{Log}[i*(g+h*x)^n])^3)/(3*h*k*n*t) + (\operatorname{Log}[e*(f*(a+b*x)^p*(c+d*x)^q]^r]*(s+t*\operatorname{Log}[i*(g+h*x)^n])^3)/(3*h*k*n*t) - (p*r*(s+t*\operatorname{Log}[i*(g+h*x)^n])^2*\operatorname{PolyLog}[2, (b*(g+h*x))/(b*g-a*h)])/ (h*k) - (q*r*(s+t*\operatorname{Log}[i*(g+h*x)^n])^2*\operatorname{PolyLog}[2, (d*(g+h*x))/(d*g-c*h)])/ (h*k) + (2*n*p*r*t*(s+t*\operatorname{Log}[i*(g+h*x)^n])* \operatorname{PolyLog}[3, (b*(g+h*x))/(b*g-a*h)])/ (h*k) + (2*n*q*r*t*(s+t*\operatorname{Log}[i*(g+h*x)^n])* \operatorname{PolyLog}[3, (d*(g+h*x))/(d*g-c*h)])/ (h*k) - (2*n^2*p*r*t^2*\operatorname{PolyLog}[4, (b*(g+h*x))/(b*g-a*h)])/ (h*k) - (2*n^2*q*r*t^2*\operatorname{PolyLog}[4, (d*(g+h*x))/(d*g-c*h)])/ (h*k)$

**Rubi [A]** time = 0.471232, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2499, 2396, 2433, 2374, 2383, 6589}

$$\frac{pr \operatorname{PolyLog}\left(2, \frac{b(g+hx)}{bg-ah}\right)\left(t \log(i(g+hx)^n) + s\right)^2}{hk} + \frac{2np \operatorname{PolyLog}\left(3, \frac{b(g+hx)}{bg-ah}\right)\left(t \log(i(g+hx)^n) + s\right)}{hk} - \frac{2n^2 p r^2 \operatorname{PolyLog}\left(4, \frac{b(g+hx)}{bg-ah}\right)\left(t \log(i(g+hx)^n) + s\right)^2}{hk}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Log}[e*(f*(a+b*x)^p*(c+d*x)^q]^r]*(s+t*\operatorname{Log}[i*(g+h*x)^n])^2)/(g*k+h*k*x), x]$

[Out]  $-(p*r*\operatorname{Log}[-((h*(a+b*x))/(b*g-a*h))]*(s+t*\operatorname{Log}[i*(g+h*x)^n])^3)/(3*h*k*n*t) - (q*r*\operatorname{Log}[-((h*(c+d*x))/(d*g-c*h))]*(s+t*\operatorname{Log}[i*(g+h*x)^n])^3)/(3*h*k*n*t) + (\operatorname{Log}[e*(f*(a+b*x)^p*(c+d*x)^q]^r]*(s+t*\operatorname{Log}[i*(g+h*x)^n])^3)/(3*h*k*n*t) - (p*r*(s+t*\operatorname{Log}[i*(g+h*x)^n])^2*\operatorname{PolyLog}[2, (b*(g+h*x))/(b*g-a*h)])/ (h*k) - (q*r*(s+t*\operatorname{Log}[i*(g+h*x)^n])^2*\operatorname{PolyLog}[2, (d*(g+h*x))/(d*g-c*h)])/ (h*k) + (2*n*p*r*t*(s+t*\operatorname{Log}[i*(g+h*x)^n])* \operatorname{PolyLog}[3, (b*(g+h*x))/(b*g-a*h)])/ (h*k) + (2*n*q*r*t*(s+t*\operatorname{Log}[i*(g+h*x)^n])* \operatorname{PolyLog}[3, (d*(g+h*x))/(d*g-c*h)])/ (h*k) - (2*n^2*p*r*t^2*\operatorname{PolyLog}[4, (b*(g+h*x))/(b*g-a*h)])/ (h*k) - (2*n^2*q*r*t^2*\operatorname{PolyLog}[4, (d*(g+h*x))/(d*g-c*h)])/ (h*k)$

+ h\*x))/(d\*g - c\*h)]/(h\*k)

### Rule 2499

Int[(Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.)^(r\_.)]\*((s\_.) + Log[(i\_.)\*((g\_.) + (h\_.)\*(x\_))^(n\_.)]\*(t\_.))^(m\_.))/((j\_.) + (k\_.)\*(x\_)), x\_Symbol] :> Simp[((s + t\*Log[i\*(g + h\*x)^n])^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)]/(k\*n\*t\*(m + 1)), x] + (-Dist[(b\*p\*r)/(k\*n\*t\*(m + 1)), Int[(s + t\*Log[i\*(g + h\*x)^n])^(m + 1)/(a + b\*x), x], x] - Dist[(d\*q\*r)/(k\*n\*t\*(m + 1)), Int[(s + t\*Log[i\*(g + h\*x)^n])^(m + 1)/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r}, x] && NeQ[b\*c - a\*d, 0] && EqQ[h\*j - g\*k, 0] && IGtQ[m, 0]

### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] :> Dist[1/e, Subst[Int[(k\*x)/d]^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_))^(m\_.)]\*((a\_.) + Log[(c\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] :> -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

### Rule 2383

Int[((a\_.) + Log[(c\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*PolyLog[k\_, (e\_.)\*(x\_))^(q\_.)]/(x\_), x\_Symbol] :> Simp[(PolyLog[k + 1, e\*x^q]\*(a + b\*Log[c\*x^n])^p)/q, x] - Dist[(b\*n\*p)/q, Int[(PolyLog[k + 1, e\*x^q]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^2}{gk+hkx} dx = \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(51(g+hx)^n\right)\right)}{3hknt}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} - \frac{qr\log\left(\dots\right)}{3hknt}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} - \frac{qr\log\left(\dots\right)}{3hknt}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} - \frac{qr\log\left(\dots\right)}{3hknt}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} - \frac{qr\log\left(\dots\right)}{3hknt}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} - \frac{qr\log\left(\dots\right)}{3hknt}$$

**Mathematica [B]** time = 7.54479, size = 22595, normalized size = 55.11

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]\*(s + t\*Log[i\*(g + h\*x)^n])^2)/(g\*k + h\*k\*x), x]

[Out] Result too large to show

---

**Maple [F]** time = 0.982, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\left(s+t\ln\left(i\left(hx+g\right)^n\right)\right)^2\right)}{h k x+g k} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)\*(s+t\*ln(i\*(h\*x+g)^n))^2/(h\*k\*x+g\*k),x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)\*(s+t\*ln(i\*(h\*x+g)^n))^2/(h\*k\*x+g\*k),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)\*(s+t\*log(i\*(h\*x+g)^n))^2/(h\*k\*x+g\*k),x, algorithm="maxima")

[Out] 1/3\*((n^2\*t^2\*log(h\*x + g)^3 + 3\*t^2\*log(h\*x + g)\*log((h\*x + g)^n)^2 - 3\*(n\*t^2\*log(i) + n\*s\*t)\*log(h\*x + g)^2 + 3\*(t^2\*log(i)^2 + 2\*s\*t\*log(i) + s^2)\*log(h\*x + g) - 3\*(n\*t^2\*log(h\*x + g)^2 - 2\*(t^2\*log(i) + s\*t)\*log(h\*x + g))\*log((h\*x + g)^n))\*log(((b\*x + a)^p)^r) + (n^2\*t^2\*log(h\*x + g)^3 + 3\*t^2\*log(h\*x + g)\*log((h\*x + g)^n)^2 - 3\*(n\*t^2\*log(i) + n\*s\*t)\*log(h\*x + g)^2 + 3\*(t^2\*log(i)^2 + 2\*s\*t\*log(i) + s^2)\*log(h\*x + g) - 3\*(n\*t^2\*log(h\*x + g)^2 - 2\*(t^2\*log(i) + s\*t)\*log(h\*x + g))\*log((h\*x + g)^n))\*log(((d\*x + c)^q)^r))/(h\*k) - integrate(-1/3\*(3\*((t^2\*log(i)^2 + 2\*s\*t\*log(i) + s^2)\*h\*log(e) + (t^2\*log(i)^2 + 2\*s\*t\*log(i) + s^2)\*h\*log(f^r))\*b\*d\*x^2 - ((p\*r + q\*r)\*b\*d\*h\*n^2\*t^2\*x^2 + b\*c\*g\*n^2\*p\*r\*t^2 + a\*d\*g\*n^2\*q\*r\*t^2 + (a\*d\*h\*n^2\*q\*r\*t^2 + (c\*h\*n^2\*p\*r\*t^2 + (p\*r + q\*r)\*d\*g\*n^2\*t^2)\*b)\*x)\*log(h\*x + g)^3 + 3\*((t^2\*log(i)^2 + 2\*s\*t\*log(i) + s^2)\*h\*log(e) + (t^2\*log(i)^2 + 2\*s\*t\*log(i) + s^2)\*h\*log(f^r))\*a\*c + 3\*((p\*r + q\*r)\*n\*t^2\*log(i) + (p\*r\*s + q\*r\*s)\*n\*t)\*b\*d\*h\*x^2 + (n\*p\*r\*t^2\*log(i) + n\*p\*r\*s\*t)\*b\*c\*g + (n\*q\*r\*t^2\*log(i) + n\*q\*r\*s\*t)\*a\*d\*g + ((n\*q\*r\*t^2\*log(i) + n\*q\*r\*s\*t)\*a\*d\*h + ((p\*r + q\*r)\*n\*t^2\*log(i) + (p\*r\*s + q\*r\*s)\*n\*t)\*d\*g + (n\*p\*r\*t^2\*log(i) + n\*p\*r\*s\*t)\*c\*h)\*b)\*x)\*log(h\*x + g)^2 + 3\*((h\*t^2\*log(e) + h\*t^2\*log(f^r))\*b\*d\*x^2 + (h\*t^2\*log(e) + h\*t^2\*log(f^r))\*a\*c + ((h\*t^2\*log(e) + h\*t^2\*log(f^r))\*b\*c + (h\*t^2\*log(e) + h\*t^2\*log(f^r))\*a\*d)\*x - ((p\*r + q\*r)\*b\*d\*h\*t^2\*x^2 + b\*c\*g\*p\*r

```

*t^2 + a*d*g*q*r*t^2 + (a*d*h*q*r*t^2 + (c*h*p*r*t^2 + (p*r + q*r)*d*g*t^2)
*b)*x)*log(h*x + g))*log((h*x + g)^n)^2 + 3*(((t^2*log(i)^2 + 2*s*t*log(i)
+ s^2)*h*log(e) + (t^2*log(i)^2 + 2*s*t*log(i) + s^2)*h*log(f^r))*b*c + ((t
^2*log(i)^2 + 2*s*t*log(i) + s^2)*h*log(e) + (t^2*log(i)^2 + 2*s*t*log(i) +
s^2)*h*log(f^r))*a*d)*x - 3*(((p*r + q*r)*t^2*log(i)^2 + p*r*s^2 + q*r*s^2
+ 2*(p*r*s + q*r*s)*t*log(i))*b*d*h*x^2 + (p*r*t^2*log(i)^2 + 2*p*r*s*t*log
(i) + p*r*s^2)*b*c*g + (q*r*t^2*log(i)^2 + 2*q*r*s*t*log(i) + q*r*s^2)*a*d
*g + ((q*r*t^2*log(i)^2 + 2*q*r*s*t*log(i) + q*r*s^2)*a*d*h + ((p*r + q*r)
*t^2*log(i)^2 + p*r*s^2 + q*r*s^2 + 2*(p*r*s + q*r*s)*t*log(i))*d*g + (p*r*
t^2*log(i)^2 + 2*p*r*s*t*log(i) + p*r*s^2)*c*h)*b)*x)*log(h*x + g) + 3*(2*(
(t^2*log(i) + s*t)*h*log(e) + (t^2*log(i) + s*t)*h*log(f^r))*b*d*x^2 + 2*((
t^2*log(i) + s*t)*h*log(e) + (t^2*log(i) + s*t)*h*log(f^r))*a*c + ((p*r + q
*r)*b*d*h*n*t^2*x^2 + b*c*g*n*p*r*t^2 + a*d*g*n*q*r*t^2 + (a*d*h*n*q*r*t^2
+ (c*h*n*p*r*t^2 + (p*r + q*r)*d*g*n*t^2)*b)*x)*log(h*x + g)^2 + 2*(((t^2*log(i)
+ s*t)*h*log(e) + (t^2*log(i) + s*t)*h*log(f^r))*b*c + ((t^2*log(i) +
s*t)*h*log(e) + (t^2*log(i) + s*t)*h*log(f^r))*a*d)*x - 2*(((p*r + q*r)*t^
2*log(i) + (p*r*s + q*r*s)*t)*b*d*h*x^2 + (p*r*t^2*log(i) + p*r*s*t)*b*c*g
+ (q*r*t^2*log(i) + q*r*s*t)*a*d*g + ((q*r*t^2*log(i) + q*r*s*t)*a*d*h + ((
(p*r + q*r)*t^2*log(i) + (p*r*s + q*r*s)*t)*d*g + (p*r*t^2*log(i) + p*r*s*t
)*c*h)*b)*x)*log(h*x + g))*log((h*x + g)^n))/(b*d*h^2*k*x^3 + a*c*g*h*k + (
a*d*h^2*k + (d*g*h*k + c*h^2*k)*b)*x^2 + (b*c*g*h*k + (d*g*h*k + c*h^2*k)*a
)*x), x)

```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\left( t^2 \log \left( (hx + g)^n i \right)^2 + 2st \log \left( (hx + g)^n i \right) + s^2 \right) \log \left( ((bx + a)^p (dx + c)^q f)^r e \right)}{h k x + g k}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x+
g*k),x, algorithm="fricas")

```

```

[Out] integral((t^2*log((h*x + g)^n*i)^2 + 2*s*t*log((h*x + g)^n*i) + s^2)*log(((
b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)*(s+t*ln(i*(h*x+g)**n))**2/(h*k*x+g*k),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(t \log\left(\left(hx + g\right)^n i\right) + s\right)^2 \log\left(\left((bx + a)^p (dx + c)^q f\right)^r e\right)}{h k x + g k} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x+g*k),x, algorithm="giac")
```

```
[Out] integrate((t*log((h*x + g)^n*i) + s)^2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```



$$3.52 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t \log(i(g+hx)^n)\right)}{gk+hkx} dx$$

**Optimal.** Leaf size=306

$$\frac{prPolyLog\left(2, \frac{b(g+hx)}{bg-ah}\right)\left(t \log(i(g+hx)^n) + s\right)}{hk} + \frac{np\text{r}tPolyLog\left(3, \frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{qrPolyLog\left(2, \frac{d(g+hx)}{dg-ch}\right)\left(t \log(i(g+hx)^n)\right)}{hk}$$

[Out]  $-(p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*(s + t*Log[i*(g + h*x)^n])^2)/(2*h*k*n*t) - (q*r*Log[-((h*(c + d*x))/(d*g - c*h))]*(s + t*Log[i*(g + h*x)^n])^2)/(2*h*k*n*t) + (Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]*(s + t*Log[i*(g + h*x)^n])^2)/(2*h*k*n*t) - (p*r*(s + t*Log[i*(g + h*x)^n])*PolyLog[2, (b*(g + h*x))/(b*g - a*h]])/(h*k) - (q*r*(s + t*Log[i*(g + h*x)^n])*PolyLog[2, (d*(g + h*x))/(d*g - c*h]])/(h*k) + (n*p*r*t*PolyLog[3, (b*(g + h*x))/(b*g - a*h]])/(h*k) + (n*q*r*t*PolyLog[3, (d*(g + h*x))/(d*g - c*h]])/(h*k)$

**Rubi [A]** time = 0.287542, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$ , Rules used = {2499, 2396, 2433, 2374, 6589}

$$\frac{prPolyLog\left(2, \frac{b(g+hx)}{bg-ah}\right)\left(t \log(i(g+hx)^n) + s\right)}{hk} + \frac{np\text{r}tPolyLog\left(3, \frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{qrPolyLog\left(2, \frac{d(g+hx)}{dg-ch}\right)\left(t \log(i(g+hx)^n)\right)}{hk}$$

Antiderivative was successfully verified.

[In] Int[(Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)\*(s + t\*Log[i\*(g + h\*x)^n]))/(g\*k + h\*k\*x), x]

[Out]  $-(p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*(s + t*Log[i*(g + h*x)^n])^2)/(2*h*k*n*t) - (q*r*Log[-((h*(c + d*x))/(d*g - c*h))]*(s + t*Log[i*(g + h*x)^n])^2)/(2*h*k*n*t) + (Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]*(s + t*Log[i*(g + h*x)^n])^2)/(2*h*k*n*t) - (p*r*(s + t*Log[i*(g + h*x)^n])*PolyLog[2, (b*(g + h*x))/(b*g - a*h]])/(h*k) - (q*r*(s + t*Log[i*(g + h*x)^n])*PolyLog[2, (d*(g + h*x))/(d*g - c*h]])/(h*k) + (n*p*r*t*PolyLog[3, (b*(g + h*x))/(b*g - a*h]])/(h*k) + (n*q*r*t*PolyLog[3, (d*(g + h*x))/(d*g - c*h]])/(h*k)$

**Rule 2499**

Int[(Log[(e.\_)\*((f.\_)\*((a.\_) + (b.\_)\*(x.\_))^(p.\_))\*((c.\_) + (d.\_)\*(x.\_))^(q.\_)]^(r.\_)]\*((s.\_) + Log[(i.\_)\*((g.\_) + (h.\_)\*(x.\_))^(n.\_)]\*(t.\_))^(m.\_))/((j.\_

```
) + (k_.)*(x_), x_Symbol] := Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)/((f_.) + (g_.
)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

### Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] := Dist[1/e, Subst[Int[(k*x)/d]^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

### Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_))^(m_.))]*((a_.) + Log[(c_.)*(x_))^(n_.)]*(b
_.))^(p_.)/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(52(g+hx)^n\right)\right)}{gk+hkx} dx &= \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt} \\
&= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt} - \frac{qr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt} \\
&= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt} - \frac{qr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt} \\
&= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt} - \frac{qr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt}
\end{aligned}$$

**Mathematica [A]** time = 2.55186, size = 436, normalized size = 1.42

$$\frac{-2pr\text{PolyLog}\left(2, \frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)+2nprt\text{PolyLog}\left(3, \frac{b(g+hx)}{bg-ah}\right)-2qr\text{PolyLog}\left(2, \frac{d(g+hx)}{dg-ch}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)}{gk+hkx}$$

Antiderivative was successfully verified.

[In] Integrate[(Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]\*(s + t\*Log[i\*(g + h\*x)^n])]/(g\*k + h\*k\*x), x]

[Out] (-2\*p\*r\*s\*Log[(h\*(a + b\*x))/(-(b\*g) + a\*h)]\*Log[g + h\*x] - 2\*q\*r\*s\*Log[(h\*(c + d\*x))/(-(d\*g) + c\*h)]\*Log[g + h\*x] + 2\*s\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]\*Log[g + h\*x] + n\*p\*r\*t\*Log[(h\*(a + b\*x))/(-(b\*g) + a\*h)]\*Log[g + h\*x]^2 + n\*q\*r\*t\*Log[(h\*(c + d\*x))/(-(d\*g) + c\*h)]\*Log[g + h\*x]^2 - n\*t\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]\*Log[g + h\*x]^2 - 2\*p\*r\*t\*Log[(h\*(a + b\*x))/(-(b\*g) + a\*h)]\*Log[g + h\*x]\*Log[i\*(g + h\*x)^n] - 2\*q\*r\*t\*Log[(h\*(c + d\*x))/(-(d\*g) + c\*h)]\*Log[g + h\*x]\*Log[i\*(g + h\*x)^n] + 2\*t\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]\*Log[g + h\*x]\*Log[i\*(g + h\*x)^n] - 2\*p\*r\*(s + t\*Log[i\*(g + h\*x)^n])\*PolyLog[2, (b\*(g + h\*x))/(b\*g - a\*h)] - 2\*q\*r\*(s + t\*Log[i\*(g + h\*x)^n])\*PolyLog[2, (d\*(g + h\*x))/(d\*g - c\*h)] + 2\*n\*p\*r\*t\*PolyLog[3, (b\*(g + h\*x))/(b\*g - a\*h)] + 2\*n\*q\*r\*t\*PolyLog[3, (d\*(g + h\*x))/(d\*g - c\*h)]/(2\*h\*k)

k)

---

**Maple [F]** time = 0.75, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)\left(s+t\ln\left(i(hx+g)^n\right)\right)}{hcx+gk} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)\*(s+t\*ln(i\*(h\*x+g)^n))/(h\*k\*x+g\*k),x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)\*(s+t\*ln(i\*(h\*x+g)^n))/(h\*k\*x+g\*k),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)\*(s+t\*log(i\*(h\*x+g)^n))/(h\*k\*x+g\*k),x, algorithm="maxima")

[Out] -1/2\*((n\*t\*log(h\*x + g)^2 - 2\*t\*log(h\*x + g)\*log((h\*x + g)^n) - 2\*(t\*log(i) + s)\*log(h\*x + g))\*log(((b\*x + a)^p)^r) + (n\*t\*log(h\*x + g)^2 - 2\*t\*log(h\*x + g)\*log((h\*x + g)^n) - 2\*(t\*log(i) + s)\*log(h\*x + g))\*log(((d\*x + c)^q)^r)/(h\*k) - integrate(-1/2\*(2\*((t\*log(i) + s)\*h\*log(e) + (t\*log(i) + s)\*h\*log(f^r))\*b\*d\*x^2 + 2\*((t\*log(i) + s)\*h\*log(e) + (t\*log(i) + s)\*h\*log(f^r))\*a\*c + ((p\*r + q\*r)\*b\*d\*h\*n\*t\*x^2 + b\*c\*g\*n\*p\*r\*t + a\*d\*g\*n\*q\*r\*t + (a\*d\*h\*n\*q\*r\*t + (c\*h\*n\*p\*r\*t + (p\*r + q\*r)\*d\*g\*n\*t)\*b)\*x)\*log(h\*x + g)^2 + 2\*((t\*log(i) + s)\*h\*log(e) + (t\*log(i) + s)\*h\*log(f^r))\*b\*c + ((t\*log(i) + s)\*h\*log(e) + (t\*log(i) + s)\*h\*log(f^r))\*a\*d)\*x - 2\*((p\*r\*s + q\*r\*s + (p\*r + q\*r)\*t\*log(i))\*b\*d\*h\*x^2 + (p\*r\*t\*log(i) + p\*r\*s)\*b\*c\*g + (q\*r\*t\*log(i) + q\*r\*s)\*a\*d\*g + ((q\*r\*t\*log(i) + q\*r\*s)\*a\*d\*h + ((p\*r\*s + q\*r\*s + (p\*r + q\*r)\*t\*log(i))\*d\*g + (p\*r\*t\*log(i) + p\*r\*s)\*c\*h)\*b)\*x)\*log(h\*x + g) + 2\*((h\*t\*log(e) + h\*t\*log(f^r))\*b\*d\*x^2 + (h\*t\*log(e) + h\*t\*log(f^r))\*a\*c + ((h\*t\*log(e) + h\*t\*log(f^r))\*b\*c + (h\*t\*log(e) + h\*t\*log(f^r))\*a\*d)\*x - ((p\*r + q\*r)\*b\*d\*h\*t\*x^2 + b\*c\*g\*p\*r\*t + a\*d\*g\*q\*r\*t + (a\*d\*h\*q\*r\*t + (c\*h\*p\*r\*t + (p\*r + q\*r)\*d\*g\*t)\*b)\*x)\*log(h\*x + g)\*log((h\*x + g)^n))/(b\*d\*h^2\*k\*x^3 + a\*c\*g\*h\*k + (a\*d\*h^2\*k + (d\*g\*h\*k + c\*h^2\*k)\*b)\*x^2 + (b\*c\*g\*h\*k + (d\*g\*h\*k + c\*h^2\*k

$k) * a) * x), x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\left( t \log \left( (hx + g)^n i \right) + s \right) \log \left( (bx + a)^p (dx + c)^q f^r e \right)}{h k x + g k}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)\*(s+t\*log(i\*(h\*x+g)^n))/(h\*k\*x+g\*k),x, algorithm="fricas")

[Out] integral((t\*log((h\*x + g)^n\*i) + s)\*log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)/(h\*k\*x + g\*k), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r)\*(s+t\*ln(i\*(h\*x+g)\*\*n))/(h\*k\*x+g\*k),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( t \log \left( (hx + g)^n i \right) + s \right) \log \left( (bx + a)^p (dx + c)^q f^r e \right)}{h k x + g k} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)\*(s+t\*log(i\*(h\*x+g)^n))/(h\*k\*x+g\*k),x, algorithm="giac")

```
[Out] integrate((t*log((h*x + g)^n*i) + s)*log((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

$$3.53 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{gk+hkx} dx$$

**Optimal.** Leaf size=172

$$\frac{pr \operatorname{PolyLog}\left(2, \frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{qr \operatorname{PolyLog}\left(2, \frac{d(g+hx)}{dg-ch}\right)}{hk} + \frac{\log(gk+hkx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{hk} - \frac{pr \log(gk+hkx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{hk}$$

[Out]  $-\left(\left(p*r*\operatorname{Log}\left[-\left(\frac{h*(a+b*x)}{b*g-a*h}\right)\right]*\operatorname{Log}[g*k+h*k*x]\right)/\left(h*k\right)\right) - \left(q*r*\operatorname{Log}\left[-\left(\frac{h*(c+d*x)}{d*g-c*h}\right)\right]*\operatorname{Log}[g*k+h*k*x]\right)/\left(h*k\right) + \left(\operatorname{Log}\left[e*\left(f*(a+b*x)\right)^p*(c+d*x)^q\right]^r*\operatorname{Log}[g*k+h*k*x]\right)/\left(h*k\right) - \left(p*r*\operatorname{PolyLog}\left[2,\left(\frac{b*(g+h*x)}{b*g-a*h}\right)\right]/\left(h*k\right) - \left(q*r*\operatorname{PolyLog}\left[2,\left(\frac{d*(g+h*x)}{d*g-c*h}\right)\right]/\left(h*k\right)\right)$

**Rubi [A]** time = 0.135941, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2494, 2394, 2393, 2391}

$$\frac{pr \operatorname{PolyLog}\left(2, \frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{qr \operatorname{PolyLog}\left(2, \frac{d(g+hx)}{dg-ch}\right)}{hk} + \frac{\log(gk+hkx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{hk} - \frac{pr \log(gk+hkx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{hk}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\operatorname{Log}\left[e*\left(f*(a+b*x)\right)^p*(c+d*x)^q\right]^r/\left(g*k+h*k*x\right), x\right]$

[Out]  $-\left(\left(p*r*\operatorname{Log}\left[-\left(\frac{h*(a+b*x)}{b*g-a*h}\right)\right]*\operatorname{Log}[g*k+h*k*x]\right)/\left(h*k\right)\right) - \left(q*r*\operatorname{Log}\left[-\left(\frac{h*(c+d*x)}{d*g-c*h}\right)\right]*\operatorname{Log}[g*k+h*k*x]\right)/\left(h*k\right) + \left(\operatorname{Log}\left[e*\left(f*(a+b*x)\right)^p*(c+d*x)^q\right]^r*\operatorname{Log}[g*k+h*k*x]\right)/\left(h*k\right) - \left(p*r*\operatorname{PolyLog}\left[2,\left(\frac{b*(g+h*x)}{b*g-a*h}\right)\right]/\left(h*k\right) - \left(q*r*\operatorname{PolyLog}\left[2,\left(\frac{d*(g+h*x)}{d*g-c*h}\right)\right]/\left(h*k\right)\right)$

#### Rule 2494

$\operatorname{Int}\left[\operatorname{Log}\left[\left(e_{.}\right)*\left(\left(f_{.}\right)*\left(\left(a_{.}\right) + \left(b_{.}\right)*\left(x_{.}\right)\right)^{\left(p_{.}\right)}*\left(\left(c_{.}\right) + \left(d_{.}\right)*\left(x_{.}\right)\right)^{\left(q_{.}\right)}\right]^{\left(r_{.}\right)}\right]/\left(\left(g_{.}\right) + \left(h_{.}\right)*\left(x_{.}\right)\right), x_{\text{Symbol}}] := \operatorname{Simp}\left[\left(\operatorname{Log}[g+h*x]*\operatorname{Log}\left[e*\left(f*(a+b*x)\right)^p*(c+d*x)^q\right]^r\right)/h, x\right] + \left(-\operatorname{Dist}\left[\left(b*p*r\right)/h, \operatorname{Int}\left[\operatorname{Log}[g+h*x]/\left(a+b*x\right), x\right], x\right) - \operatorname{Dist}\left[\left(d*q*r\right)/h, \operatorname{Int}\left[\operatorname{Log}[g+h*x]/\left(c+d*x\right), x\right], x\right)\right] /; \operatorname{FreeQ}\left[\{a, b, c, d, e, f, g, h, p, q, r\}, x\right] \&\& \operatorname{NeQ}\left[b*c - a*d, 0\right]$

#### Rule 2394

$\operatorname{Int}\left[\left(\left(a_{.}\right) + \operatorname{Log}\left[\left(c_{.}\right)*\left(\left(d_{.}\right) + \left(e_{.}\right)*\left(x_{.}\right)\right]^{\left(n_{.}\right)}*\left(b_{.}\right)\right)\right]/\left(\left(f_{.}\right) + \left(g_{.}\right)*\left(x_{.}\right)\right), x_{\text{Symbol}}] := \operatorname{Simp}\left[\left(\operatorname{Log}\left[\left(e*(f+g*x)\right)/\left(e*f-d*g\right)\right]*\left(a+b*\operatorname{Log}\left[c*(d+e*x)\right]\right)\right)$

)^n))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned} \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{gk+hx} dx &= \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \log(gk+hx)}{hk} - \frac{(bpr) \int \frac{\log(gk+hx)}{a+bx} dx}{hk} - \frac{(dqr) \int \frac{\log(gk+hx)}{c+dx} dx}{hk} \\ &= -\frac{pr \log\left(-\frac{h(a+bx)}{bg-ah}\right) \log(gk+hx)}{hk} - \frac{qr \log\left(-\frac{h(c+dx)}{dg-ch}\right) \log(gk+hx)}{hk} + \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \log(gk+hx)}{hk} \\ &= -\frac{pr \log\left(-\frac{h(a+bx)}{bg-ah}\right) \log(gk+hx)}{hk} - \frac{qr \log\left(-\frac{h(c+dx)}{dg-ch}\right) \log(gk+hx)}{hk} + \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \log(gk+hx)}{hk} \\ &= -\frac{pr \log\left(-\frac{h(a+bx)}{bg-ah}\right) \log(gk+hx)}{hk} - \frac{qr \log\left(-\frac{h(c+dx)}{dg-ch}\right) \log(gk+hx)}{hk} + \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \log(gk+hx)}{hk} \end{aligned}$$

**Mathematica [A]** time = 0.0887067, size = 166, normalized size = 0.97

$$\frac{pr \text{PolyLog}\left(2, \frac{h(a+bx)}{ah-bg}\right) + qr \text{PolyLog}\left(2, \frac{h(c+dx)}{ch-dg}\right) + \log(g+hx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - pr \log(a+bx) \log(g+hx)}{hk}$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(g\*k + h\*k\*x), x]

[Out] (-(p\*r\*Log[a + b\*x]\*Log[g + h\*x]) - q\*r\*Log[c + d\*x]\*Log[g + h\*x] + Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]\*Log[g + h\*x] + p\*r\*Log[a + b\*x]\*Log[(b\*(g + h



$*x)/(b*g - a*h)] + q*r*\text{Log}[c + d*x]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] + p*r*\text{PolyLog}[2, (h*(a + b*x))/(-b*g) + a*h)] + q*r*\text{PolyLog}[2, (h*(c + d*x))/(-d*g) + c*h)]/(h*k)$

**Maple [F]** time = 0.491, size = 0, normalized size = 0.

$$\int \frac{\ln \left( e \left( f (bx + a)^p (dx + c)^q \right)^r \right)}{h k x + g k} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k), x)`

[Out] `int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k), x)`

**Maxima [A]** time = 1.26247, size = 275, normalized size = 1.6

$$\frac{\left( \frac{\left( \log(bx+a) \log\left(\frac{b h x+a h}{b g-a h}+1\right)+\text{Li}_2\left(-\frac{b h x+a h}{b g-a h}\right)\right) f p}{h k} + \frac{\left( \log(dx+c) \log\left(\frac{d h x+c h}{d g-c h}+1\right)+\text{Li}_2\left(-\frac{d h x+c h}{d g-c h}\right)\right) f q}{h k} \right) r}{f} - \frac{(f p \log(bx+a) + f q \log(dx+c)) r \log(f h k)}{f h k}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k), x, algorithm="maxima")`

[Out] `((log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))*f*p/(h*k) + (log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilog(-(d*h*x + c*h)/(d*g - c*h)))*f*q/(h*k))*r/f - (f*p*log(b*x + a) + f*q*log(d*x + c))*r*log(h*k*x + g*k)/(f*h*k) + log(h*k*x + g*k)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)}{h k x + g k}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx + a)^p(dx + c)^q f\right)^r e\right)}{h k x + g k} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

$$3.54 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)(s+t\log(i(g+hx)^n))} dx$$

**Optimal.** Leaf size=50

$$\text{Unintegrable}\left(\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)(t\log(i(g+hx)^n)+s)}, x\right)$$

[Out] Unintegrable[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/((g\*k + h\*k\*x)\*(s + t\*Log[i\*(g + h\*x)^n])), x]

**Rubi [A]** time = 0.0532358, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)(s+t\log(i(g+hx)^n))} dx$$

Verification is Not applicable to the result.

[In] Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/((g\*k + h\*k\*x)\*(s + t\*Log[i\*(g + h\*x)^n])), x]

[Out] Defer[Int][Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/((g\*k + h\*k\*x)\*(s + t\*Log[i\*(g + h\*x)^n])), x]

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)(s+t\log(54(g+hx)^n))} dx = \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)(s+t\log(54(g+hx)^n))} dx$$

**Mathematica [A]** time = 0.362222, size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)(s+t\log(i(g+hx)^n))} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*(g + h*x)^n])), x]
```

```
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*(g + h*x)^n])), x]
```

**Maple [A]** time = 0.723, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{(h k x + g k)\left(s + t \ln\left(i(hx+g)^n\right)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n)), x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n)), x)
```

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{(h k x + g k)\left(t \log\left(\frac{(h x + g)^n}{i}\right) + s\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n)), x, algorithm="maxima")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x + g)^n*i) + s)), x)
```

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)}{hksx + gks + (hktx + gkt) \log \left( (hx + g)^n i \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n)),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*s*x + g*k*s + (h*k*t*x + g*k*t)*log((h*x + g)^n*i)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)*n)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)}{(h k x + g k) \left( t \log \left( (h x + g)^n i \right) + s \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n)),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x + g)^n*i) + s)), x)
```

$$3.55 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)(s+t\log(i(g+hx)^n))^2} dx$$

**Optimal.** Leaf size=50

$$\text{Unintegrable}\left(\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)(t\log(i(g+hx)^n)+s)^2}, x\right)$$

[Out] Unintegrable[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/((g\*k + h\*k\*x)\*(s + t\*Log[i\*(g + h\*x)^n]))^2, x]

**Rubi [A]** time = 0.0525536, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)(s+t\log(i(g+hx)^n))^2} dx$$

Verification is Not applicable to the result.

[In] Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/((g\*k + h\*k\*x)\*(s + t\*Log[i\*(g + h\*x)^n]))^2, x]

[Out] Defer[Int][Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/((g\*k + h\*k\*x)\*(s + t\*Log[i\*(g + h\*x)^n]))^2, x]

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)(s+t\log(55(g+hx)^n))^2} dx = \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)(s+t\log(55(g+hx)^n))^2} dx$$

**Mathematica [A]** time = 2.74292, size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)(s+t\log(i(g+hx)^n))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/((g\*k + h\*k\*x)\*(s + t\*Log[i\*(g + h\*x)^n])^2), x]

[Out] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/((g\*k + h\*k\*x)\*(s + t\*Log[i\*(g + h\*x)^n])^2), x]

**Maple [A]** time = 0.739, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{(h k x + g k)\left(s + t \ln\left(i(hx+g)^n\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(h\*k\*x+g\*k)/(s+t\*ln(i\*(h\*x+g)^n))^2,x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(h\*k\*x+g\*k)/(s+t\*ln(i\*(h\*x+g)^n))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{\log\left(\left((bx+a)^p\right)^r\right) + \log\left(\left((dx+c)^q\right)^r\right) + \log(e) + \log\left(f^r\right)}{h k n t^2 \log\left((hx+g)^n\right) + (k n t^2 \log(i) + k n s t) h} + \int \frac{\log\left(\left((bx+a)^p\right)^r\right) + \log\left(\left((dx+c)^q\right)^r\right) + \log(e) + \log\left(f^r\right)}{(k n t^2 \log(i) + k n s t) b d h x^2 + (k n t^2 \log(i) + k n s t) a c h x + (k n t^2 \log(i) + k n s t) a d h} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/(h\*k\*x+g\*k)/(s+t\*log(i\*(h\*x+g)^n))^2,x, algorithm="maxima")

[Out] -(log(((b\*x + a)^p)^r) + log(((d\*x + c)^q)^r) + log(e) + log(f^r))/(h\*k\*n\*t^2\*log((h\*x + g)^n) + (k\*n\*t^2\*log(i) + k\*n\*s\*t)\*h) + integrate((b\*c\*p\*r + a\*d\*q\*r + (p\*r + q\*r)\*b\*d\*x)/((k\*n\*t^2\*log(i) + k\*n\*s\*t)\*b\*d\*h\*x^2 + (k\*n\*t^2\*log(i) + k\*n\*s\*t)\*a\*c\*h + ((k\*n\*t^2\*log(i) + k\*n\*s\*t)\*b\*c\*h + (k\*n\*t^2\*log(i) + k\*n\*s\*t)\*a\*d\*h)\*x + (b\*d\*h\*k\*n\*t^2\*x^2 + a\*c\*h\*k\*n\*t^2 + (b\*c\*h\*k\*n\*t^2 + a\*d\*h\*k\*n\*t^2)\*x)\*log((h\*x + g)^n)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)}{hks^2x + gks^2 + (hkt^2x + gkt^2) \log \left( (hx + g)^n i \right)^2 + 2(hkstx + gkst) \log \left( (hx + g)^n i \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*s^2*x + g*k*s^2 + (h*k*t^2*x + g*k*t^2)*log((h*x + g)^n*i)^2 + 2*(h*k*s*t*x + g*k*s*t)*log((h*x + g)^n*i)), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)*n))**2,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)}{(hcx + gk) \left( t \log \left( (hx + g)^n i \right) + s \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n))^2,x, algorithm="giac")
```



```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x + g)^n*i) + s)^2), x)
```

$$3.56 \quad \int \frac{\log^3\left(i(j(hx)^t)^u\right) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} dx$$

**Optimal.** Leaf size=328

$$-6prt^2u^2\text{PolyLog}\left(4, -\frac{bx}{a}\right) \log\left(i(j(hx)^t)^u\right) - pr\text{PolyLog}\left(2, -\frac{bx}{a}\right) \log^3\left(i(j(hx)^t)^u\right) + 3prt\text{PolyLog}\left(3, -\frac{bx}{a}\right) \log^2\left(i(j(hx)^t)^u\right)$$

```
[Out] -(p*r*Log[i*(j*(h*x)^t)^u]^4*Log[1 + (b*x)/a])/(4*t*u) + (Log[i*(j*(h*x)^t)^u]^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(4*t*u) - (q*r*Log[i*(j*(h*x)^t)^u]^4*Log[1 + (d*x)/c])/(4*t*u) - p*r*Log[i*(j*(h*x)^t)^u]^3*PolyLog[2, -(b*x)/a] - q*r*Log[i*(j*(h*x)^t)^u]^3*PolyLog[2, -(d*x)/c] + 3*p*r*t*u*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -(b*x)/a] + 3*q*r*t*u*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -(d*x)/c] - 6*p*r*t^2*u^2*Log[i*(j*(h*x)^t)^u]*PolyLog[4, -(b*x)/a] - 6*q*r*t^2*u^2*Log[i*(j*(h*x)^t)^u]*PolyLog[4, -(d*x)/c] + 6*p*r*t^3*u^3*PolyLog[5, -(b*x)/a] + 6*q*r*t^3*u^3*PolyLog[5, -(d*x)/c]
```

**Rubi [A]** time = 1.25157, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2499, 2317, 2374, 2383, 6589, 2445}

$$-6prt^2u^2\text{PolyLog}\left(4, -\frac{bx}{a}\right) \log\left(i(j(hx)^t)^u\right) - pr\text{PolyLog}\left(2, -\frac{bx}{a}\right) \log^3\left(i(j(hx)^t)^u\right) + 3prt\text{PolyLog}\left(3, -\frac{bx}{a}\right) \log^2\left(i(j(hx)^t)^u\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/x,x]
```

```
[Out] -(p*r*Log[i*(j*(h*x)^t)^u]^4*Log[1 + (b*x)/a])/(4*t*u) + (Log[i*(j*(h*x)^t)^u]^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(4*t*u) - (q*r*Log[i*(j*(h*x)^t)^u]^4*Log[1 + (d*x)/c])/(4*t*u) - p*r*Log[i*(j*(h*x)^t)^u]^3*PolyLog[2, -(b*x)/a] - q*r*Log[i*(j*(h*x)^t)^u]^3*PolyLog[2, -(d*x)/c] + 3*p*r*t*u*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -(b*x)/a] + 3*q*r*t*u*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -(d*x)/c] - 6*p*r*t^2*u^2*Log[i*(j*(h*x)^t)^u]*PolyLog[4, -(b*x)/a] - 6*q*r*t^2*u^2*Log[i*(j*(h*x)^t)^u]*PolyLog[4, -(d*x)/c] + 6*p*r*t^3*u^3*PolyLog[5, -(b*x)/a] + 6*q*r*t^3*u^3*PolyLog[5, -(d*x)/c]
```

**Rule 2499**

```
Int[(Log[(e._)*((f._)*((a._) + (b._)*(x._))^(p._))*((c._) + (d._)*(x._))^(q._))]^(r._)]*((s._) + Log[(i._)*((g._) + (h._)*(x._))^(n._)]*(t._))^(m._))/((j._)
```

```
) + (k_.)*(x_), x_Symbol] := Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

### Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_)), x_Symb
ol] := Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

### Rule 2374

```
Int[(Log[(d_.)*((e_.) + (f_.)*(x_)^(m_.))]*(a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.)/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

### Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.))]/(x_), x_Symbol] := Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_.)]*(b_.))^(p_.
)*(u_.), x_Symbol] := Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)]]^p, x],
c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)]]^p, x]]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\log^3 \left( 56 (j(hx)^t)^u \right) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{x} dx &= \text{Subst} \left( \int \frac{\log^3 \left( 56 j^u (hx)^{tu} \right) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{x} dx, 5 \right. \\
&= \text{Subst} \left( \text{Subst} \left( \int \frac{\log^3 \left( 56 h^{tu} j^u x^{tu} \right) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{x} \right. \right. \\
&= \frac{\log^4 \left( 56 (j(hx)^t)^u \right) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4tu} - \text{Subst} \left( \text{Subst} \left( \int \frac{\log^3 \left( 56 h^{tu} j^u x^{tu} \right) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{x} \right. \right. \\
&= -\frac{pr \log^4 \left( 56 (j(hx)^t)^u \right) \log \left( 1 + \frac{bx}{a} \right)}{4tu} + \frac{\log^4 \left( 56 (j(hx)^t)^u \right) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4tu} \\
&= -\frac{pr \log^4 \left( 56 (j(hx)^t)^u \right) \log \left( 1 + \frac{bx}{a} \right)}{4tu} + \frac{\log^4 \left( 56 (j(hx)^t)^u \right) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4tu} \\
&= -\frac{pr \log^4 \left( 56 (j(hx)^t)^u \right) \log \left( 1 + \frac{bx}{a} \right)}{4tu} + \frac{\log^4 \left( 56 (j(hx)^t)^u \right) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4tu} \\
&= -\frac{pr \log^4 \left( 56 (j(hx)^t)^u \right) \log \left( 1 + \frac{bx}{a} \right)}{4tu} + \frac{\log^4 \left( 56 (j(hx)^t)^u \right) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4tu} \\
&= -\frac{pr \log^4 \left( 56 (j(hx)^t)^u \right) \log \left( 1 + \frac{bx}{a} \right)}{4tu} + \frac{\log^4 \left( 56 (j(hx)^t)^u \right) \log \left( e \left( f(a + bx)^p (c + dx)^q \right)^r \right)}{4tu}
\end{aligned}$$

**Mathematica [B]** time = 1.82702, size = 1241, normalized size = 3.78

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Log[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/x,x
]
```

```
[Out] p*r*t^3*u^3*Log[x]*Log[h*x]^3*Log[a + b*x] - p*r*t^3*u^3*Log[h*x]^4*Log[a +
b*x] - 3*p*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] +
3*p*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] + 3*p*r*t*u*Log
```

```

[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[a + b*x] - 3*p*r*t*u*Log[h*x]^2*Log
[i*(j*(h*x)^t)^u]^2*Log[a + b*x] - p*r*Log[x]*Log[i*(j*(h*x)^t)^u]^3*Log[a
+ b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[a + b*x] + (p*r*t^3*u^3*Lo
g[h*x]^4*Log[1 + (b*x)/a])/4 - p*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*
Log[1 + (b*x)/a] + (3*p*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (b
x)/a])/2 - p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (b*x)/a] + q*r*t^3*u
^3*Log[x]*Log[h*x]^3*Log[c + d*x] - q*r*t^3*u^3*Log[h*x]^4*Log[c + d*x] - 3
*q*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] + 3*q*r*t^
2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] + 3*q*r*t*u*Log[x]*Log[h
*x]*Log[i*(j*(h*x)^t)^u]^2*Log[c + d*x] - 3*q*r*t*u*Log[h*x]^2*Log[i*(j*(h
x)^t)^u]^2*Log[c + d*x] - q*r*Log[x]*Log[i*(j*(h*x)^t)^u]^3*Log[c + d*x] +
q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[c + d*x] - t^3*u^3*Log[x]*Log[h*x]^
3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (3*t^3*u^3*Log[h*x]^4*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r])/4 + 3*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t
)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*t^2*u^2*Log[h*x]^3*Log[i*(j*(h
x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 3*t*u*Log[x]*Log[h*x]*Log[
i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (3*t*u*Log[h*x]^2
*Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 + Log[x]*Lo
g[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (q*r*t^3*u^3*Lo
g[h*x]^4*Log[1 + (d*x)/c])/4 - q*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*
Log[1 + (d*x)/c] + (3*q*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (d
x)/c])/2 - q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (d*x)/c] - p*r*Log[i
*(j*(h*x)^t)^u]^3*PolyLog[2, -((b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^3*PolyL
og[2, -((d*x)/c)] + 3*p*r*t*u*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((b*x)/a)]
+ 3*q*r*t*u*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((d*x)/c)] - 6*p*r*t^2*u^2*
Log[i*(j*(h*x)^t)^u]*PolyLog[4, -((b*x)/a)] - 6*q*r*t^2*u^2*Log[i*(j*(h*x)^
t)^u]*PolyLog[4, -((d*x)/c)] + 6*p*r*t^3*u^3*PolyLog[5, -((b*x)/a)] + 6*q*r
*t^3*u^3*PolyLog[5, -((d*x)/c)]

```

---

**Maple [F]** time = 2.213, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(i\left(j\left(hx\right)^t\right)^u\right)\right)^3 \ln\left(e\left(f\left(bx+a\right)^p\left(dx+c\right)^q\right)^r\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(i\*(j\*(h\*x)^t)^u)^3\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x)

[Out] int(ln(i\*(j\*(h\*x)^t)^u)^3\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(i\*(j\*(h\*x)^t)^u)^3\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x, algorithm="maxima")

[Out] 
$$-1/4*(t^3*u^3*\log(x)^4 - 4*(t^2*u^2*\log((h^t)^u) + t^2*u^2*\log(i) + t^2*u^2*\log(j^u))*\log(x)^3 - 4*\log(x)*\log((x^t)^u)^3 + 6*(t*u*\log((h^t)^u)^2 + t*u*\log(i)^2 + 2*t*u*\log(i)*\log(j^u) + t*u*\log(j^u)^2 + 2*(t*u*\log(i) + t*u*\log(j^u))*\log((h^t)^u))*\log(x)^2 + 6*(t*u*\log(x)^2 - 2*(\log((h^t)^u) + \log(i) + \log(j^u))*\log(x))*\log((x^t)^u)^2 - 4*(t^2*u^2*\log(x)^3 - 3*(t*u*\log((h^t)^u) + t*u*\log(i) + t*u*\log(j^u))*\log(x)^2 + 3*(2*(\log(i) + \log(j^u))*\log((h^t)^u) + \log((h^t)^u)^2 + \log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log(x))*\log((x^t)^u) - 4*(3*(\log(i) + \log(j^u))*\log((h^t)^u)^2 + \log((h^t)^u)^3 + \log(i)^3 + 3*\log(i)^2*\log(j^u) + 3*\log(i)*\log(j^u)^2 + \log(j^u)^3 + 3*(\log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log((h^t)^u))*\log(x))*\log(((b*x + a)^p)^r) - 1/4*(t^3*u^3*\log(x)^4 - 4*(t^2*u^2*\log((h^t)^u) + t^2*u^2*\log(i) + t^2*u^2*\log(j^u))*\log(x)^3 - 4*\log(x)*\log((x^t)^u)^3 + 6*(t*u*\log((h^t)^u)^2 + t*u*\log(i)^2 + 2*t*u*\log(i)*\log(j^u) + t*u*\log(j^u)^2 + 2*(t*u*\log(i) + t*u*\log(j^u))*\log((h^t)^u))*\log(x)^2 + 6*(t*u*\log(x)^2 - 2*(\log((h^t)^u) + \log(i) + \log(j^u))*\log(x))*\log((x^t)^u)^2 - 4*(t^2*u^2*\log(x)^3 - 3*(t*u*\log((h^t)^u) + t*u*\log(i) + t*u*\log(j^u))*\log(x)^2 + 3*(2*(\log(i) + \log(j^u))*\log((h^t)^u) + \log((h^t)^u)^2 + \log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log(x))*\log((x^t)^u) - 4*(3*(\log(i) + \log(j^u))*\log((h^t)^u)^2 + \log((h^t)^u)^3 + \log(i)^3 + 3*\log(i)^2*\log(j^u) + 3*\log(i)*\log(j^u)^2 + \log(j^u)^3 + 3*(\log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log((h^t)^u))*\log(x))*\log(((d*x + c)^q)^r) - \text{integrate}(-1/4*(4*((3*(\log(i) + \log(j^u))*\log((h^t)^u)^2 + \log((h^t)^u)^3 + \log(i)^3 + 3*\log(i)^2*\log(j^u) + 3*\log(i)*\log(j^u)^2 + \log(j^u)^3 + 3*(\log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log((h^t)^u))*\log(e) + (3*(\log(i) + \log(j^u))*\log((h^t)^u)^2 + \log((h^t)^u)^3 + \log(i)^3 + 3*\log(i)^2*\log(j^u) + 3*\log(i)*\log(j^u)^2 + \log(j^u)^3 + 3*(\log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log((h^t)^u))*\log(f^r))*b*d*x^2 + ((p*r*t^3*u^3 + q*r*t^3*u^3)*b*d*x^2 + (b*c*p*r*t^3*u^3 + a*d*q*r*t^3*u^3)*x)*\log(x)^4 - 4*((p*r*t^2*u^2 + q*r*t^2*u^2)*\log((h^t)^u) + (p*r*t^2*u^2 + q*r*t^2*u^2)*\log(i) + (p*r*t^2*u^2 + q*r*t^2*u^2)*\log(j^u))*b*d*x^2 + ((p*r*t^2*u^2*\log((h^t)^u) + p*r*t^2*u^2*\log(i) + p*r*t^2*u^2*\log(j^u))*b*c + (q*r*t^2*u^2*\log((h^t)^u) + q*r*t^2*u^2*\log(i) + q*r*t^2*u^2*\log(j^u))*a*d)*x)*\log(x)^3 + 4*(b*d*x^2*(\log(e) + \log(f^r)) + a*c*(\log(e) + \log(f^r)) + (b*c*(\log(e) + \log(f^r)) + a*d*(\log(e) + \log(f^r))))*x - ((p*r + q*r)*b*d*x^2 + (b*c*p*r + a*d*q*r)*x)*\log(x))*\log((x^t)^u)^3 + 4*((3*(\log(i) + \log(j^u))*\log((h^t)^u)^2 + \log((h^t)^u)^3 + \log(i)^3 + 3*\log(i)^2*\log(j^u) + 3*\log(i)*\log(j^u)^2$$

$$\begin{aligned}
& 2 + \log(j^u)^3 + 3*(\log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log((h^t)^u) \\
& )*\log(e) + (3*(\log(i) + \log(j^u))*\log((h^t)^u)^2 + \log((h^t)^u)^3 + \log(i)^3 \\
& + 3*\log(i)^2*\log(j^u) + 3*\log(i)*\log(j^u)^2 + \log(j^u)^3 + 3*(\log(i)^2 + \\
& 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log((h^t)^u))*\log(f^r))*a*c + 6*((p*r*t*u \\
& + q*r*t*u)*\log((h^t)^u)^2 + (p*r*t*u + q*r*t*u)*\log(i)^2 + 2*(p*r*t*u + q*r \\
& *t*u)*\log(i)*\log(j^u) + (p*r*t*u + q*r*t*u)*\log(j^u)^2 + 2*((p*r*t*u + q*r \\
& *t*u)*\log(i) + (p*r*t*u + q*r*t*u)*\log(j^u))*\log((h^t)^u))*b*d*x^2 + ((p*r*t \\
& *u*\log((h^t)^u)^2 + p*r*t*u*\log(i)^2 + 2*p*r*t*u*\log(i)*\log(j^u) + p*r*t*u \\
& *\log(j^u)^2 + 2*(p*r*t*u*\log(i) + p*r*t*u*\log(j^u))*\log((h^t)^u))*b*c + (q*r \\
& *t*u*\log((h^t)^u)^2 + q*r*t*u*\log(i)^2 + 2*q*r*t*u*\log(i)*\log(j^u) + q*r*t \\
& *u*\log(j^u)^2 + 2*(q*r*t*u*\log(i) + q*r*t*u*\log(j^u))*\log((h^t)^u))*a*d)*x) \\
& *\log(x)^2 + 6*(2*((\log((h^t)^u) + \log(i) + \log(j^u))*\log(e) + (\log((h^t)^u) \\
& + \log(i) + \log(j^u))*\log(f^r))*b*d*x^2 + 2*((\log((h^t)^u) + \log(i) + \log(j^ \\
& u))*\log(e) + (\log((h^t)^u) + \log(i) + \log(j^u))*\log(f^r))*a*c + ((p*r*t*u + \\
& q*r*t*u)*b*d*x^2 + (b*c*p*r*t*u + a*d*q*r*t*u)*x)*\log(x)^2 + 2*(((\log((h^t \\
& )^u) + \log(i) + \log(j^u))*\log(e) + (\log((h^t)^u) + \log(i) + \log(j^u))*\log(f \\
& ^r))*b*c + ((\log((h^t)^u) + \log(i) + \log(j^u))*\log(e) + (\log((h^t)^u) + \log \\
& (i) + \log(j^u))*\log(f^r))*a*d)*x - 2*((p*r + q*r)*\log((h^t)^u) + (p*r + q*r \\
& )*\log(i) + (p*r + q*r)*\log(j^u))*b*d*x^2 + ((p*r*\log((h^t)^u) + p*r*\log(i) \\
& + p*r*\log(j^u))*b*c + (q*r*\log((h^t)^u) + q*r*\log(i) + q*r*\log(j^u))*a*d)* \\
& x)*\log(x))*\log((h^t)^u)^2 + 4*((3*(\log(i) + \log(j^u))*\log((h^t)^u)^2 + \log \\
& ((h^t)^u)^3 + \log(i)^3 + 3*\log(i)^2*\log(j^u) + 3*\log(i)*\log(j^u)^2 + \log(j^ \\
& u)^3 + 3*(\log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log((h^t)^u))*\log(e) + \\
& (3*(\log(i) + \log(j^u))*\log((h^t)^u)^2 + \log((h^t)^u)^3 + \log(i)^3 + 3*\log( \\
& i)^2*\log(j^u) + 3*\log(i)*\log(j^u)^2 + \log(j^u)^3 + 3*(\log(i)^2 + 2*\log(i)* \\
& \log(j^u) + \log(j^u)^2)*\log((h^t)^u))*\log(f^r))*b*c + ((3*(\log(i) + \log(j^u)) \\
& )*\log((h^t)^u)^2 + \log((h^t)^u)^3 + \log(i)^3 + 3*\log(i)^2*\log(j^u) + 3*\log(i) \\
& )*\log(j^u)^2 + \log(j^u)^3 + 3*(\log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)* \\
& \log((h^t)^u))*\log(e) + (3*(\log(i) + \log(j^u))*\log((h^t)^u)^2 + \log((h^t)^u)^ \\
& 3 + \log(i)^3 + 3*\log(i)^2*\log(j^u) + 3*\log(i)*\log(j^u)^2 + \log(j^u)^3 + 3*( \\
& \log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log((h^t)^u))*\log(f^r))*a*d)*x - \\
& 4*((p*r + q*r)*\log((h^t)^u)^3 + (p*r + q*r)*\log(i)^3 + 3*(p*r + q*r)*\log( \\
& i)^2*\log(j^u) + 3*(p*r + q*r)*\log(i)*\log(j^u)^2 + (p*r + q*r)*\log(j^u)^3 + \\
& 3*((p*r + q*r)*\log(i) + (p*r + q*r)*\log(j^u))*\log((h^t)^u)^2 + 3*((p*r + q*r) \\
& )*\log(i)^2 + 2*(p*r + q*r)*\log(i)*\log(j^u) + (p*r + q*r)*\log(j^u)^2)*\log(( \\
& h^t)^u))*b*d*x^2 + ((p*r*\log((h^t)^u)^3 + p*r*\log(i)^3 + 3*p*r*\log(i)^2*\log \\
& (j^u) + 3*p*r*\log(i)*\log(j^u)^2 + p*r*\log(j^u)^3 + 3*(p*r*\log(i) + p*r*\log( \\
& j^u))*\log((h^t)^u)^2 + 3*(p*r*\log(i)^2 + 2*p*r*\log(i)*\log(j^u) + p*r*\log(j^ \\
& u)^2)*\log((h^t)^u))*b*c + (q*r*\log((h^t)^u)^3 + q*r*\log(i)^3 + 3*q*r*\log(i) \\
& ^2*\log(j^u) + 3*q*r*\log(i)*\log(j^u)^2 + q*r*\log(j^u)^3 + 3*(q*r*\log(i) + q*r \\
& *\log(j^u))*\log((h^t)^u)^2 + 3*(q*r*\log(i)^2 + 2*q*r*\log(i)*\log(j^u) + q*r* \\
& \log(j^u)^2)*\log((h^t)^u))*a*d)*x)*\log(x) + 4*(3*((2*(\log(i) + \log(j^u))*\log \\
& ((h^t)^u) + \log((h^t)^u)^2 + \log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log \\
& (e) + (2*(\log(i) + \log(j^u))*\log((h^t)^u) + \log((h^t)^u)^2 + \log(i)^2 + 2* \\
& \log(i)*\log(j^u) + \log(j^u)^2)*\log(f^r))*b*d*x^2 - ((p*r*t^2*u^2 + q*r*t^2*u^
\end{aligned}$$

```

2)*b*d*x^2 + (b*c*p*r*t^2*u^2 + a*d*q*r*t^2*u^2)*x)*log(x)^3 + 3*((2*(log(i)
) + log(j^u))*log((h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u)
+ log(j^u)^2)*log(e) + (2*(log(i) + log(j^u))*log((h^t)^u) + log((h^t)^u)^2
+ log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log(f^r))*a*c + 3*((p*r*t*u
+ q*r*t*u)*log((h^t)^u) + (p*r*t*u + q*r*t*u)*log(i) + (p*r*t*u + q*r*t*u)*
log(j^u))*b*d*x^2 + ((p*r*t*u*log((h^t)^u) + p*r*t*u*log(i) + p*r*t*u*log(j
^u))*b*c + (q*r*t*u*log((h^t)^u) + q*r*t*u*log(i) + q*r*t*u*log(j^u))*a*d)*
x)*log(x)^2 + 3*((2*(log(i) + log(j^u))*log((h^t)^u) + log((h^t)^u)^2 + lo
g(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log(e) + (2*(log(i) + log(j^u))*lo
g((h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*lo
g(f^r))*b*c + ((2*(log(i) + log(j^u))*log((h^t)^u) + log((h^t)^u)^2 + log(i)
)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log(e) + (2*(log(i) + log(j^u))*log((
h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log(f
^r))*a*d)*x - 3*((p*r + q*r)*log((h^t)^u)^2 + (p*r + q*r)*log(i)^2 + 2*(p*
r + q*r)*log(i)*log(j^u) + (p*r + q*r)*log(j^u)^2 + 2*((p*r + q*r)*log(i) +
(p*r + q*r)*log(j^u))*log((h^t)^u))*b*d*x^2 + ((p*r*log((h^t)^u)^2 + p*r*log
(i)^2 + 2*p*r*log(i)*log(j^u) + p*r*log(j^u)^2 + 2*(p*r*log(i) + p*r*log(
j^u))*log((h^t)^u))*b*c + (q*r*log((h^t)^u)^2 + q*r*log(i)^2 + 2*q*r*log(i)
*log(j^u) + q*r*log(j^u)^2 + 2*(q*r*log(i) + q*r*log(j^u))*log((h^t)^u))*a*d
)*x)*log(x))*log((x^t)^u))/(b*d*x^3 + a*c*x + (b*c + a*d)*x^2), x)

```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right) \log \left( \left( (hx)^t j^u i \right)^3 \right)}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(i\*(j\*(h\*x)^t)^u)^3\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x, algo  
rithm="fricas")

[Out] integral(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)\*log(((h\*x)^t\*j)^u\*i)^3/x, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(ln(i*(j*(h*x)**t)**u)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right) \log\left(\left((hx)^t j\right)^u i\right)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^3/x, x)
```

$$3.57 \quad \int \frac{\log^2\left(i(j(hx)^t)^u\right) \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x} dx$$

**Optimal.** Leaf size=262

$$-pr \text{PolyLog}\left(2, -\frac{bx}{a}\right) \log^2\left(i(j(hx)^t)^u\right) + 2prt u \text{PolyLog}\left(3, -\frac{bx}{a}\right) \log\left(i(j(hx)^t)^u\right) - 2prt^2 u^2 \text{PolyLog}\left(4, -\frac{bx}{a}\right) - qr \text{Poly}$$

[Out]  $-(p*r*\text{Log}[i*(j*(h*x)^t)^u]^3*\text{Log}[1 + (b*x)/a])/(3*t*u) + (\text{Log}[i*(j*(h*x)^t)^u]^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(3*t*u) - (q*r*\text{Log}[i*(j*(h*x)^t)^u]^3*\text{Log}[1 + (d*x)/c])/(3*t*u) - p*r*\text{Log}[i*(j*(h*x)^t)^u]^2*\text{PolyLog}[2, -((b*x)/a)] - q*r*\text{Log}[i*(j*(h*x)^t)^u]^2*\text{PolyLog}[2, -((d*x)/c)] + 2*p*r*t*u*\text{Log}[i*(j*(h*x)^t)^u]*\text{PolyLog}[3, -((b*x)/a)] + 2*q*r*t*u*\text{Log}[i*(j*(h*x)^t)^u]*\text{PolyLog}[3, -((d*x)/c)] - 2*p*r*t^2*u^2*\text{PolyLog}[4, -((b*x)/a)] - 2*q*r*t^2*u^2*\text{PolyLog}[4, -((d*x)/c)]$

**Rubi [A]** time = 0.910214, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2499, 2317, 2374, 2383, 6589, 2445}

$$-pr \text{PolyLog}\left(2, -\frac{bx}{a}\right) \log^2\left(i(j(hx)^t)^u\right) + 2prt u \text{PolyLog}\left(3, -\frac{bx}{a}\right) \log\left(i(j(hx)^t)^u\right) - 2prt^2 u^2 \text{PolyLog}\left(4, -\frac{bx}{a}\right) - qr \text{Poly}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Log}[i*(j*(h*x)^t)^u]^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/x, x]$

[Out]  $-(p*r*\text{Log}[i*(j*(h*x)^t)^u]^3*\text{Log}[1 + (b*x)/a])/(3*t*u) + (\text{Log}[i*(j*(h*x)^t)^u]^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(3*t*u) - (q*r*\text{Log}[i*(j*(h*x)^t)^u]^3*\text{Log}[1 + (d*x)/c])/(3*t*u) - p*r*\text{Log}[i*(j*(h*x)^t)^u]^2*\text{PolyLog}[2, -((b*x)/a)] - q*r*\text{Log}[i*(j*(h*x)^t)^u]^2*\text{PolyLog}[2, -((d*x)/c)] + 2*p*r*t*u*\text{Log}[i*(j*(h*x)^t)^u]*\text{PolyLog}[3, -((b*x)/a)] + 2*q*r*t*u*\text{Log}[i*(j*(h*x)^t)^u]*\text{PolyLog}[3, -((d*x)/c)] - 2*p*r*t^2*u^2*\text{PolyLog}[4, -((b*x)/a)] - 2*q*r*t^2*u^2*\text{PolyLog}[4, -((d*x)/c)]$

**Rule 2499**

$\text{Int}[(\text{Log}[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))]^(r_.)]*((s_.) + \text{Log}[(i_.)*((g_.) + (h_.)*(x_.))^(n_.)]*(t_.))^(m_.)]/((j_.) + (k_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}(((s + t*\text{Log}[i*(g + h*x)^n])^(m + 1)*\text{Log}[(e*(f*(a + b*x)^p*(c + d*x)^q)^r])^3*\text{Log}[1 + (b*x)/a])/(3*t*u) + (\text{Log}[i*(j*(h*x)^t)^u]^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(3*t*u) - (q*r*\text{Log}[i*(j*(h*x)^t)^u]^3*\text{Log}[1 + (d*x)/c])/(3*t*u) - p*r*\text{Log}[i*(j*(h*x)^t)^u]^2*\text{PolyLog}[2, -((b*x)/a)] - q*r*\text{Log}[i*(j*(h*x)^t)^u]^2*\text{PolyLog}[2, -((d*x)/c)] + 2*p*r*t*u*\text{Log}[i*(j*(h*x)^t)^u]*\text{PolyLog}[3, -((b*x)/a)] + 2*q*r*t*u*\text{Log}[i*(j*(h*x)^t)^u]*\text{PolyLog}[3, -((d*x)/c)] - 2*p*r*t^2*u^2*\text{PolyLog}[4, -((b*x)/a)] - 2*q*r*t^2*u^2*\text{PolyLog}[4, -((d*x)/c)]$

```
e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r}, x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

### Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e, Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]
```

### Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d*e, 1]
```

### Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x], c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\log^2 \left( 57 (j(hx)^t)^u \right) \log \left( e (f(a + bx)^p (c + dx)^q)^r \right)}{x} dx &= \text{Subst} \left( \int \frac{\log^2 \left( 57 j^u (hx)^{tu} \right) \log \left( e (f(a + bx)^p (c + dx)^q)^r \right)}{x} dx, 5 \right. \\
&= \text{Subst} \left( \text{Subst} \left( \int \frac{\log^2 \left( 57 h^{tu} j^u x^{tu} \right) \log \left( e (f(a + bx)^p (c + dx)^q)^r \right)}{x} \right. \right. \\
&= \frac{\log^3 \left( 57 (j(hx)^t)^u \right) \log \left( e (f(a + bx)^p (c + dx)^q)^r \right)}{3tu} - \text{Subst} \left( \text{Subst} \left( \int \frac{\log^2 \left( 57 h^{tu} j^u x^{tu} \right) \log \left( e (f(a + bx)^p (c + dx)^q)^r \right)}{x} \right. \right. \\
&= -\frac{pr \log^3 \left( 57 (j(hx)^t)^u \right) \log \left( 1 + \frac{bx}{a} \right)}{3tu} + \frac{\log^3 \left( 57 (j(hx)^t)^u \right) \log \left( e (f(a + bx)^p (c + dx)^q)^r \right)}{3tu} \\
&= -\frac{pr \log^3 \left( 57 (j(hx)^t)^u \right) \log \left( 1 + \frac{bx}{a} \right)}{3tu} + \frac{\log^3 \left( 57 (j(hx)^t)^u \right) \log \left( e (f(a + bx)^p (c + dx)^q)^r \right)}{3tu} \\
&= -\frac{pr \log^3 \left( 57 (j(hx)^t)^u \right) \log \left( 1 + \frac{bx}{a} \right)}{3tu} + \frac{\log^3 \left( 57 (j(hx)^t)^u \right) \log \left( e (f(a + bx)^p (c + dx)^q)^r \right)}{3tu} \\
&= -\frac{pr \log^3 \left( 57 (j(hx)^t)^u \right) \log \left( 1 + \frac{bx}{a} \right)}{3tu} + \frac{\log^3 \left( 57 (j(hx)^t)^u \right) \log \left( e (f(a + bx)^p (c + dx)^q)^r \right)}{3tu}
\end{aligned}$$

**Mathematica [B]** time = 0.913282, size = 839, normalized size = 3.2

$$pr^2 u^2 \log(a + bx) \log^3(hx) - \frac{1}{3} pr^2 u^2 \log\left(\frac{bx}{a} + 1\right) \log^3(hx) + qr^2 u^2 \log(c + dx) \log^3(hx) - \frac{2}{3} t^2 u^2 \log\left(e (f(a + bx)^p (c + dx)^q)^r\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Log[i\*(j\*(h\*x)^t)^u]^2\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r])/x,x  
]

[Out] -(p\*r\*t^2\*u^2\*Log[x]\*Log[h\*x]^2\*Log[a + b\*x]) + p\*r\*t^2\*u^2\*Log[h\*x]^3\*Log[a + b\*x] + 2\*p\*r\*t\*u\*Log[x]\*Log[h\*x]\*Log[i\*(j\*(h\*x)^t)^u]\*Log[a + b\*x] - 2\*p\*r\*t\*u\*Log[h\*x]^2\*Log[i\*(j\*(h\*x)^t)^u]\*Log[a + b\*x] - p\*r\*Log[x]\*Log[i\*(j\*(h\*x)^t)^u]^2\*Log[a + b\*x] + p\*r\*Log[h\*x]\*Log[i\*(j\*(h\*x)^t)^u]^2\*Log[a + b\*x] - (p\*r\*t^2\*u^2\*Log[h\*x]^3\*Log[1 + (b\*x)/a])/3 + p\*r\*t\*u\*Log[h\*x]^2\*Log[i\*(j\*(h\*x)^t)^u]\*Log[1 + (b\*x)/a] - p\*r\*Log[h\*x]\*Log[i\*(j\*(h\*x)^t)^u]^2\*Log[

$$\begin{aligned}
& 1 + (b*x)/a - q*r*t^2*u^2*\text{Log}[x]*\text{Log}[h*x]^2*\text{Log}[c + d*x] + q*r*t^2*u^2*\text{Log} \\
& [h*x]^3*\text{Log}[c + d*x] + 2*q*r*t*u*\text{Log}[x]*\text{Log}[h*x]*\text{Log}[i*(j*(h*x)^t)^u]*\text{Log}[c \\
& + d*x] - 2*q*r*t*u*\text{Log}[h*x]^2*\text{Log}[i*(j*(h*x)^t)^u]*\text{Log}[c + d*x] - q*r*\text{Log}[ \\
& x]*\text{Log}[i*(j*(h*x)^t)^u]^2*\text{Log}[c + d*x] + q*r*\text{Log}[h*x]*\text{Log}[i*(j*(h*x)^t)^u]^ \\
& 2*\text{Log}[c + d*x] + t^2*u^2*\text{Log}[x]*\text{Log}[h*x]^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q \\
& )^r] - (2*t^2*u^2*\text{Log}[h*x]^3*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/3 - 2*t* \\
& u*\text{Log}[x]*\text{Log}[h*x]*\text{Log}[i*(j*(h*x)^t)^u]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r \\
& + t*u*\text{Log}[h*x]^2*\text{Log}[i*(j*(h*x)^t)^u]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r \\
& + \text{Log}[x]*\text{Log}[i*(j*(h*x)^t)^u]^2*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r - (q* \\
& r*t^2*u^2*\text{Log}[h*x]^3*\text{Log}[1 + (d*x)/c])/3 + q*r*t*u*\text{Log}[h*x]^2*\text{Log}[i*(j*(h*x) \\
& )^t)^u]*\text{Log}[1 + (d*x)/c] - q*r*\text{Log}[h*x]*\text{Log}[i*(j*(h*x)^t)^u]^2*\text{Log}[1 + (d*x) \\
& )/c] - p*r*\text{Log}[i*(j*(h*x)^t)^u]^2*\text{PolyLog}[2, -((b*x)/a)] - q*r*\text{Log}[i*(j*(h*x) \\
& )^t)^u]^2*\text{PolyLog}[2, -((d*x)/c)] + 2*p*r*t*u*\text{Log}[i*(j*(h*x)^t)^u]*\text{PolyLog}[ \\
& 3, -((b*x)/a)] + 2*q*r*t*u*\text{Log}[i*(j*(h*x)^t)^u]*\text{PolyLog}[3, -((d*x)/c)] - 2* \\
& p*r*t^2*u^2*\text{PolyLog}[4, -((b*x)/a)] - 2*q*r*t^2*u^2*\text{PolyLog}[4, -((d*x)/c)]
\end{aligned}$$

**Maple [F]** time = 1.8, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(i\left(j\left(hx^t\right)^u\right)\right)\right)^2 \ln\left(e\left(f\left(bx+a\right)^p\left(dx+c\right)^q\right)^r\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(i\*(j\*(h\*x)^t)^u)^2\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x)

[Out] int(ln(i\*(j\*(h\*x)^t)^u)^2\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(i\*(j\*(h\*x)^t)^u)^2\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x, algo  
rithm="maxima")

[Out] 1/3\*(t^2\*u^2\*log(x)^3 - 3\*(t\*u\*log((h^t)^u) + t\*u\*log(i) + t\*u\*log(j^u))\*lo  
g(x)^2 + 3\*log(x)\*log((x^t)^u)^2 - 3\*(t\*u\*log(x)^2 - 2\*(log((h^t)^u) + log(  
i) + log(j^u))\*log(x))\*log((x^t)^u) + 3\*(2\*(log(i) + log(j^u))\*log((h^t)^u)

$$\begin{aligned}
& + \log((h^t)^u)^2 + \log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2*\log(x))*\log( \\
& ((b*x + a)^p)^r) + 1/3*(t^2*u^2*\log(x)^3 - 3*(t*u*\log((h^t)^u) + t*u*\log(i) \\
& + t*u*\log(j^u))*\log(x)^2 + 3*\log(x)*\log((x^t)^u)^2 - 3*(t*u*\log(x)^2 - 2*( \\
& \log((h^t)^u) + \log(i) + \log(j^u))*\log(x))*\log((x^t)^u) + 3*(2*(\log(i) + \log \\
& (j^u))*\log((h^t)^u) + \log((h^t)^u)^2 + \log(i)^2 + 2*\log(i)*\log(j^u) + \log(j \\
& ^u)^2)*\log(x))*\log(((d*x + c)^q)^r) - \text{integrate}(-1/3*(3*((2*(\log(i) + \log(j \\
& ^u))*\log((h^t)^u) + \log((h^t)^u)^2 + \log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u \\
& )^2)*\log(e) + (2*(\log(i) + \log(j^u))*\log((h^t)^u) + \log((h^t)^u)^2 + \log(i) \\
& ^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log(f^r))*b*d*x^2 - ((p*r*t^2*u^2 + q* \\
& r*t^2*u^2)*b*d*x^2 + (b*c*p*r*t^2*u^2 + a*d*q*r*t^2*u^2)*x)*\log(x)^3 + 3*(( \\
& 2*(\log(i) + \log(j^u))*\log((h^t)^u) + \log((h^t)^u)^2 + \log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log(e) + (2*(\log(i) + \log(j^u))*\log((h^t)^u) + \log((h^t)^u)^2 + \log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log(f^r))*a*c + 3*(( \\
& p*r*t*u + q*r*t*u)*\log((h^t)^u) + (p*r*t*u + q*r*t*u)*\log(i) + (p*r*t*u + q \\
& *r*t*u)*\log(j^u))*b*d*x^2 + ((p*r*t*u*\log((h^t)^u) + p*r*t*u*\log(i) + p*r*t \\
& *u*\log(j^u))*b*c + (q*r*t*u*\log((h^t)^u) + q*r*t*u*\log(i) + q*r*t*u*\log(j^u \\
& ))*a*d)*x)*\log(x)^2 + 3*(b*d*x^2*(\log(e) + \log(f^r)) + a*c*(\log(e) + \log(f^ \\
& r)) + (b*c*(\log(e) + \log(f^r)) + a*d*(\log(e) + \log(f^r))))*x - ((p*r + q*r)* \\
& b*d*x^2 + (b*c*p*r + a*d*q*r)*x)*\log(x))*\log((x^t)^u)^2 + 3*((2*(\log(i) + \\
& \log(j^u))*\log((h^t)^u) + \log((h^t)^u)^2 + \log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log(e) + (2*(\log(i) + \log(j^u))*\log((h^t)^u) + \log((h^t)^u)^2 + \log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log(f^r))*b*c + ((2*(\log(i) + \log(j^u))*\log((h^t)^u) + \log((h^t)^u)^2 + \log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log(e) + (2*(\log(i) + \log(j^u))*\log((h^t)^u) + \log((h^t)^u)^2 + \log(i)^2 + 2*\log(i)*\log(j^u) + \log(j^u)^2)*\log(f^r))*a*d)*x - 3*((p*r + q*r)*\log((h^t)^u)^2 + (p*r + q*r)*\log(i)^2 + 2*(p*r + q*r)*\log(i)*\log(j^u) + (p*r + q*r)*\log(j^u)^2 + 2*((p*r + q*r)*\log(i) + (p*r + q*r)*\log(j^u))*\log((h^t)^u))*b*d*x^2 + ((p*r*\log((h^t)^u)^2 + p*r*\log(i)^2 + 2*p*r*\log(i)*\log(j^u) + p*r*\log(j^u)^2 + 2*(p*r*\log(i) + p*r*\log(j^u))*\log((h^t)^u))*b*c + (q*r*\log((h^t)^u)^2 + q*r*\log(i)^2 + 2*q*r*\log(i)*\log(j^u) + q*r*\log(j^u)^2 + 2*(q*r*\log(i) + q*r*\log(j^u))*\log((h^t)^u))*a*d)*x)*\log(x) + 3*(2*((\log((h^t)^u) + \log(i) + \log(j^u))*\log(e) + (\log((h^t)^u) + \log(i) + \log(j^u))*\log(f^r))*b*d*x^2 + 2*((\log((h^t)^u) + \log(i) + \log(j^u))*\log(e) + (\log((h^t)^u) + \log(i) + \log(j^u))*\log(f^r))*a*c + ((p*r*t*u + q*r*t*u)*b*d*x^2 + (b*c*p*r*t*u + a*d*q*r*t*u)*x)*\log(x)^2 + 2*(((\log((h^t)^u) + \log(i) + \log(j^u))*\log(e) + (\log((h^t)^u) + \log(i) + \log(j^u))*\log(f^r))*b*c + ((\log((h^t)^u) + \log(i) + \log(j^u))*\log(e) + (\log((h^t)^u) + \log(i) + \log(j^u))*\log(f^r))*a*d)*x - 2*((p*r + q*r)*\log((h^t)^u) + (p*r + q*r)*\log(i) + (p*r + q*r)*\log(j^u))*b*d*x^2 + ((p*r*\log((h^t)^u) + p*r*\log(i) + p*r*\log(j^u))*b*c + (q*r*\log((h^t)^u) + q*r*\log(i) + q*r*\log(j^u))*a*d)*x)*\log(x))*\log((x^t)^u))/(b*d*x^3 + a*c*x + (b*c + a*d)*x^2), x)
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right) \log \left( \left( (hx)^t j \right)^u i \right)^2}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(i\*(j\*(h\*x)^t)^u)^2\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x, algorithm="fricas")

[Out] integral(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)\*log(((h\*x)^t\*j)^u\*i)^2/x, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(i\*(j\*(h\*x)\*\*t)\*\*u)\*\*2\*ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r)/x,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right) \log \left( \left( (hx)^t j \right)^u i \right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(i\*(j\*(h\*x)^t)^u)^2\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x, algorithm="giac")

[Out] integrate(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)\*log(((h\*x)^t\*j)^u\*i)^2/x, x)

$$3.58 \quad \int \frac{\log\left(i(j(hx)^t)^u\right) \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x} dx$$

**Optimal.** Leaf size=194

$$-prPolyLog\left(2, -\frac{bx}{a}\right) \log\left(i(j(hx)^t)^u\right) + prtPolyLog\left(3, -\frac{bx}{a}\right) - qrPolyLog\left(2, -\frac{dx}{c}\right) \log\left(i(j(hx)^t)^u\right) + qrtPolyLog$$

```
[Out] -(p*r*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (b*x)/a])/(2*t*u) + (Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(2*t*u) - (q*r*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (d*x)/c])/(2*t*u) - p*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((d*x)/c)] + p*r*t*u*PolyLog[3, -((b*x)/a)] + q*r*t*u*PolyLog[3, -((d*x)/c)]
```

---

**Rubi [A]** time = 0.603361, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$ , Rules used = {2499, 2317, 2374, 6589, 2445}

$$-prPolyLog\left(2, -\frac{bx}{a}\right) \log\left(i(j(hx)^t)^u\right) + prtPolyLog\left(3, -\frac{bx}{a}\right) - qrPolyLog\left(2, -\frac{dx}{c}\right) \log\left(i(j(hx)^t)^u\right) + qrtPolyLog$$

Antiderivative was successfully verified.

```
[In] Int[(Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/x,x]
```

```
[Out] -(p*r*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (b*x)/a])/(2*t*u) + (Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(2*t*u) - (q*r*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (d*x)/c])/(2*t*u) - p*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((d*x)/c)] + p*r*t*u*PolyLog[3, -((b*x)/a)] + q*r*t*u*PolyLog[3, -((d*x)/c)]
```

### Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))^r_.)]*(s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)^(m_.)]/((j_.) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
```



$x]$  && NeQ[ $b*c - a*d, 0]$  && EqQ[ $h*j - g*k, 0]$  && IGtQ[ $m, 0]$

### Rule 2317

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[1 + (e\*x)/d]\*(a + b\*Log[c\*x^n])^p)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[1 + (e\*x)/d]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

### Rule 2374

Int[(Log[(d\_.)\*((e\_.) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 2445

Int[((a\_.) + Log[(c\_.)\*((d\_.)\*((e\_.) + (f\_.)\*(x\_)^(m\_.))^(n\_.)]\*(b\_.))^(p\_.)\*(u\_.), x\_Symbol] := Subst[Int[u\*(a + b\*Log[c\*d^n\*(e + f\*x)^(m\*n)]]^p, x], c\*d^n\*(e + f\*x)^(m\*n), c\*(d\*(e + f\*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[IntHide[u\*(a + b\*Log[c\*d^n\*(e + f\*x)^(m\*n)]]^p, x]

### Rubi steps

$$\begin{aligned}
\int \frac{\log\left(58(j(hx)^t)^u\right) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} dx &= \text{Subst}\left(\int \frac{\log\left(58j^u(hx)^{tu}\right) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} dx, 58j\right) \\
&= \text{Subst}\left(\text{Subst}\left(\int \frac{\log\left(58h^{tu}j^u x^{tu}\right) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} dx, h\right), j\right) \\
&= \frac{\log^2\left(58(j(hx)^t)^u\right) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2tu} - \text{Subst}\left(\text{Subst}\left(\int \frac{\log\left(58h^{tu}j^u x^{tu}\right) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} dx, h\right), j\right) \\
&= -\frac{pr \log^2\left(58(j(hx)^t)^u\right) \log\left(1 + \frac{bx}{a}\right)}{2tu} + \frac{\log^2\left(58(j(hx)^t)^u\right) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2tu} \\
&= -\frac{pr \log^2\left(58(j(hx)^t)^u\right) \log\left(1 + \frac{bx}{a}\right)}{2tu} + \frac{\log^2\left(58(j(hx)^t)^u\right) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2tu} \\
&= -\frac{pr \log^2\left(58(j(hx)^t)^u\right) \log\left(1 + \frac{bx}{a}\right)}{2tu} + \frac{\log^2\left(58(j(hx)^t)^u\right) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2tu}
\end{aligned}$$

**Mathematica [B]** time = 0.448249, size = 451, normalized size = 2.32

$$-pr \text{PolyLog}\left(2, -\frac{bx}{a}\right) \log\left(i(j(hx)^t)^u\right) + prt u \text{PolyLog}\left(3, -\frac{bx}{a}\right) - qr \text{PolyLog}\left(2, -\frac{dx}{c}\right) \log\left(i(j(hx)^t)^u\right) + qrt u \text{PolyLog}\left(3, -\frac{dx}{c}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Log[i\*(j\*(h\*x)^t)^u]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r])/x,x]

[Out] p\*r\*t\*u\*Log[x]\*Log[h\*x]\*Log[a + b\*x] - p\*r\*t\*u\*Log[h\*x]^2\*Log[a + b\*x] - p\*r\*Log[x]\*Log[i\*(j\*(h\*x)^t)^u]\*Log[a + b\*x] + p\*r\*Log[h\*x]\*Log[i\*(j\*(h\*x)^t)^u]\*Log[a + b\*x] + (p\*r\*t\*u\*Log[h\*x]^2\*Log[1 + (b\*x)/a])/2 - p\*r\*Log[h\*x]\*Log[i\*(j\*(h\*x)^t)^u]\*Log[1 + (b\*x)/a] + q\*r\*t\*u\*Log[x]\*Log[h\*x]\*Log[c + d\*x] - q\*r\*t\*u\*Log[h\*x]^2\*Log[c + d\*x] - q\*r\*Log[x]\*Log[i\*(j\*(h\*x)^t)^u]\*Log[c + d\*x] + q\*r\*Log[h\*x]\*Log[i\*(j\*(h\*x)^t)^u]\*Log[c + d\*x] - t\*u\*Log[x]\*Log[h\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r + (t\*u\*Log[h\*x]^2\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r])/2 + Log[x]\*Log[i\*(j\*(h\*x)^t)^u]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r + (q\*r\*t\*u\*Log[h\*x]^2\*Log[1 + (d\*x)/c])/2 - q\*r\*Log[h\*x]\*Log[i\*(j\*(h\*x)^t)^u]\*Log[1 + (d\*x)/c] - p\*r\*Log[i\*(j\*(h\*x)^t)^u]\*PolyLog[2, -(b\*x)/a] - q\*r\*Log[i\*(j\*(h\*x)^t)^u]\*PolyLog[2, -(d\*x)/c] + p\*r\*t\*u\*PolyL

og[3, -((b\*x)/a)] + q\*r\*t\*u\*PolyLog[3, -((d\*x)/c)]

**Maple [F]** time = 1.784, size = 0, normalized size = 0.

$$\int \frac{\ln\left(i\left(j(hx)^t\right)^u\right) \ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(i\*(j\*(h\*x)^t)^u)\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x)

[Out] int(ln(i\*(j\*(h\*x)^t)^u)\*ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(i\*(j\*(h\*x)^t)^u)\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x, algorithm="maxima")

[Out]  $-1/2*(t*u*\log(x)^2 - 2*(\log((h^t)^u) + \log(i) + \log(j^u))*\log(x) - 2*\log(x)*\log((x^t)^u))*\log(((b*x + a)^p)^r) - 1/2*(t*u*\log(x)^2 - 2*(\log((h^t)^u) + \log(i) + \log(j^u))*\log(x) - 2*\log(x)*\log((x^t)^u))*\log(((d*x + c)^q)^r) - \text{integrate}(-1/2*(2*((\log((h^t)^u) + \log(i) + \log(j^u))*\log(e) + (\log((h^t)^u) + \log(i) + \log(j^u))*\log(f^r))*b*d*x^2 + 2*((\log((h^t)^u) + \log(i) + \log(j^u))*\log(e) + (\log((h^t)^u) + \log(i) + \log(j^u))*\log(f^r))*a*c + ((p*r*t*u + q*r*t*u)*b*d*x^2 + (b*c*p*r*t*u + a*d*q*r*t*u)*x)*\log(x)^2 + 2*((\log((h^t)^u) + \log(i) + \log(j^u))*\log(e) + (\log((h^t)^u) + \log(i) + \log(j^u))*\log(f^r))*b*c + ((\log((h^t)^u) + \log(i) + \log(j^u))*\log(e) + (\log((h^t)^u) + \log(i) + \log(j^u))*\log(f^r))*a*d)*x - 2*((p*r + q*r)*\log((h^t)^u) + (p*r + q*r)*\log(i) + (p*r + q*r)*\log(j^u))*b*d*x^2 + ((p*r*\log((h^t)^u) + p*r*\log(i) + p*r*\log(j^u))*b*c + (q*r*\log((h^t)^u) + q*r*\log(i) + q*r*\log(j^u))*a*d)*x)*\log(x) + 2*(b*d*x^2*(\log(e) + \log(f^r)) + a*c*(\log(e) + \log(f^r)) + (b*c*(\log(e) + \log(f^r)) + a*d*(\log(e) + \log(f^r)))*x - ((p*r + q*r)*b*d*x^2 + (b*c*p*r + a*d*q*r)*x)*\log(x))*\log((x^t)^u)/(b*d*x^3 + a*c*x + (b*c + a*d)*x^2), x)$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right) \log \left( \left( (hx)^t j \right)^u i \right)}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(i\*(j\*(h\*x)^t)^u)\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x, algorithm="fricas")

[Out] integral(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)\*log(((h\*x)^t\*j)^u\*i)/x, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(i\*(j\*(h\*x)\*\*t)\*\*u)\*ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r)/x,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right) \log \left( \left( (hx)^t j \right)^u i \right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(i\*(j\*(h\*x)^t)^u)\*log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x, algorithm="giac")

[Out] integrate(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)\*log(((h\*x)^t\*j)^u\*i)/x, x)

$$3.59 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} dx$$

**Optimal.** Leaf size=81

$$-pr\text{PolyLog}\left(2, -\frac{bx}{a}\right) - qr\text{PolyLog}\left(2, -\frac{dx}{c}\right) + \log(x) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - pr \log(x) \log\left(\frac{bx}{a} + 1\right) - qr \log(x) \log\left(\frac{dx}{c} + 1\right)$$

[Out]  $-(p*r*\text{Log}[x]*\text{Log}[1 + (b*x)/a]) + \text{Log}[x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]$   
 $- q*r*\text{Log}[x]*\text{Log}[1 + (d*x)/c] - p*r*\text{PolyLog}[2, -((b*x)/a)] - q*r*\text{PolyLog}[2, -((d*x)/c)]$

**Rubi [A]** time = 0.0657538, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$ , Rules used = {2494, 2317, 2391}

$$-pr\text{PolyLog}\left(2, -\frac{bx}{a}\right) - qr\text{PolyLog}\left(2, -\frac{dx}{c}\right) + \log(x) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - pr \log(x) \log\left(\frac{bx}{a} + 1\right) - qr \log(x) \log\left(\frac{dx}{c} + 1\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/x, x]$

[Out]  $-(p*r*\text{Log}[x]*\text{Log}[1 + (b*x)/a]) + \text{Log}[x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]$   
 $- q*r*\text{Log}[x]*\text{Log}[1 + (d*x)/c] - p*r*\text{PolyLog}[2, -((b*x)/a)] - q*r*\text{PolyLog}[2, -((d*x)/c)]$

#### Rule 2494

$\text{Int}[\text{Log}[(e_*)*((f_*)((a_*) + (b_*)(x_))^{(p_*)}*((c_*) + (d_*)(x_))^{(q_*)})^{(r_*)}]/((g_*) + (h_*)(x_)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[g + h*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/h, x] + (-\text{Dist}[(b*p*r)/h, \text{Int}[\text{Log}[g + h*x]/(a + b*x), x], x] - \text{Dist}[(d*q*r)/h, \text{Int}[\text{Log}[g + h*x]/(c + d*x), x], x]) /;$  FreeQ[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 2317

$\text{Int}[(a_*) + \text{Log}[(c_*)(x_))^{(n_*)}*(b_*)^{(p_*)}]/((d_*) + (e_*)(x_)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[1 + (e*x)/d]*(a + b*\text{Log}[c*x^n])^p)/e, x] - \text{Dist}[(b*n*p)/e, \text{Int}[(\text{Log}[1 + (e*x)/d]*(a + b*\text{Log}[c*x^n])^{(p-1)})/x, x], x] /;$  FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

Rule 2391

```
Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} dx &= \log(x) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - (bpr) \int \frac{\log(x)}{a+bx} dx - (dqr) \int \frac{\log(x)}{c+dx} dx \\ &= -pr \log(x) \log\left(1 + \frac{bx}{a}\right) + \log(x) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - qr \log(x) \log\left(1 + \frac{dx}{c}\right) \\ &= -pr \log(x) \log\left(1 + \frac{bx}{a}\right) + \log(x) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - qr \log(x) \log\left(1 + \frac{dx}{c}\right) \end{aligned}$$

**Mathematica [A]** time = 0.0630908, size = 78, normalized size = 0.96

$$-pr \text{PolyLog}\left(2, -\frac{bx}{a}\right) - qr \text{PolyLog}\left(2, -\frac{dx}{c}\right) + \log(x) \left( \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - pr \log\left(\frac{bx}{a} + 1\right) - qr \log\left(\frac{dx}{c} + 1\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/x,x]
```

```
[Out] Log[x]*(-(p*r*Log[1 + (b*x)/a]) + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - q*r*Log[1 + (d*x)/c]) - p*r*PolyLog[2, -((b*x)/a)] - q*r*PolyLog[2, -((d*x)/c)]
```

**Maple [F]** time = 0.123, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

---

**Maxima [A]** time = 1.20753, size = 170, normalized size = 2.1

$$-\frac{(fp \log(bx + a) + fq \log(dx + c))r \log(x)}{f} + \log\left(\left((bx + a)^p(dx + c)^q f\right)^r e\right) \log(x) + \frac{\left(\log(bx + a) \log\left(-\frac{bx+a}{a} + 1\right) + \operatorname{dilog}\left(\frac{bx+a}{a}\right)\right) f^p + \left(\log(dx + c) \log\left(-\frac{dx+c}{c} + 1\right) + \operatorname{dilog}\left(\frac{dx+c}{c}\right)\right) f^q}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x, algorithm="maxima")

[Out] -(f\*p\*log(b\*x + a) + f\*q\*log(d\*x + c))\*r\*log(x)/f + log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)\*log(x) + ((log(b\*x + a)\*log(-(b\*x + a)/a + 1) + dilog((b\*x + a)/a))\*f\*p + (log(d\*x + c)\*log(-(d\*x + c)/c + 1) + dilog((d\*x + c)/c))\*f\*q)\*r/f

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\log\left(\left((bx + a)^p(dx + c)^q f\right)^r e\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x,x, algorithm="fricas")

[Out] integral(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)/x, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r)/x,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/x, x)
```



$$3.60 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x \log\left(i\left(j(hx)^t\right)^u\right)} dx$$

**Optimal.** Leaf size=41

$$\text{CannotIntegrate}\left[\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x \log\left(i\left(j(hx)^t\right)^u\right)}, x\right]$$

[Out] CannotIntegrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(x\*Log[i\*(j\*(h\*x)^t)^u]), x]

**Rubi [A]** time = 0.544652, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x \log\left(i\left(j(hx)^t\right)^u\right)} dx$$

Verification is Not applicable to the result.

[In] Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(x\*Log[i\*(j\*(h\*x)^t)^u]), x]

[Out] Defer[Int][Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(x\*Log[i\*(j\*(h\*x)^t)^u]), x]

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x \log\left(60\left(j(hx)^t\right)^u\right)} dx = \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x \log\left(60\left(j(hx)^t\right)^u\right)} dx$$

**Mathematica [A]** time = 0.444816, size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x \log\left(i\left(j(hx)^t\right)^u\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(x\*Log[i\*(j\*(h\*x)^t)^u]),x]

[Out] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(x\*Log[i\*(j\*(h\*x)^t)^u]), x]

**Maple [A]** time = 1.772, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{x \ln\left(i\left(j(hx)^t\right)^u\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x/ln(i\*(j\*(h\*x)^t)^u),x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x/ln(i\*(j\*(h\*x)^t)^u),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x \log\left(\left((hx)^t j\right)^u i\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x/log(i\*(j\*(h\*x)^t)^u),x, algorithm="maxima")

[Out] integrate(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)/(x\*log(((h\*x)^t\*j)^u\*i)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x \log\left(\left((hx)^t j\right)^u i\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x/ln(i*(j*(h*x)**t)**u),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x \log\left(\left((hx)^t j\right)^u i\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)), x)
```

$$3.61 \quad \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x \log^2\left(i\left(j(hx)^t\right)^u\right)} dx$$

**Optimal.** Leaf size=41

$$\text{CannotIntegrate}\left(\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x \log^2\left(i\left(j(hx)^t\right)^u\right)}, x\right)$$

[Out] CannotIntegrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(x\*Log[i\*(j\*(h\*x)^t)^u]^2), x]

**Rubi [A]** time = 0.42531, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x \log^2\left(i\left(j(hx)^t\right)^u\right)} dx$$

Verification is Not applicable to the result.

[In] Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(x\*Log[i\*(j\*(h\*x)^t)^u]^2), x]

[Out] Defer[Int][Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(x\*Log[i\*(j\*(h\*x)^t)^u]^2), x]

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x \log^2\left(61\left(j(hx)^t\right)^u\right)} dx = \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x \log^2\left(61\left(j(hx)^t\right)^u\right)} dx$$

**Mathematica [A]** time = 2.46198, size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x \log^2\left(i\left(j(hx)^t\right)^u\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(x\*Log[i\*(j\*(h\*x)^t)^u]^2), x]

[Out] Integrate[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]/(x\*Log[i\*(j\*(h\*x)^t)^u]^2), x]

**Maple [A]** time = 1.78, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{x\left(\ln\left(i\left(j(hx)^t\right)^u\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x/ln(i\*(j\*(h\*x)^t)^u)^2,x)

[Out] int(ln(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x/ln(i\*(j\*(h\*x)^t)^u)^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{\log\left(\left((bx+a)^p\right)^r\right) + \log\left(\left((dx+c)^q\right)^r\right) + \log(e) + \log(f^r)}{tu \log\left((h^t)^u\right) + tu \log(i) + tu \log(j^u) + tu \log(x^t)^u} + \int \frac{b dx^2 + (tu \log\left((h^t)^u\right) + tu \log(i) + tu \log(j^u))}{(tu \log\left((h^t)^u\right) + tu \log(i) + tu \log(j^u))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x/log(i\*(j\*(h\*x)^t)^u)^2,x, algorithm="maxima")

[Out] -(log(((b\*x + a)^p)^r) + log(((d\*x + c)^q)^r) + log(e) + log(f^r))/(t\*u\*log((h^t)^u) + t\*u\*log(i) + t\*u\*log(j^u) + t\*u\*log((x^t)^u)) + integrate((b\*c\*p\*r + a\*d\*q\*r + (p\*r + q\*r)\*b\*d\*x)/((t\*u\*log((h^t)^u) + t\*u\*log(i) + t\*u\*log(j^u))\*b\*d\*x^2 + (t\*u\*log((h^t)^u) + t\*u\*log(i) + t\*u\*log(j^u))\*a\*c + ((t\*u\*log((h^t)^u) + t\*u\*log(i) + t\*u\*log(j^u))\*b\*c + (t\*u\*log((h^t)^u) + t\*u\*log(i) + t\*u\*log(j^u))\*a\*d)\*x + (b\*d\*t\*u\*x^2 + a\*c\*t\*u + (b\*c\*t\*u + a\*d\*t\*u)\*x)\*log((x^t)^u), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)}{x \log \left( \left( (hx)^t j \right)^u i \right)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x/log(i\*(j\*(h\*x)^t)^u)^2,x, algorithm="fricas")

[Out] integral(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)/(x\*log(((h\*x)^t\*j)^u\*i)^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*(f\*(b\*x+a)\*\*p\*(d\*x+c)\*\*q)\*\*r)/x/ln(i\*(j\*(h\*x)\*\*t)\*\*u)\*\*2,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( \left( (bx + a)^p (dx + c)^q f \right)^r e \right)}{x \log \left( \left( (hx)^t j \right)^u i \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*(f\*(b\*x+a)^p\*(d\*x+c)^q)^r)/x/log(i\*(j\*(h\*x)^t)^u)^2,x, algorithm="giac")

[Out] integrate(log(((b\*x + a)^p\*(d\*x + c)^q\*f)^r\*e)/(x\*log(((h\*x)^t\*j)^u\*i)^2), x)

$$3.62 \quad \int \frac{\log(x) \log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable}\left(\frac{\log(x) \log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}, x\right)$$

[Out] Unintegrable[(Log[x]\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^3)/x, x]

**Rubi [A]** time = 0.021354, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\log(x) \log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(Log[x]\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^3)/x, x]

[Out] Defer[Int] [(Log[x]\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^3)/x, x]

Rubi steps

$$\int \frac{\log(x) \log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx = \int \frac{\log(x) \log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx$$

**Mathematica [A]** time = 5.09395, size = 0, normalized size = 0.

$$\int \frac{\log(x) \log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Log[x]\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^3)/x, x]

[Out] Integrate[(Log[x]\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^3)/x, x]

**Maple [A]** time = 0.385, size = 0, normalized size = 0.

$$\int \frac{\ln(x)}{x} \left( \ln \left( \frac{bx+a}{(-ad+bc)x} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(x)\*ln((b\*x+a)/(-a\*d+b\*c)/x)^3/x,x)

[Out] int(ln(x)\*ln((b\*x+a)/(-a\*d+b\*c)/x)^3/x,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} \log(bx+a)^3 \log(x)^2 - \int \frac{2(bx+a) \log(x)^4 + 6(bx \log(bc-ad) + a \log(bc-ad)) \log(x)^3 + 3((3bx+2a) \log(x)^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)\*log((b\*x+a)/(-a\*d+b\*c)/x)^3/x,x, algorithm="maxima")

[Out] 1/2\*log(b\*x + a)^3\*log(x)^2 - integrate(1/2\*(2\*(b\*x + a)\*log(x)^4 + 6\*(b\*x\*log(b\*c - a\*d) + a\*log(b\*c - a\*d))\*log(x)^3 + 3\*((3\*b\*x + 2\*a)\*log(x)^2 + 2\*(b\*x\*log(b\*c - a\*d) + a\*log(b\*c - a\*d))\*log(x))\*log(b\*x + a)^2 + 6\*(b\*x\*log(b\*c - a\*d)^2 + a\*log(b\*c - a\*d)^2)\*log(x)^2 - 6\*((b\*x + a)\*log(x)^3 + 2\*(b\*x\*log(b\*c - a\*d) + a\*log(b\*c - a\*d))\*log(x)^2 + (b\*x\*log(b\*c - a\*d)^2 + a\*log(b\*c - a\*d)^2)\*log(x))\*log(b\*x + a) + 2\*(b\*x\*log(b\*c - a\*d)^3 + a\*log(b\*c - a\*d)^3)\*log(x))/(b\*x^2 + a\*x), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log(x) \log \left( \frac{bx+a}{(bc-ad)x} \right)^3}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="fricas")
```

```
[Out] integral(log(x)*log((b*x + a)/((b*c - a*d)*x))^3/x, x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\frac{3a \int \frac{\log(x)^2 \log\left(\frac{a}{-adx+bcx} + \frac{bx}{-adx+bcx}\right)^2}{ax+bx^2} dx}{2} + \frac{\log(x)^2 \log\left(\frac{a+bx}{x(-ad+bc)}\right)^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)**3/x,x)
```

```
[Out] 3*a*Integral(log(x)**2*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))**2/(a*x + b*x**2), x)/2 + log(x)**2*log((a + b*x)/(x*(-a*d + b*c)))*3/2
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x) \log\left(\frac{bx+a}{(bc-ad)x}\right)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="giac")
```

```
[Out] integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))^3/x, x)
```

$$3.63 \quad \int \frac{\log(x) \log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable}\left(\frac{\log(x) \log^2\left(\frac{a+bx}{x(bc-ad)}\right)}{x}, x\right)$$

[Out] Unintegrable[(Log[x]\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^2)/x, x]

**Rubi [A]** time = 0.0213293, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\log(x) \log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(Log[x]\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^2)/x, x]

[Out] Defer[Int] [(Log[x]\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^2)/x, x]

Rubi steps

$$\int \frac{\log(x) \log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx = \int \frac{\log(x) \log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx$$

**Mathematica [A]** time = 5.08442, size = 0, normalized size = 0.

$$\int \frac{\log(x) \log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Log[x]\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^2)/x, x]

[Out] Integrate[(Log[x]\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^2)/x, x]

**Maple [A]** time = 0.209, size = 0, normalized size = 0.

$$\int \frac{\ln(x)}{x} \left( \ln \left( \frac{bx+a}{(-ad+bc)x} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(x)\*ln((b\*x+a)/(-a\*d+b\*c)/x)^2/x,x)

[Out] int(ln(x)\*ln((b\*x+a)/(-a\*d+b\*c)/x)^2/x,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} \log(bx+a)^2 \log(x)^2 - \int -\frac{(bx+a) \log(x)^3 + 2(bx \log(bc-ad) + a \log(bc-ad)) \log(x)^2 - ((3bx+2a) \log(x)^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)\*log((b\*x+a)/(-a\*d+b\*c)/x)^2/x,x, algorithm="maxima")

[Out] 1/2\*log(b\*x + a)^2\*log(x)^2 - integrate(-((b\*x + a)\*log(x)^3 + 2\*(b\*x\*log(b\*c - a\*d) + a\*log(b\*c - a\*d))\*log(x)^2 - ((3\*b\*x + 2\*a)\*log(x)^2 + 2\*(b\*x\*log(b\*c - a\*d) + a\*log(b\*c - a\*d))\*log(x))\*log(b\*x + a) + (b\*x\*log(b\*c - a\*d))^2 + a\*log(b\*c - a\*d)^2)\*log(x))/(b\*x^2 + a\*x), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log(x) \log \left( \frac{bx+a}{(bc-ad)x} \right)^2}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)\*log((b\*x+a)/(-a\*d+b\*c)/x)^2/x,x, algorithm="fricas")

[Out] `integral(log(x)*log((b*x + a)/((b*c - a*d)*x))^2/x, x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$a \int \frac{\log(x)^2 \log\left(\frac{a}{-adx+bcx} + \frac{bx}{-adx+bcx}\right)}{ax + bx^2} dx + \frac{\log(x)^2 \log\left(\frac{a+bx}{x(-ad+bc)}\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)**2/x,x)`

[Out] `a*Integral(log(x)**2*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))/(a*x + b*x**2), x) + log(x)**2*log((a + b*x)/(x*(-a*d + b*c)))**2/2`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x) \log\left(\frac{bx+a}{(bc-ad)x}\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="giac")`

[Out] `integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))^2/x, x)`

$$3.64 \quad \int \frac{\log(x) \log\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx$$

**Optimal.** Leaf size=82

$$\text{PolyLog}\left(3, -\frac{a}{bx}\right) + \log(x)\text{PolyLog}\left(2, -\frac{a}{bx}\right) + \frac{1}{2} \log^2(x) \log\left(\frac{a}{x(bc-ad)} + \frac{b}{bc-ad}\right) - \frac{1}{2} \log^2(x) \log\left(\frac{a}{bx} + 1\right)$$

[Out]  $-(\text{Log}[1 + a/(b*x)]*\text{Log}[x]^2)/2 + (\text{Log}[b/(b*c - a*d) + a/((b*c - a*d)*x)]*\text{Log}[x]^2)/2 + \text{Log}[x]*\text{PolyLog}[2, -(a/(b*x))] + \text{PolyLog}[3, -(a/(b*x))]$

**Rubi [A]** time = 0.176511, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2380, 2375, 2337, 2374, 6589}

$$\text{PolyLog}\left(3, -\frac{a}{bx}\right) + \log(x)\text{PolyLog}\left(2, -\frac{a}{bx}\right) + \frac{1}{2} \log^2(x) \log\left(\frac{a}{x(bc-ad)} + \frac{b}{bc-ad}\right) - \frac{1}{2} \log^2(x) \log\left(\frac{a}{bx} + 1\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Log}[x]*\text{Log}[(a + b*x)/((b*c - a*d)*x)])]/x, x]$

[Out]  $-(\text{Log}[1 + a/(b*x)]*\text{Log}[x]^2)/2 + (\text{Log}[b/(b*c - a*d) + a/((b*c - a*d)*x)]*\text{Log}[x]^2)/2 + \text{Log}[x]*\text{PolyLog}[2, -(a/(b*x))] + \text{PolyLog}[3, -(a/(b*x))]$

### Rule 2380

$\text{Int}[\text{Log}[(d_*)*(u_)^{(r_*)}]*((a_*) + \text{Log}[(c_*)*(x_)^{(n_*)}])*(b_*)^{(p_*)}*((g_*)*(x_)^{(q_*)}), x\_Symbol] \rightarrow \text{Int}[(g*x)^q*\text{Log}[d*\text{ExpandToSum}[u, x]^r]*(a + b*\text{Log}[c*x^n])^p, x] /;$  FreeQ[{a, b, c, d, g, r, n, p, q}, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

### Rule 2375

$\text{Int}[(\text{Log}[(d_*)*((e_*) + (f_*)*(x_)^{(m_*)})^{(r_*)}])*(a_*) + \text{Log}[(c_*)*(x_)^{(n_*)}])*(b_*)^{(p_*)}/(x_), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[d*(e + f*x^m)^r]*(a + b*\text{Log}[c*x^n])^{(p+1)})/(b*n*(p+1)), x] - \text{Dist}[(f*m*r)/(b*n*(p+1)), \text{Int}[(x^{(m-1)}*(a + b*\text{Log}[c*x^n])^{(p+1)})/(e + f*x^m), x], x] /;$  FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d\*e, 1]

### Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_)^(m_.)))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

### Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d*e, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned} \int \frac{\log(x) \log\left(\frac{a+bx}{(bc-ad)x}\right)}{x} dx &= \int \frac{\log\left(\frac{b}{bc-ad} + \frac{a}{(bc-ad)x}\right) \log(x)}{x} dx \\ &= \frac{1}{2} \log\left(\frac{b}{bc-ad} + \frac{a}{(bc-ad)x}\right) \log^2(x) + \frac{a \int \frac{\log^2(x)}{\left(\frac{b}{bc-ad} + \frac{a}{(bc-ad)x}\right)x^2} dx}{2(bc-ad)} \\ &= -\frac{1}{2} \log\left(1 + \frac{a}{bx}\right) \log^2(x) + \frac{1}{2} \log\left(\frac{b}{bc-ad} + \frac{a}{(bc-ad)x}\right) \log^2(x) + \int \frac{\log\left(1 + \frac{a}{bx}\right) \log(x)}{x} dx \\ &= -\frac{1}{2} \log\left(1 + \frac{a}{bx}\right) \log^2(x) + \frac{1}{2} \log\left(\frac{b}{bc-ad} + \frac{a}{(bc-ad)x}\right) \log^2(x) + \log(x) \text{Li}_2\left(-\frac{a}{bx}\right) - \int \frac{\text{Li}_2\left(-\frac{a}{bx}\right)}{x} dx \\ &= -\frac{1}{2} \log\left(1 + \frac{a}{bx}\right) \log^2(x) + \frac{1}{2} \log\left(\frac{b}{bc-ad} + \frac{a}{(bc-ad)x}\right) \log^2(x) + \log(x) \text{Li}_2\left(-\frac{a}{bx}\right) + \text{Li}_3\left(-\frac{a}{bx}\right) \end{aligned}$$

**Mathematica [A]** time = 5.02716, size = 66, normalized size = 0.8

$$\text{PolyLog}\left(3, -\frac{bx}{a}\right) - \log(x) \text{PolyLog}\left(2, -\frac{bx}{a}\right) + \frac{1}{6} \log^2(x) \left(3 \log\left(\frac{a+bx}{bcx-adx}\right) - 3 \log\left(\frac{bx}{a} + 1\right) + \log(x)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Log[x]\*Log[(a + b\*x)/((b\*c - a\*d)\*x)])/x,x]

[Out] (Log[x]^2\*(Log[x] - 3\*Log[1 + (b\*x)/a] + 3\*Log[(a + b\*x)/(b\*c\*x - a\*d\*x)]))/6 - Log[x]\*PolyLog[2, -((b\*x)/a)] + PolyLog[3, -((b\*x)/a)]

**Maple [C]** time = 0.121, size = 450, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(x)\*ln((b\*x+a)/(-a\*d+b\*c)/x)/x,x)

[Out]  $\frac{1}{2} \ln(x)^2 \ln(b*x+a) - \frac{1}{3} \ln(x)^3 - \frac{1}{4} I \ln(x)^2 \operatorname{csgn}(I/x) \operatorname{csgn}(I*(b*x+a)/(a*d-b*c)) \operatorname{csgn}(I/x*(b*x+a)/(a*d-b*c)) + \frac{1}{2} I \ln(x)^2 \operatorname{csgn}(I*(b*x+a)/(a*d-b*c)) \operatorname{csgn}(I*(b*x+a)/(a*d-b*c))^{-2} - \frac{1}{4} I \ln(x)^2 \operatorname{csgn}(I*(b*x+a)/(a*d-b*c)) \operatorname{csgn}(I/x*(b*x+a)/(a*d-b*c))^{-3} + \frac{1}{4} I \ln(x)^2 \operatorname{csgn}(I*(b*x+a)/(a*d-b*c)) \operatorname{csgn}(I/x*(b*x+a)/(a*d-b*c))^{-2} + \frac{1}{4} I \ln(x)^2 \operatorname{csgn}(I/x*(b*x+a)/(a*d-b*c))^{-3} - \frac{1}{2} I \ln(x)^2 \operatorname{csgn}(I/x*(b*x+a)/(a*d-b*c))^{-2} - \frac{1}{4} I \ln(x)^2 \operatorname{csgn}(I*(b*x+a)/(a*d-b*c)) \operatorname{csgn}(I/(a*d-b*c)) \operatorname{csgn}(I*(b*x+a)/(a*d-b*c)) + \frac{1}{4} I \ln(x)^2 \operatorname{csgn}(I/x) \operatorname{csgn}(I/x*(b*x+a)/(a*d-b*c))^{-2} + \frac{1}{4} I \ln(x)^2 \operatorname{csgn}(I/(a*d-b*c)) \operatorname{csgn}(I*(b*x+a)/(a*d-b*c))^{-2} - \frac{1}{2} \ln(x)^2 \ln(a*d-b*c) - \frac{1}{2} \ln(x)^2 \ln(1+x*b/a) - \ln(x) \operatorname{polylog}(2, -x*b/a) + \operatorname{polylog}(3, -x*b/a)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)\*log((b\*x+a)/(-a\*d+b\*c)/x)/x,x, algorithm="maxima")

[Out] Exception raised: TypeError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)\*log((b\*x+a)/(-a\*d+b\*c)/x)/x,x, algorithm="fricas")

[Out] integral(log(x)\*log((b\*x + a)/((b\*c - a\*d)\*x))/x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a \int \frac{\log(x)^2}{ax+bx^2} dx}{2} + \frac{\log(x)^2 \log\left(\frac{a+bx}{x(-ad+bc)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(x)\*ln((b\*x+a)/(-a\*d+b\*c)/x)/x,x)

[Out] a\*Integral(log(x)\*\*2/(a\*x + b\*x\*\*2), x)/2 + log(x)\*\*2\*log((a + b\*x)/(x\*(-a\*d + b\*c)))/2

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)\*log((b\*x+a)/(-a\*d+b\*c)/x)/x,x, algorithm="giac")

[Out] integrate(log(x)\*log((b\*x + a)/((b\*c - a\*d)\*x))/x, x)



$$3.65 \quad \int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable}\left(\frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)}, x\right)$$

[Out] Unintegrable[Log[x]/(x\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]), x]

**Rubi [A]** time = 0.0216231, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[Log[x]/(x\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]), x]

[Out] Defer[Int][Log[x]/(x\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]), x]

Rubi steps

$$\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} dx = \int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

**Mathematica [A]** time = 5.0975, size = 0, normalized size = 0.

$$\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[Log[x]/(x\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]),x]

[Out] Integrate[Log[x]/(x\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]), x]

**Maple [A]** time = 0.089, size = 0, normalized size = 0.

$$\int \frac{\ln(x)}{x} \left( \ln \left( \frac{bx+a}{(-ad+bc)x} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(x)/x/ln((b\*x+a)/(-a\*d+b\*c)/x),x)

[Out] int(ln(x)/x/ln((b\*x+a)/(-a\*d+b\*c)/x),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)}{x \log\left(\frac{bx+a}{(bc-ad)x}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)/x/log((b\*x+a)/(-a\*d+b\*c)/x),x, algorithm="maxima")

[Out] integrate(log(x)/(x\*log((b\*x + a)/((b\*c - a\*d)\*x))), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log(x)}{x \log\left(\frac{bx+a}{(bc-ad)x}\right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)/x/log((b\*x+a)/(-a\*d+b\*c)/x),x, algorithm="fricas")

[Out] `integral(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)}{x \log\left(\frac{a}{-adx+bcx} + \frac{bx}{-adx+bcx}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x), x)`

[Out] `Integral(log(x)/(x*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))), x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)}{x \log\left(\frac{bx+a}{(bc-ad)x}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x), x, algorithm="giac")`

[Out] `integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)`

$$3.66 \quad \int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable}\left(\frac{\log(x)}{x \log^2\left(\frac{a+bx}{x(bc-ad)}\right)}, x\right)$$

[Out] Unintegrable[Log[x]/(x\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^2), x]

**Rubi [A]** time = 0.0219153, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[Log[x]/(x\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^2), x]

[Out] Defer[Int][Log[x]/(x\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^2), x]

Rubi steps

$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx = \int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

**Mathematica [A]** time = 33.1028, size = 0, normalized size = 0.

$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[Log[x]/(x\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^2), x]

[Out] Integrate[Log[x]/(x\*Log[(a + b\*x)/((b\*c - a\*d)\*x)]^2), x]

**Maple [A]** time = 0.327, size = 0, normalized size = 0.

$$\int \frac{\ln(x)}{x} \left( \ln \left( \frac{bx + a}{(-ad + bc)x} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(x)/x/ln((b\*x+a)/(-a\*d+b\*c)/x)^2, x)

[Out] int(ln(x)/x/ln((b\*x+a)/(-a\*d+b\*c)/x)^2, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-\frac{(bx + a) \log(x)}{a \log(bc - ad) - a \log(bx + a) + a \log(x)} - \int -\frac{bx \log(x) + bx + a}{ax \log(bc - ad) - ax \log(bx + a) + ax \log(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)/x/log((b\*x+a)/(-a\*d+b\*c)/x)^2, x, algorithm="maxima")

[Out] -(b\*x + a)\*log(x)/(a\*log(b\*c - a\*d) - a\*log(b\*x + a) + a\*log(x)) - integrate(-(b\*x\*log(x) + b\*x + a)/(a\*x\*log(b\*c - a\*d) - a\*x\*log(b\*x + a) + a\*x\*log(x)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log(x)}{x \log \left( \frac{bx+a}{(bc-ad)x} \right)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="fricas")
```

```
[Out] integral(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))^2), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\frac{a \log(x) + bx \log(x)}{a \log\left(\frac{a+bx}{x(-ad+bc)}\right)} - \frac{\int \frac{b}{\log\left(\frac{a}{-adx+bcx} + \frac{bx}{-adx+bcx}\right)} dx + \int \frac{a}{x \log\left(\frac{a}{-adx+bcx} + \frac{bx}{-adx+bcx}\right)} dx + \int \frac{b \log(x)}{\log\left(\frac{a}{-adx+bcx} + \frac{bx}{-adx+bcx}\right)} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)**2,x)
```

```
[Out] (a*log(x) + b*x*log(x))/(a*log((a + b*x)/(x*(-a*d + b*c)))) - (Integral(b/1
og(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x)), x) + Integral(a/(x*log(a/(-a
*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))), x) + Integral(b*log(x)/log(a/(-a*d*
x + b*c*x) + b*x/(-a*d*x + b*c*x))), x))/a
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)}{x \log\left(\frac{bx+a}{(bc-ad)x}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="giac")
```

```
[Out] integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))^2), x)
```

$$3.67 \quad \int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(h(f+gx)^m)}{(a+bx)(c+dx)} dx$$

**Optimal.** Leaf size=620

$$\frac{6mn^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{PolyLog}\left(4, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad} - \frac{m \log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad} + \frac{3mn \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad}$$

[Out] (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^4\*Log[(b\*c - a\*d)/(b\*(c + d\*x))])/(4\*(b\*c - a\*d)\*n) + (Log[e\*((a + b\*x)/(c + d\*x))^n]^4\*Log[h\*(f + g\*x)^m])/(4\*(b\*c - a\*d)\*n) - (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^4\*Log[1 - ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/(4\*(b\*c - a\*d)\*n) + (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^3\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*c - a\*d) - (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^3\*PolyLog[2, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/(b\*c - a\*d) - (3\*m\*n\*Log[e\*((a + b\*x)/(c + d\*x))^n]^2\*PolyLog[3, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*c - a\*d) + (3\*m\*n\*Log[e\*((a + b\*x)/(c + d\*x))^n]^2\*PolyLog[3, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/(b\*c - a\*d) + (6\*m\*n^2\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*PolyLog[4, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*c - a\*d) - (6\*m\*n^2\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*PolyLog[4, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/(b\*c - a\*d) - (6\*m\*n^3\*PolyLog[5, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*c - a\*d) + (6\*m\*n^3\*PolyLog[5, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/(b\*c - a\*d)

**Rubi [A]** time = 1.13263, antiderivative size = 649, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {2507, 2489, 2488, 2506, 2508, 6610, 2503}

$$\frac{6mn^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{PolyLog}\left(4, 1 - \frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad} - \frac{m \log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{PolyLog}\left(2, 1 - \frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad} + \frac{3mn \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad}$$

Antiderivative was successfully verified.

[In] Int[(Log[e\*((a + b\*x)/(c + d\*x))^n]^3\*Log[h\*(f + g\*x)^m])/((a + b\*x)\*(c + d\*x)), x]

[Out] (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^4\*Log[(b\*c - a\*d)/(b\*(c + d\*x))])/(4\*(b\*c - a\*d)\*n) - (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^4\*Log[((b\*c - a\*d)\*(f + g\*x))/((b\*f - a\*g)\*(c + d\*x))])/(4\*(b\*c - a\*d)\*n) + (Log[e\*((a + b\*x)/(c + d\*x))^n]^4\*Log[h\*(f + g\*x)^m])/(4\*(b\*c - a\*d)\*n) + (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^3\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*c - a\*d) - (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^3\*PolyLog[2, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/(b\*c - a\*d) - (3\*m\*n\*Log[e\*((a + b\*x)/(c + d\*x))^n]^2\*PolyLog[3, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*c - a\*d) + (3\*m\*n\*Log[e\*((a + b\*x)/(c + d\*x))^n]^2\*PolyLog[3, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/(b\*c - a\*d) + (6\*m\*n^2\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*PolyLog[4, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*c - a\*d) - (6\*m\*n^2\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*PolyLog[4, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/(b\*c - a\*d) - (6\*m\*n^3\*PolyLog[5, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*c - a\*d) + (6\*m\*n^3\*PolyLog[5, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/(b\*c - a\*d)

```

))n]3*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))]/(b*c - a*d) - (m*Log[e*(
(a + b*x)/(c + d*x))n]3*PolyLog[2, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*
g)*(c + d*x))]/(b*c - a*d) - (3*m*n*Log[e*((a + b*x)/(c + d*x))n]2*PolyL
og[3, 1 - (b*c - a*d)/(b*(c + d*x))]/(b*c - a*d) + (3*m*n*Log[e*((a + b*x)
/(c + d*x))n]2*PolyLog[3, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d
*x))]/(b*c - a*d) + (6*m*n2*Log[e*((a + b*x)/(c + d*x))n]*PolyLog[4, 1 -
(b*c - a*d)/(b*(c + d*x))]/(b*c - a*d) - (6*m*n2*Log[e*((a + b*x)/(c + d
*x))n]*PolyLog[4, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))]/(b
*c - a*d) - (6*m*n3*PolyLog[5, 1 - (b*c - a*d)/(b*(c + d*x))]/(b*c - a*d)
+ (6*m*n3*PolyLog[5, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))
]/(b*c - a*d)

```

### Rule 2507

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))(p_.)*((c_.) + (d_.)*(x_))(q_.)
(r_.)](s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))(t_.)](u_.)]*(v_), x_S
ymbol] :=> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[(k*Log[i*(j*(g +
h*x)t]u)*Log[e*(f*(a + b*x)p(c + d*x)q]r](s + 1)]/(p*r*(s + 1)*(b*c
- a*d)), x] - Dist[(k*h*t*u)/(p*r*(s + 1)*(b*c - a*d)), Int[Log[e*(f*(a +
b*x)p(c + d*x)q]r](s + 1)/(g + h*x), x], x] /; FreeQ[k, x] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
EqQ[p + q, 0] && NeQ[s, -1]

```

### Rule 2489

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))(p_.)*((c_.) + (d_.)*(x_))(q_.)
(r_.)](s_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :=> Dist[d/h, Int[Log[e*(f*(a +
b*x)p(c + d*x)q]r]s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)p(c + d*x)q]r]s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]

```

### Rule 2488

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))(p_.)*((c_.) + (d_.)*(x_))(q_.)
(r_.)](s_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :=> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)p(c + d*x)q]r]s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)p
*(c + d*x)q]r](s - 1)]/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]

```

### Rule 2506

```

Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))(p_.)*((c_.) + (d_.)*(x_)

```



```
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] := With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

### Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] := With[{g = Simplify[(v*(c +
d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] - Dist[h*p*r
*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]
```

### Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] := With[{g = Coeff[Simplify[1/(u*(a + b*x))],
x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q]^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
))))]/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x))))]/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(h(f+gx)^m)}{(a+bx)(c+dx)} dx &= \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(h(f+gx)^m)}{4(bc-ad)n} - \frac{(gm) \int \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx} dx}{4(bc-ad)n} \\
&= \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(h(f+gx)^m)}{4(bc-ad)n} - \frac{(dm) \int \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} dx}{4(bc-ad)n} + \frac{((df-cg)m) \int}{4(bc-ad)n} \\
&= \frac{m \log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m \log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n} + \frac{\log^4}{4(bc-ad)n} \\
&= \frac{m \log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m \log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n} + \frac{\log^4}{4(bc-ad)n} \\
&= \frac{m \log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m \log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n} + \frac{\log^4}{4(bc-ad)n} \\
&= \frac{m \log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m \log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n} + \frac{\log^4}{4(bc-ad)n} \\
&= \frac{m \log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m \log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n} + \frac{\log^4}{4(bc-ad)n}
\end{aligned}$$

**Mathematica [B]** time = 21.5443, size = 18164, normalized size = 29.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/((a + b*x)*(c + d*x)),x]
```

[Out] Result too large to show

**Maple [F]** time = 7.167, size = 0, normalized size = 0.

$$\int \frac{\ln\left(h(gx+f)^m\right)}{(bx+a)(dx+c)} \left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="maxima")
```

```
[Out] -1/4*(n^3*log(b*x + a)^4 + n^3*log(d*x + c)^4 - 4*n^2*log(b*x + a)^3*log(e)
+ 6*n*log(b*x + a)^2*log(e)^2 - 4*(n^3*log(b*x + a) - n^2*log(e))*log(d*x
+ c)^3 - 4*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n)^3 + 4*(log(b*x +
a) - log(d*x + c))*log((d*x + c)^n)^3 - 4*log(b*x + a)*log(e)^3 + 6*(n^3*lo
g(b*x + a)^2 - 2*n^2*log(b*x + a)*log(e) + n*log(e)^2)*log(d*x + c)^2 + 6*(
n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))*log(d*x +
c) - 2*log(b*x + a)*log(e))*log((b*x + a)^n)^2 + 6*(n*log(b*x + a)^2 + n*log
(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))*log(d*x + c) - 2*(log(b*x + a)
- log(d*x + c))*log((b*x + a)^n) - 2*log(b*x + a)*log(e))*log((d*x + c)^n)^
2 - 4*(n^3*log(b*x + a)^3 - 3*n^2*log(b*x + a)^2*log(e) + 3*n*log(b*x + a)*
log(e)^2 - log(e)^3)*log(d*x + c) - 4*(n^2*log(b*x + a)^3 - n^2*log(d*x + c
)^3 - 3*n*log(b*x + a)^2*log(e) + 3*(n^2*log(b*x + a) - n*log(e))*log(d*x +
c)^2 + 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*log(b*x + a)*
log(e) + log(e)^2)*log(d*x + c))*log((b*x + a)^n) + 4*(n^2*log(b*x + a)^3 -
n^2*log(d*x + c)^3 - 3*n*log(b*x + a)^2*log(e) + 3*(n^2*log(b*x + a) - n*log
(e))*log(d*x + c)^2 + 3*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n)^2
+ 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*log(b*x + a)*log(e)
+ log(e)^2)*log(d*x + c) - 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log
(b*x + a) - log(e))*log(d*x + c) - 2*log(b*x + a)*log(e))*log((b*x + a)^n
))*log((d*x + c)^n))*log((g*x + f)^m)/(b*c - a*d) + integrate(1/4*(4*b*c*f*
log(e)^3*log(h) - 4*a*d*f*log(e)^3*log(h) + (b*d*g*m*n^3*x^2 + a*c*g*m*n^3
+ (b*c*g*m*n^3 + a*d*g*m*n^3)*x)*log(b*x + a)^4 + (b*d*g*m*n^3*x^2 + a*c*g*
m*n^3 + (b*c*g*m*n^3 + a*d*g*m*n^3)*x)*log(d*x + c)^4 - 4*(b*d*g*m*n^2*x^2*
log(e) + a*c*g*m*n^2*log(e) + (b*c*g*m*n^2*log(e) + a*d*g*m*n^2*log(e))*x)*
log(b*x + a)^3 + 4*(b*d*g*m*n^2*x^2*log(e) + a*c*g*m*n^2*log(e) + (b*c*g*m*
n^2*log(e) + a*d*g*m*n^2*log(e))*x - (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 + (b*c*
g*m*n^3 + a*d*g*m*n^3)*x)*log(b*x + a))*log(d*x + c)^3 + 4*(b*c*f*log(h) -
```

$$\begin{aligned}
& a*d*f*\log(h) + (b*c*g*\log(h) - a*d*g*\log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*\log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*\log(d*x + c))*\log((b*x + a)^n)^3 - 4*(b*c*f*\log(h) - a*d*f*\log(h) + (b*c*g*\log(h) - a*d*g*\log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*\log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*\log(d*x + c))*\log((d*x + c)^n)^3 + 6*(b*d*g*m*n*x^2*\log(e)^2 + a*c*g*m*n*\log(e)^2 + (b*c*g*m*n*\log(e)^2 + a*d*g*m*n*\log(e)^2)*x)*\log(b*x + a)^2 + 6*(b*d*g*m*n*x^2*\log(e)^2 + a*c*g*m*n*\log(e)^2 + (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 + (b*c*g*m*n^3 + a*d*g*m*n^3)*x)*\log(b*x + a)^2 + (b*c*g*m*n*\log(e)^2 + a*d*g*m*n*\log(e)^2)*x - 2*(b*d*g*m*n^2*x^2*\log(e) + a*c*g*m*n^2*\log(e) + (b*c*g*m*n^2*\log(e) + a*d*g*m*n^2*\log(e))*x)*\log(b*x + a))*\log(d*x + c)^2 + 6*(2*b*c*f*\log(e)*\log(h) - 2*a*d*f*\log(e)*\log(h) + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*\log(b*x + a)^2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*\log(d*x + c)^2 + 2*(b*c*g*\log(e)*\log(h) - a*d*g*\log(e)*\log(h))*x - 2*(b*d*g*m*x^2*\log(e) + a*c*g*m*\log(e) + (b*c*g*m*\log(e) + a*d*g*m*\log(e))*x)*\log(b*x + a) + 2*(b*d*g*m*x^2*\log(e) + a*c*g*m*\log(e) + (b*c*g*m*\log(e) + a*d*g*m*\log(e))*x - (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*\log(b*x + a))*\log(d*x + c))*\log((b*x + a)^n)^2 + 6*(2*b*c*f*\log(e)*\log(h) - 2*a*d*f*\log(e)*\log(h) + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*\log(b*x + a)^2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*\log(d*x + c)^2 + 2*(b*c*g*\log(e)*\log(h) - a*d*g*\log(e)*\log(h))*x - 2*(b*d*g*m*x^2*\log(e) + a*c*g*m*\log(e) + (b*c*g*m*\log(e) + a*d*g*m*\log(e))*x)*\log(b*x + a) + 2*(b*d*g*m*x^2*\log(e) + a*c*g*m*\log(e) + (b*c*g*m*\log(e) + a*d*g*m*\log(e))*x - (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*\log(b*x + a))*\log(d*x + c) + 2*(b*c*f*\log(h) - a*d*f*\log(h) + (b*c*g*\log(h) - a*d*g*\log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*\log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*\log(d*x + c))*\log((b*x + a)^n)*\log((d*x + c)^n)^2 + 4*(b*c*g*\log(e)^3*\log(h) - a*d*g*\log(e)^3*\log(h))*x - 4*(b*d*g*m*x^2*\log(e)^3 + a*c*g*m*\log(e)^3 + (b*c*g*m*\log(e)^3 + a*d*g*m*\log(e)^3)*x)*\log(b*x + a) + 4*(b*d*g*m*x^2*\log(e)^3 + a*c*g*m*\log(e)^3 - (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 + (b*c*g*m*n^3 + a*d*g*m*n^3)*x)*\log(b*x + a)^3 + 3*(b*d*g*m*n^2*x^2*\log(e) + a*c*g*m*n^2*\log(e) + (b*c*g*m*n^2*\log(e) + a*d*g*m*n^2*\log(e))*x)*\log(b*x + a)^2 + (b*c*g*m*\log(e)^3 + a*d*g*m*\log(e)^3)*x - 3*(b*d*g*m*n*x^2*\log(e)^2 + a*c*g*m*n*\log(e)^2 + (b*c*g*m*n*\log(e)^2 + a*d*g*m*n*\log(e)^2)*x)*\log(b*x + a))*\log(d*x + c) + 4*(3*b*c*f*\log(e)^2*\log(h) - 3*a*d*f*\log(e)^2*\log(h) - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*\log(b*x + a)^3 + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*\log(d*x + c)^3 + 3*(b*d*g*m*n*x^2*\log(e) + a*c*g*m*n*\log(e) + (b*c*g*m*n*\log(e) + a*d*g*m*n*\log(e))*x)*\log(b*x + a)^2 + 3*(b*d*g*m*n*x^2*\log(e) + a*c*g*m*n*\log(e) + (b*c*g*m*n*\log(e) + a*d*g*m*n*\log(e))*x - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*\log(b*x + a))*\log(d*x + c)^2 + 3*(b*c*g*\log(e)^2*\log(h) - a*d*g*\log(e)^2*\log(h))*x - 3*(b*d*g*m*x^2*\log(e)^2 + a*c*g*m*\log(e)^2 + (b*c*g*m*\log(e)^2 + a*d*g*m*\log(e)^2)*x)*\log(b*x + a) + 3*(b*d*g*m*x^2*\log(e)^2 + a*c*g*m*\log(e)^2 + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c
\end{aligned}$$

```

*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log
(e)^2)*x - 2*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) +
a*d*g*m*n*log(e))*x)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n) - 4*(3*b
*c*f*log(e)^2*log(h) - 3*a*d*f*log(e)^2*log(h) - (b*d*g*m*n^2*x^2 + a*c*g*m
*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^3 + (b*d*g*m*n^2*x^2 + a
*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(d*x + c)^3 + 3*(b*d*g*m*n*x
^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x)*log
(b*x + a)^2 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e)
) + a*d*g*m*n*log(e))*x - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a
*d*g*m*n^2)*x)*log(b*x + a))*log(d*x + c)^2 + 3*(b*c*f*log(h) - a*d*f*log(h)
) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a
*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*l
og(d*x + c))*log((b*x + a)^n)^2 + 3*(b*c*g*log(e)^2*log(h) - a*d*g*log(e)^2
*log(h))*x - 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*log(e)^2 + (b*c*g*m*log(e)^2
+ a*d*g*m*log(e)^2)*x)*log(b*x + a) + 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*lo
g(e)^2 + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*lo
g(b*x + a)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log(e)^2)*x - 2*(b*d*g*m*n*x^2*l
og(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x)*log(b*x
+ a))*log(d*x + c) + 3*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b
*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d
*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c
*g*log(e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m
*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^
2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m
n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c))*
log((b*x + a)^n))*log((d*x + c)^n))/(a*b*c^2*f - a^2*c*d*f + (b^2*c*d*g - a
*b*d^2*g)*x^3 - (a*b*d^2*f + a^2*d^2*g - (c*d*f + c^2*g)*b^2)*x^2 + (b^2*c^
2*f + a*b*c^2*g - (d^2*f + c*d*g)*a^2)*x), x)

```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( (gx + f)^m h \right) \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^3}{bdx^2 + ac + (bc + ad)x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="fricas")

```

```

[Out] integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^3/(b*d*x^2 + a*c

```

+ (b\*c + a\*d)\*x), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*((b\*x+a)/(d\*x+c))\*\*n)\*\*3\*ln(h\*(g\*x+f)\*\*m)/(b\*x+a)/(d\*x+c), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((gx+f)^m h\right) \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3}{(bx+a)(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*((b\*x+a)/(d\*x+c))<sup>n</sup>)<sup>3</sup>\*log(h\*(g\*x+f)<sup>m</sup>)/(b\*x+a)/(d\*x+c), x,  
algorithm="giac")

[Out] integrate(log((g\*x + f)<sup>m</sup>\*h)\*log(e\*((b\*x + a)/(d\*x + c))<sup>n</sup>)<sup>3</sup>/((b\*x + a)\*(d\*x + c)), x)

$$3.68 \quad \int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(h(f+gx)^m)}{(a+bx)(c+dx)} dx$$

**Optimal.** Leaf size=496

$$\frac{m \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad} + \frac{2mn \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{PolyLog}\left(3, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad} + \frac{m \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad}$$

[Out] (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^3\*Log[(b\*c - a\*d)/(b\*(c + d\*x))])/(3\*(b\*c - a\*d)\*n) + (Log[e\*((a + b\*x)/(c + d\*x))^n]^3\*Log[h\*(f + g\*x)^m])/(3\*(b\*c - a\*d)\*n) - (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^3\*Log[1 - ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/(3\*(b\*c - a\*d)\*n) + (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^2\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*c - a\*d) - (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^2\*PolyLog[2, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/(b\*c - a\*d) - (2\*m\*n\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*PolyLog[3, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*c - a\*d) + (2\*m\*n\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*PolyLog[3, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/(b\*c - a\*d) + (2\*m\*n^2\*PolyLog[4, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*c - a\*d) - (2\*m\*n^2\*PolyLog[4, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/(b\*c - a\*d)

**Rubi [A]** time = 0.818883, antiderivative size = 517, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {2507, 2489, 2488, 2506, 2508, 6610, 2503}

$$\frac{m \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{PolyLog}\left(2, 1 - \frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad} + \frac{2mn \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{PolyLog}\left(3, 1 - \frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad} + \frac{m \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad}$$

Antiderivative was successfully verified.

[In] Int[(Log[e\*((a + b\*x)/(c + d\*x))^n]^2\*Log[h\*(f + g\*x)^m])/((a + b\*x)\*(c + d\*x)), x]

[Out] (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^3\*Log[(b\*c - a\*d)/(b\*(c + d\*x))])/(3\*(b\*c - a\*d)\*n) - (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^3\*Log[((b\*c - a\*d)\*(f + g\*x))/((b\*f - a\*g)\*(c + d\*x))])/(3\*(b\*c - a\*d)\*n) + (Log[e\*((a + b\*x)/(c + d\*x))^n]^3\*Log[h\*(f + g\*x)^m])/(3\*(b\*c - a\*d)\*n) + (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^2\*PolyLog[2, 1 - (b\*c - a\*d)/(b\*(c + d\*x))])/(b\*c - a\*d) - (m\*Log[e\*((a + b\*x)/(c + d\*x))^n]^2\*PolyLog[2, 1 - ((b\*c - a\*d)\*(f + g\*x))/((b\*f - a\*g)\*(c + d\*x))])/(b\*c - a\*d)

```
g)*(c + d*x)))]/(b*c - a*d) - (2*m*n*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog
[3, 1 - (b*c - a*d)/(b*(c + d*x)))]/(b*c - a*d) + (2*m*n*Log[e*((a + b*x)/(
c + d*x))^n]*PolyLog[3, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))
])/ (b*c - a*d) + (2*m*n^2*PolyLog[4, 1 - (b*c - a*d)/(b*(c + d*x)))]/(b*c -
a*d) - (2*m*n^2*PolyLog[4, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d
*x)))]/(b*c - a*d)
```

### Rule 2507

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :=> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[(k*Log[i*(j*(g +
h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1))/(p*r*(s + 1)*(b*c
- a*d)), x] - Dist[(k*h*t*u)/(p*r*(s + 1)*(b*c - a*d)), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
EqQ[p + q, 0] && NeQ[s, -1]
```

### Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :=> Dist[d/h, Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s)/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s)/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

### Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :=> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

### Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))
^(q_.)]^(r_.)]^(s_.)*(u_), x_Symbol] :=> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f
```



, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]

### Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] := With[{g = Simplify[(v*(c +
d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^(s - 1))/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] := With[{g = Coeff[Simplify[1/(u*(a + b*x))],
x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q]^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
))))]/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q]^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x))))]/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(h(f+gx)^m)}{(a+bx)(c+dx)} dx &= \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(h(f+gx)^m)}{3(bc-ad)n} - \frac{(gm) \int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx} dx}{3(bc-ad)n} \\
&= \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(h(f+gx)^m)}{3(bc-ad)n} - \frac{(dm) \int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} dx}{3(bc-ad)n} + \frac{((df-cg)m) \int}{3(bc-ad)n} \\
&= \frac{m \log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m \log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n} + \frac{\log^3}{3(bc-ad)n} \\
&= \frac{m \log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m \log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n} + \frac{\log^3}{3(bc-ad)n} \\
&= \frac{m \log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m \log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n} + \frac{\log^3}{3(bc-ad)n} \\
&= \frac{m \log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m \log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n} + \frac{\log^3}{3(bc-ad)n}
\end{aligned}$$

**Mathematica [B]** time = 10.071, size = 9211, normalized size = 18.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/((a + b*x)*(c + d*x)),x]
```

```
[Out] Result too large to show
```

**Maple [F]** time = 5.039, size = 0, normalized size = 0.

$$\int \frac{\ln\left(h(gx+f)^m\right)}{(bx+a)(dx+c)} \left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="maxima")
```

```
[Out] 1/3*(n^2*log(b*x + a)^3 - n^2*log(d*x + c)^3 - 3*n*log(b*x + a)^2*log(e) +
3*(n^2*log(b*x + a) - n*log(e))*log(d*x + c)^2 + 3*(log(b*x + a) - log(d*x
+ c))*log((b*x + a)^n)^2 + 3*(log(b*x + a) - log(d*x + c))*log((d*x + c)^n)
^2 + 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*log(b*x + a)*log
(e) + log(e)^2)*log(d*x + c) - 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(
n*log(b*x + a) - log(e))*log(d*x + c) - 2*log(b*x + a)*log(e))*log((b*x + a
)^n) + 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))
*log(d*x + c) - 2*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n) - 2*log(b*x
+ a)*log(e))*log((d*x + c)^n)*log((g*x + f)^m)/(b*c - a*d) - integrate(-
1/3*(3*b*c*f*log(e)^2*log(h) - 3*a*d*f*log(e)^2*log(h) - (b*d*g*m*n^2*x^2 +
a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^3 + (b*d*g*m*n^2
*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(d*x + c)^3 + 3*(b*d
*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e)
)*x)*log(b*x + a)^2 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m
*n*log(e) + a*d*g*m*n*log(e))*x - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m
*n^2 + a*d*g*m*n^2)*x)*log(b*x + a))*log(d*x + c)^2 + 3*(b*c*f*log(h) - a*d
*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c
*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g
*m)*x)*log(d*x + c))*log((b*x + a)^n)^2 + 3*(b*c*f*log(h) - a*d*f*log(h) +
(b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g
*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(d
*x + c))*log((d*x + c)^n)^2 + 3*(b*c*g*log(e)^2*log(h) - a*d*g*log(e)^2*log
(h))*x - 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*log(e)^2 + (b*c*g*m*log(e)^2 + a
*d*g*m*log(e)^2)*x)*log(b*x + a) + 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*log(e)
^2 + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x
+ a)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log(e)^2)*x - 2*(b*d*g*m*n*x^2*log(e)
+ a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x)*log(b*x + a
))*log(d*x + c) + 3*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b*d*g
*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m
```

```

*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*log(e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n) - 3*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*log(e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c) + 2*(b*c*f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((b*x + a)^n))*log((d*x + c)^n))/(a*b*c^2*f - a^2*c*d*f + (b^2*c*d*g - a*b*d^2*g)*x^3 - (a*b*d^2*f + a^2*d^2*g - (c*d*f + c^2*g)*b^2)*x^2 + (b^2*c^2*f + a*b*c^2*g - (d^2*f + c*d*g)*a^2)*x), x)

```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( (gx + f)^m h \right) \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2}{bdx^2 + ac + (bc + ad)x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="fricas")

```

```

[Out] integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^2/(b*d*x^2 + a*c
+ (b*c + a*d)*x), x)

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**2*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx + f\right)^m h\right) \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}{(bx+a)(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))n)2*log(h*(g*x+f)m)/(b*x+a)/(d*x+c),x,
algorithm="giac")
```

```
[Out] integrate(log((g*x + f)m*h)*log(e*((b*x + a)/(d*x + c))n)2/((b*x + a)*(d
*x + c)), x)
```

$$3.69 \quad \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(h(f+gx)^m)}{(a+bx)(c+dx)} dx$$

**Optimal.** Leaf size=371

$$\frac{m \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad} + \frac{m \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad} + \frac{mn \text{PolyLog}\left(3, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}$$

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])/(2*(b*c - a*d)*n) + (Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/(2*(b*c - a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(2*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(b*c - a*d) - (m*n*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) + (m*n*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(b*c - a*d)
```

**Rubi [A]** time = 0.557658, antiderivative size = 384, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 6, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$ , Rules used = {2507, 2489, 2488, 2506, 6610, 2503}

$$\frac{m \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{PolyLog}\left(2, 1 - \frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad} + \frac{m \text{PolyLog}\left(2, 1 - \frac{bc-ad}{b(c+dx)}\right) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad} + \frac{mn \text{PolyLog}\left(3, 1 - \frac{(f+g)}{(c+d)}\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m])/((a + b*x)*(c + d*x)), x]
```

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])/(2*(b*c - a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))])/(2*(b*c - a*d)*n) + (Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/(2*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[2, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))])/(b*c - a*d) - (m*n*PolyLog[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) + (m*n*PolyLog[3, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))])/(b*c - a*d)
```

d\*x)))]/(b\*c - a\*d)

### Rule 2507

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*Log[(i\_.)\*((j\_.)\*((g\_.) + (h\_.)\*(x\_))^(t\_.))^(u\_.)]\*(v\_), x\_Symbol] := With[{k = Simplify[v\*(a + b\*x)\*(c + d\*x)]}, Simp[(k\*Log[i\*(j\*(g + h\*x)^t]^u)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s + 1))/(p\*r\*(s + 1)\*(b\*c - a\*d)), x] - Dist[(k\*h\*t\*u)/(p\*r\*(s + 1)\*(b\*c - a\*d)), Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s + 1)/(g + h\*x), x], x] /; FreeQ[k, x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && NeQ[s, -1]

### Rule 2489

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Dist[d/h, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s/(c + d\*x), x], x] - Dist[(d\*g - c\*h)/h, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s/((c + d\*x)\*(g + h\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && NeQ[b\*g - a\*h, 0] && NeQ[d\*g - c\*h, 0] && IGtQ[s, 1]

### Rule 2488

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := -Simp[(Log[-(b\*c - a\*d)/(d\*(a + b\*x))])\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d)/h, Int[(Log[-(b\*c - a\*d)/(d\*(a + b\*x))])\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

### Rule 2506

Int[Log[v\_]\*Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*(u\_), x\_Symbol] := With[{g = Simplify[((v - 1)\*(c + d\*x))/(a + b\*x)], h = Simplify[u\*(a + b\*x)\*(c + d\*x)]}, -Simp[(h\*PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(b\*c - a\*d), x] + Dist[h\*p\*r\*s, Int[(PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v,

x}], Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] := With[{g = Coeff[Simplify[1/(u*(a + b*x))],
x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
))))]/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q]^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x))))]/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
x]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(h(f+gx)^m)}{(a+bx)(c+dx)} dx &= \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(h(f+gx)^m)}{2(bc-ad)n} - \frac{(gm) \int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx} dx}{2(bc-ad)n} \\
 &= \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(h(f+gx)^m)}{2(bc-ad)n} - \frac{(dm) \int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} dx}{2(bc-ad)n} + \frac{((df-cg)m) \int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} dx}{2(bc-ad)n} \\
 &= \frac{m \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} - \frac{m \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n} + \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n} \\
 &= \frac{m \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} - \frac{m \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n} + \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n} \\
 &= \frac{m \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} - \frac{m \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n} + \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n}
 \end{aligned}$$

**Mathematica [B]** time = 6.02764, size = 1408, normalized size = 3.8

result too large to display

Antiderivative was successfully verified.



[In] Integrate[(Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[h\*(f + g\*x)^m])/((a + b\*x)\*(c + d\*x)),x]

[Out]  $(m*n*\text{Log}[(a + b*x)/(c + d*x)]^2*\text{Log}[(b*c - a*d)/(b*c + b*d*x)] - m*n*\text{Log}[a/b + x]^2*\text{Log}[f + g*x] - m*n*\text{Log}[c/d + x]^2*\text{Log}[f + g*x] + 2*m*n*\text{Log}[a/b + x]*\text{Log}[a + b*x]*\text{Log}[f + g*x] - 2*m*n*\text{Log}[c/d + x]*\text{Log}[a + b*x]*\text{Log}[f + g*x] + 2*m*n*\text{Log}[c/d + x]*\text{Log}[(d*(a + b*x))/(-b*c + a*d)]*\text{Log}[f + g*x] + 2*m*\text{Log}[a/b + x]*\text{Log}[e*((a + b*x)/(c + d*x))^n]*\text{Log}[f + g*x] - 2*m*\text{Log}[c/d + x]*\text{Log}[e*((a + b*x)/(c + d*x))^n]*\text{Log}[f + g*x] - 2*m*\text{Log}[a + b*x]*\text{Log}[e*((a + b*x)/(c + d*x))^n]*\text{Log}[f + g*x] - 2*m*n*\text{Log}[a/b + x]*\text{Log}[(a + b*x)/(c + d*x)]*\text{Log}[f + g*x] + 2*m*n*\text{Log}[c/d + x]*\text{Log}[(a + b*x)/(c + d*x)]*\text{Log}[f + g*x] + m*n*\text{Log}[(a + b*x)/(c + d*x)]^2*\text{Log}[f + g*x] - 2*m*n*\text{Log}[a/b + x]*\text{Log}[c + d*x]*\text{Log}[f + g*x] + 2*m*n*\text{Log}[c/d + x]*\text{Log}[c + d*x]*\text{Log}[f + g*x] + 2*m*\text{Log}[e*((a + b*x)/(c + d*x))^n]*\text{Log}[c + d*x]*\text{Log}[f + g*x] + 2*m*n*\text{Log}[a/b + x]*\text{Log}[(b*(c + d*x))/(b*c - a*d)]*\text{Log}[f + g*x] - 2*m*\text{Log}[a/b + x]*\text{Log}[e*((a + b*x)/(c + d*x))^n]*\text{Log}[(b*(f + g*x))/(b*f - a*g)] + 2*m*n*\text{Log}[a/b + x]*\text{Log}[(a + b*x)/(c + d*x)]*\text{Log}[(b*(f + g*x))/(b*f - a*g)] + 2*m*\text{Log}[c/d + x]*\text{Log}[e*((a + b*x)/(c + d*x))^n]*\text{Log}[(d*(f + g*x))/(d*f - c*g)] - 2*m*n*\text{Log}[c/d + x]*\text{Log}[(a + b*x)/(c + d*x)]*\text{Log}[(d*(f + g*x))/(d*f - c*g)] - m*n*\text{Log}[(a + b*x)/(c + d*x)]^2*\text{Log}[(b*c - a*d)*(f + g*x)/((b*f - a*g)*(c + d*x))] + n*\text{Log}[a/b + x]^2*\text{Log}[h*(f + g*x)^m] + n*\text{Log}[c/d + x]^2*\text{Log}[h*(f + g*x)^m] - 2*n*\text{Log}[a/b + x]*\text{Log}[a + b*x]*\text{Log}[h*(f + g*x)^m] + 2*n*\text{Log}[c/d + x]*\text{Log}[a + b*x]*\text{Log}[h*(f + g*x)^m] - 2*n*\text{Log}[c/d + x]*\text{Log}[(d*(a + b*x))/(-b*c + a*d)]*\text{Log}[h*(f + g*x)^m] + 2*\text{Log}[a + b*x]*\text{Log}[e*((a + b*x)/(c + d*x))^n]*\text{Log}[h*(f + g*x)^m] + 2*n*\text{Log}[a/b + x]*\text{Log}[c + d*x]*\text{Log}[h*(f + g*x)^m] - 2*n*\text{Log}[c/d + x]*\text{Log}[c + d*x]*\text{Log}[h*(f + g*x)^m] - 2*\text{Log}[e*((a + b*x)/(c + d*x))^n]*\text{Log}[c + d*x]*\text{Log}[h*(f + g*x)^m] - 2*n*\text{Log}[a/b + x]*\text{Log}[(b*(c + d*x))/(b*c - a*d)]*\text{Log}[h*(f + g*x)^m] + 2*n*(m*\text{Log}[f + g*x] - \text{Log}[h*(f + g*x)^m])*PolyLog[2, (d*(a + b*x))/(-b*c + a*d)] - 2*m*(\text{Log}[e*((a + b*x)/(c + d*x))^n] - n*\text{Log}[(a + b*x)/(c + d*x)])*PolyLog[2, (g*(a + b*x))/(-b*f + a*g)] + 2*m*n*\text{Log}[(a + b*x)/(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*m*n*\text{Log}[(a + b*x)/(c + d*x)]*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))] + 2*m*n*\text{Log}[f + g*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*n*\text{Log}[h*(f + g*x)^m]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 2*m*\text{Log}[e*((a + b*x)/(c + d*x))^n]*PolyLog[2, (g*(c + d*x))/(-d*f + c*g)] - 2*m*n*\text{Log}[(a + b*x)/(c + d*x)]*PolyLog[2, (g*(c + d*x))/(-d*f + c*g)] - 2*m*n*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] + 2*m*n*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))]/(2*b*c - 2*a*d)$

---

**Maple [F]** time = 4.115, size = 0, normalized size = 0.

$$\int \frac{\ln\left(h(gx+f)^m\right)}{(bx+a)(dx+c)} \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*((b\*x+a)/(d\*x+c))^n)\*ln(h\*(g\*x+f)^m)/(b\*x+a)/(d\*x+c), x)

[Out] int(ln(e\*((b\*x+a)/(d\*x+c))^n)\*ln(h\*(g\*x+f)^m)/(b\*x+a)/(d\*x+c), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*((b\*x+a)/(d\*x+c))^n)\*log(h\*(g\*x+f)^m)/(b\*x+a)/(d\*x+c), x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/2*(n*\log(b*x + a)^2 + n*\log(d*x + c)^2 - 2*(n*\log(b*x + a) - \log(e))*\log \\ & (d*x + c) - 2*(\log(b*x + a) - \log(d*x + c))*\log((b*x + a)^n) + 2*(\log(b*x + \\ & a) - \log(d*x + c))*\log((d*x + c)^n) - 2*\log(b*x + a)*\log(e))*\log((g*x + f) \\ & ^m)/(b*c - a*d) + \text{integrate}(1/2*(2*b*c*f*\log(e))*\log(h) - 2*a*d*f*\log(e))*\log \\ & (h) + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*\log(b*x + a)^2 \\ & + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*\log(d*x + c)^2 \\ & + 2*(b*c*g*\log(e))*\log(h) - a*d*g*\log(e))*\log(h))*x - 2*(b*d*g*m*x^2*\log(e) + \\ & a*c*g*m*\log(e) + (b*c*g*m*\log(e) + a*d*g*m*\log(e))*x)*\log(b*x + a) + 2*(b*d \\ & g*m*x^2*\log(e) + a*c*g*m*\log(e) + (b*c*g*m*\log(e) + a*d*g*m*\log(e))*x - ( \\ & b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*\log(b*x + a))*\log(d*x \\ & + c) + 2*(b*c*f*\log(h) - a*d*f*\log(h) + (b*c*g*\log(h) - a*d*g*\log(h))*x - \\ & (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*\log(b*x + a) + (b*d*g*m*x^2 \\ & + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*\log(d*x + c))*\log((b*x + a)^n) - 2*(b*c \\ & f*\log(h) - a*d*f*\log(h) + (b*c*g*\log(h) - a*d*g*\log(h))*x - (b*d*g*m*x^2 \\ & + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*\log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + \\ & (b*c*g*m + a*d*g*m)*x)*\log(d*x + c))*\log((d*x + c)^n))/(a*b*c^2*f - a^2*c*d \\ & *f + (b^2*c*d*g - a*b*d^2*g)*x^3 - (a*b*d^2*f + a^2*d^2*g - (c*d*f + c^2*g) \\ & *b^2)*x^2 + (b^2*c^2*f + a*b*c^2*g - (d^2*f + c*d*g)*a^2)*x, x) \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( (gx + f)^m h \right) \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{bdx^2 + ac + (bc + ad)x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( (gx + f)^m h \right) \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{(bx + a)(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x + c)), x)
```

$$3.70 \quad \int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

**Optimal.** Leaf size=106

$$\frac{b \text{Unintegrable}\left(\frac{\log(h(f+gx)^m)}{(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}, x\right)}{bc - ad} - \frac{d \text{Unintegrable}\left(\frac{\log(h(f+gx)^m)}{(c+dx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}, x\right)}{bc - ad}$$

[Out] (b\*Unintegrable[Log[h\*(f + g\*x)^m]/((a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x])/(b\*c - a\*d) - (d\*Unintegrable[Log[h\*(f + g\*x)^m]/((c + d\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x])/(b\*c - a\*d)

---

**Rubi [A]** time = 0.500247, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Int[Log[h\*(f + g\*x)^m]/((a + b\*x)\*(c + d\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] (b\*Defer[Int][Log[h\*(f + g\*x)^m]/((a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x])/(b\*c - a\*d) - (d\*Defer[Int][Log[h\*(f + g\*x)^m]/((c + d\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x])/(b\*c - a\*d)

Rubi steps

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \int \left( \frac{b \log(h(f+gx)^m)}{(bc-ad)(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} - \frac{d \log(h(f+gx)^m)}{(bc-ad)(c+dx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} \right) dx$$

$$= \frac{b \int \frac{\log(h(f+gx)^m)}{(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx}{bc-ad} - \frac{d \int \frac{\log(h(f+gx)^m)}{(c+dx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx}{bc-ad}$$

**Mathematica [A]** time = 1.90844, size = 0, normalized size = 0.

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[Log[h\*(f + g\*x)^m]/((a + b\*x)\*(c + d\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] Integrate[Log[h\*(f + g\*x)^m]/((a + b\*x)\*(c + d\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

**Maple [A]** time = 38.288, size = 0, normalized size = 0.

$$\int \frac{\ln(h(gx+f)^m)}{(bx+a)(dx+c)} \left( \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(h\*(g\*x+f)^m)/(b\*x+a)/(d\*x+c)/ln(e\*((b\*x+a)/(d\*x+c))^n), x)

[Out] int(ln(h\*(g\*x+f)^m)/(b\*x+a)/(d\*x+c)/ln(e\*((b\*x+a)/(d\*x+c))^n), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((gx + f)^m h\right)}{(bx + a)(dx + c) \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(h\*(g\*x+f)^m)/(b\*x+a)/(d\*x+c)/log(e\*((b\*x+a)/(d\*x+c))^n), x, algorithm="maxima")

[Out] integrate(log((g\*x + f)^m\*h)/((b\*x + a)\*(d\*x + c)\*log(e\*((b\*x + a)/(d\*x + c))^n)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left((gx + f)^m h\right)}{\left(bdx^2 + ac + (bc + ad)x\right) \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(h\*(g\*x+f)^m)/(b\*x+a)/(d\*x+c)/log(e\*((b\*x+a)/(d\*x+c))^n), x, algorithm="fricas")

[Out] integral(log((g\*x + f)^m\*h)/((b\*d\*x^2 + a\*c + (b\*c + a\*d)\*x)\*log(e\*((b\*x + a)/(d\*x + c))^n)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(h\*(g\*x+f)\*\*m)/(b\*x+a)/(d\*x+c)/ln(e\*((b\*x+a)/(d\*x+c))\*\*n), x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx + f\right)^m h\right)}{(bx + a)(dx + c) \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

$$3.71 \quad \int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx) \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

**Optimal.** Leaf size=92

$$\frac{gm\text{Unintegrable}\left(\frac{1}{(f+gx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}, x\right)}{n(bc-ad)} - \frac{\log(h(f+gx)^m)}{n(bc-ad) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

[Out]  $-(\text{Log}[h*(f + g*x)^m]/((b*c - a*d)*n*\text{Log}[e*((a + b*x)/(c + d*x))^n])) + (g*m$   
 $*\text{Unintegrable}[1/((f + g*x)*\text{Log}[e*((a + b*x)/(c + d*x))^n]], x)/((b*c - a*d$   
 $)*n)$

**Rubi [A]** time = 0.128186, antiderivative size = 0, normalized size of antiderivative = 0.,  
 number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ ,  
 Rules used = {}

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx) \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[\text{Log}[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2, x]$

[Out]  $-(\text{Log}[h*(f + g*x)^m]/((b*c - a*d)*n*\text{Log}[e*((a + b*x)/(c + d*x))^n])) + (g*m$   
 $*\text{Defer}[\text{Int}][1/((f + g*x)*\text{Log}[e*((a + b*x)/(c + d*x))^n]], x)/((b*c - a*d)*$   
 $n)$

Rubi steps

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx) \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = -\frac{\log(h(f+gx)^m)}{(bc-ad)n \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{(gm) \int \frac{1}{(f+gx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx}{(bc-ad)n}$$



**Mathematica [A]** time = 1.28668, size = 0, normalized size = 0.

$$\int \frac{\log(h(f + gx)^m)}{(a + bx)(c + dx) \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[Log[h\*(f + g\*x)^m]/((a + b\*x)\*(c + d\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n]^2), x]

[Out] Integrate[Log[h\*(f + g\*x)^m]/((a + b\*x)\*(c + d\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n]^2), x]

**Maple [A]** time = 48.165, size = 0, normalized size = 0.

$$\int \frac{\ln(h(gx + f)^m)}{(bx + a)(dx + c)} \left( \ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(h\*(g\*x+f)^m)/(b\*x+a)/(d\*x+c)/ln(e\*((b\*x+a)/(d\*x+c))^n)^2, x)

[Out] int(ln(h\*(g\*x+f)^m)/(b\*x+a)/(d\*x+c)/ln(e\*((b\*x+a)/(d\*x+c))^n)^2, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$g^m \int \frac{1}{bcfn \log(e) - adfn \log(e) + (bcgn \log(e) - adgn \log(e))x + (bcfn - adfn + (bcgn - adgn)x) \log((bx + a)^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(h\*(g\*x+f)^m)/(b\*x+a)/(d\*x+c)/log(e\*((b\*x+a)/(d\*x+c))^n)^2, x, algorithm="maxima")

[Out] g\*m\*integrate(1/(b\*c\*f\*n\*log(e) - a\*d\*f\*n\*log(e) + (b\*c\*g\*n\*log(e) - a\*d\*g\*n\*log(e))\*x + (b\*c\*f\*n - a\*d\*f\*n + (b\*c\*g\*n - a\*d\*g\*n)\*x)\*log((b\*x + a)^n)

- (b\*c\*f\*n - a\*d\*f\*n + (b\*c\*g\*n - a\*d\*g\*n)\*x)\*log((d\*x + c)^n), x) - (log((g\*x + f)^m) + log(h))/(b\*c\*n\*log(e) - a\*d\*n\*log(e) + (b\*c\*n - a\*d\*n)\*log((b\*x + a)^n) - (b\*c\*n - a\*d\*n)\*log((d\*x + c)^n))

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log\left(\left(gx + f\right)^m h\right)}{\left(bdx^2 + ac + (bc + ad)x\right) \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(h\*(g\*x+f)^m)/(b\*x+a)/(d\*x+c)/log(e\*((b\*x+a)/(d\*x+c))^n)^2,x, algorithm="fricas")

[Out] integral(log((g\*x + f)^m\*h)/((b\*d\*x^2 + a\*c + (b\*c + a\*d)\*x)\*log(e\*((b\*x + a)/(d\*x + c))^n)^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(h\*(g\*x+f)\*\*m)/(b\*x+a)/(d\*x+c)/ln(e\*((b\*x+a)/(d\*x+c))\*\*n)\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx + f\right)^m h\right)}{(bx + a)(dx + c) \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,  
algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(e*((b*x + a)/(d*x + c  
))^n)^2), x)
```

$$3.72 \quad \int \frac{\log\left(1 - \frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} dx$$

**Optimal.** Leaf size=112

$$\frac{b \text{CannotIntegrate}\left(\frac{\log\left(1 - \frac{a+bx}{c+dx}\right)}{(a+bx) \log^2\left(\frac{a+bx}{c+dx}\right)}, x\right)}{bc - ad} - \frac{d \text{CannotIntegrate}\left(\frac{\log\left(1 - \frac{a+bx}{c+dx}\right)}{(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)}, x\right)}{bc - ad}$$

[Out] (b\*CannotIntegrate[Log[1 - (a + b\*x)/(c + d\*x)]/((a + b\*x)\*Log[(a + b\*x)/(c + d\*x)]^2), x)]/(b\*c - a\*d) - (d\*CannotIntegrate[Log[1 - (a + b\*x)/(c + d\*x)]/((c + d\*x)\*Log[(a + b\*x)/(c + d\*x)]^2), x)]/(b\*c - a\*d)

**Rubi [A]** time = 0.519287, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\log\left(1 - \frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[Log[1 - (a + b\*x)/(c + d\*x)]/((a + b\*x)\*(c + d\*x)\*Log[(a + b\*x)/(c + d\*x)]^2), x]

[Out] (b\*Defer[Int][Log[1 - (a + b\*x)/(c + d\*x)]/((a + b\*x)\*Log[(a + b\*x)/(c + d\*x)]^2), x)]/(b\*c - a\*d) - (d\*Defer[Int][Log[1 - (a + b\*x)/(c + d\*x)]/((c + d\*x)\*Log[(a + b\*x)/(c + d\*x)]^2), x)]/(b\*c - a\*d)

Rubi steps

$$\begin{aligned} \int \frac{\log\left(1 - \frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} dx &= \int \left( \frac{b \log\left(1 - \frac{a+bx}{c+dx}\right)}{(bc - ad)(a + bx) \log^2\left(\frac{a+bx}{c+dx}\right)} - \frac{d \log\left(1 - \frac{a+bx}{c+dx}\right)}{(bc - ad)(c + dx) \log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx \\ &= \frac{b \int \frac{\log\left(1 - \frac{a+bx}{c+dx}\right)}{(a+bx) \log^2\left(\frac{a+bx}{c+dx}\right)} dx}{bc - ad} - \frac{d \int \frac{\log\left(1 - \frac{a+bx}{c+dx}\right)}{(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} dx}{bc - ad} \end{aligned}$$

**Mathematica [A]** time = 0.631728, size = 0, normalized size = 0.

$$\int \frac{\log\left(1 - \frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[Log[1 - (a + b\*x)/(c + d\*x)]/((a + b\*x)\*(c + d\*x)\*Log[(a + b\*x)/(c + d\*x)]^2), x]

[Out] Integrate[Log[1 - (a + b\*x)/(c + d\*x)]/((a + b\*x)\*(c + d\*x)\*Log[(a + b\*x)/(c + d\*x)]^2), x]

**Maple [A]** time = 1.07, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)} \ln\left(1 + \frac{-bx-a}{dx+c}\right) \left(\ln\left(\frac{bx+a}{dx+c}\right)\right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(1+(-b\*x-a)/(d\*x+c))/(b\*x+a)/(d\*x+c)/ln((b\*x+a)/(d\*x+c))^2,x)

[Out] int(ln(1+(-b\*x-a)/(d\*x+c))/(b\*x+a)/(d\*x+c)/ln((b\*x+a)/(d\*x+c))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{\log(-(b-d)x-a+c) - \log(bx+a)}{(bc-ad)\log(bx+a) - (bc-ad)\log(dx+c)} - \int \frac{1}{((bd-d^2)x^2 + ac - c^2 + (bc+ad-2cd)x)\log(bx+a) - ((bd-d^2)x^2 + ac - c^2 + (bc+ad-2cd)x)\log(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(1+(-b\*x-a)/(d\*x+c))/(b\*x+a)/(d\*x+c)/log((b\*x+a)/(d\*x+c))^2,x,  
algorithm="maxima")

[Out] -(log(-(b-d)\*x - a + c) - log(b\*x + a))/((b\*c - a\*d)\*log(b\*x + a) - (b\*c - a\*d)\*log(d\*x + c)) - integrate(-1/(((b\*d - d^2)\*x^2 + a\*c - c^2 + (b\*c + a\*d - 2\*c\*d)\*x)\*log(b\*x + a) - ((b\*d - d^2)\*x^2 + a\*c - c^2 + (b\*c + a\*d - 2\*c\*d)\*x)\*log(d\*x + c)), x)

$2*c*d)*x)*\log(d*x + c)), x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( -\frac{(b-d)x+a-c}{dx+c} \right)}{\left( (bdx^2 + ac + (bc + ad)x) \log \left( \frac{bx+a}{dx+c} \right) \right)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] integral(log(-((b - d)*x + a - c)/(d*x + c))/((b*d*x^2 + a*c + (b*c + a*d)*
x)*log((b*x + a)/(d*x + c))^2), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( -\frac{bx+a}{dx+c} + 1 \right)}{(bx+a)(dx+c) \log \left( \frac{bx+a}{dx+c} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] integrate(log(-(b*x + a)/(d*x + c) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/  
(d*x + c))^2), x)
```

$$3.73 \quad \int \frac{\log\left(1 - \frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} dx$$

**Optimal.** Leaf size=112

$$\frac{b \text{CannotIntegrate}\left(\frac{\log\left(1 - \frac{c+dx}{a+bx}\right)}{(a+bx) \log^2\left(\frac{a+bx}{c+dx}\right)}, x\right)}{bc - ad} - \frac{d \text{CannotIntegrate}\left(\frac{\log\left(1 - \frac{c+dx}{a+bx}\right)}{(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)}, x\right)}{bc - ad}$$

[Out] (b\*CannotIntegrate[Log[1 - (c + d\*x)/(a + b\*x)]/((a + b\*x)\*Log[(a + b\*x)/(c + d\*x])^2), x])/(b\*c - a\*d) - (d\*CannotIntegrate[Log[1 - (c + d\*x)/(a + b\*x)]/((c + d\*x)\*Log[(a + b\*x)/(c + d\*x])^2), x])/(b\*c - a\*d)

**Rubi [A]** time = 0.483652, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\log\left(1 - \frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[Log[1 - (c + d\*x)/(a + b\*x)]/((a + b\*x)\*(c + d\*x)\*Log[(a + b\*x)/(c + d\*x])^2), x]

[Out] (b\*Defer[Int][Log[1 - (c + d\*x)/(a + b\*x)]/((a + b\*x)\*Log[(a + b\*x)/(c + d\*x])^2), x])/(b\*c - a\*d) - (d\*Defer[Int][Log[1 - (c + d\*x)/(a + b\*x)]/((c + d\*x)\*Log[(a + b\*x)/(c + d\*x])^2), x])/(b\*c - a\*d)

Rubi steps

$$\begin{aligned} \int \frac{\log\left(1 - \frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} dx &= \int \left( \frac{b \log\left(1 - \frac{c+dx}{a+bx}\right)}{(bc - ad)(a + bx) \log^2\left(\frac{a+bx}{c+dx}\right)} - \frac{d \log\left(1 - \frac{c+dx}{a+bx}\right)}{(bc - ad)(c + dx) \log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx \\ &= \frac{b \int \frac{\log\left(1 - \frac{c+dx}{a+bx}\right)}{(a+bx) \log^2\left(\frac{a+bx}{c+dx}\right)} dx}{bc - ad} - \frac{d \int \frac{\log\left(1 - \frac{c+dx}{a+bx}\right)}{(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} dx}{bc - ad} \end{aligned}$$



**Mathematica [A]** time = 0.574993, size = 0, normalized size = 0.

$$\int \frac{\log\left(1 - \frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[Log[1 - (c + d\*x)/(a + b\*x)]/((a + b\*x)\*(c + d\*x)\*Log[(a + b\*x)/(c + d\*x)]^2), x]

[Out] Integrate[Log[1 - (c + d\*x)/(a + b\*x)]/((a + b\*x)\*(c + d\*x)\*Log[(a + b\*x)/(c + d\*x)]^2), x]

**Maple [A]** time = 1.008, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)} \ln\left(1 + \frac{-dx-c}{bx+a}\right) \left(\ln\left(\frac{bx+a}{dx+c}\right)\right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(1+(-d\*x-c)/(b\*x+a))/(b\*x+a)/(d\*x+c)/ln((b\*x+a)/(d\*x+c))^2, x)

[Out] int(ln(1+(-d\*x-c)/(b\*x+a))/(b\*x+a)/(d\*x+c)/ln((b\*x+a)/(d\*x+c))^2, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{\log((b-d)x+a-c) - \log(bx+a)}{(bc-ad)\log(bx+a) - (bc-ad)\log(dx+c)} - \int \frac{1}{((b^2-bd)x^2 + a^2 - ac + (a(2b-d) - bc)x)\log(bx+a) - ((b^2 - b^2d) - b^2c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(1+(-d\*x-c)/(b\*x+a))/(b\*x+a)/(d\*x+c)/log((b\*x+a)/(d\*x+c))^2, x, algorithm="maxima")

[Out] -(log((b - d)\*x + a - c) - log(b\*x + a))/((b\*c - a\*d)\*log(b\*x + a) - (b\*c - a\*d)\*log(d\*x + c)) - integrate(-1/(((b^2 - b\*d)\*x^2 + a^2 - a\*c + (a\*(2\*b - d) - b\*c)\*x)\*log(b\*x + a) - ((b^2 - b\*d)\*x^2 + a^2 - a\*c + (a\*(2\*b - d) - b\*c)\*x)), x)

$b*c*x)*\log(d*x + c)), x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log\left(\frac{(b-d)x+a-c}{bx+a}\right)}{\left((bdx^2 + ac + (bc + ad)x)\log\left(\frac{bx+a}{dx+c}\right)\right)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(1+(-d\*x-c)/(b\*x+a))/(b\*x+a)/(d\*x+c)/log((b\*x+a)/(d\*x+c))^2,x,  
algorithm="fricas")

[Out] integral(log(((b - d)\*x + a - c)/(b\*x + a))/((b\*d\*x^2 + a\*c + (b\*c + a\*d)\*x)\*log((b\*x + a)/(d\*x + c))^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(1+(-d\*x-c)/(b\*x+a))/(b\*x+a)/(d\*x+c)/ln((b\*x+a)/(d\*x+c))^2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(-\frac{dx+c}{bx+a} + 1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(1+(-d\*x-c)/(b\*x+a))/(b\*x+a)/(d\*x+c)/log((b\*x+a)/(d\*x+c))^2,x,  
algorithm="giac")

```
[Out] integrate(log(-(d*x + c)/(b*x + a) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/  
(d*x + c))^2), x)
```

$$3.74 \quad \int \left( \frac{1}{(c+dx)(-a+c+(-b+d)x) \log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx$$

**Optimal.** Leaf size=45

$$\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(bc-ad) \log\left(\frac{a+bx}{c+dx}\right)}$$

[Out]  $-(\text{Log}[1 - (a + b*x)/(c + d*x)]/((b*c - a*d)*\text{Log}[(a + b*x)/(c + d*x)]))$

**Rubi [F]** time = 0.515643, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \left( \frac{1}{(c+dx)(-a+c+(-b+d)x) \log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[1/((c + d*x)*(-a + c + (-b + d)*x)*\text{Log}[(a + b*x)/(c + d*x]]) + \text{Log}[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*\text{Log}[(a + b*x)/(c + d*x)]^2), x]$

[Out]  $\text{Defer}[\text{Int}[1/((c + d*x)*(-a + c + (-b + d)*x)*\text{Log}[(a + b*x)/(c + d*x]])], x] + (b*\text{Defer}[\text{Int}[\text{Log}[1 - (a + b*x)/(c + d*x)]/((a + b*x)*\text{Log}[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d) - (d*\text{Defer}[\text{Int}[\text{Log}[1 - (a + b*x)/(c + d*x)]/((c + d*x)*\text{Log}[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)$

Rubi steps

$$\int \left( \frac{1}{(c+dx)(-a+c+(-b+d)x) \log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1 - \frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx = \int \frac{1}{(c+dx)(-a+c+(-b+d)x) \log\left(\frac{a+bx}{c+dx}\right)} dx$$

$$= \int \frac{1}{(c+dx)(-a+c+(-b+d)x) \log\left(\frac{a+bx}{c+dx}\right)} dx$$

$$= \frac{b \int \frac{\log\left(1 - \frac{a+bx}{c+dx}\right)}{(a+bx) \log^2\left(\frac{a+bx}{c+dx}\right)} dx}{bc-ad} - \frac{d \int \frac{\log\left(1 - \frac{a+bx}{c+dx}\right)}{(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} dx}{bc-ad}$$

**Mathematica [A]** time = 0.311511, size = 44, normalized size = 0.98

$$\frac{\log\left(1 - \frac{a+bx}{c+dx}\right)}{(ad-bc) \log\left(\frac{a+bx}{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]) + Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2), x]
```

```
[Out] Log[1 - (a + b*x)/(c + d*x)]/((-b*c) + a*d)*Log[(a + b*x)/(c + d*x)]
```

---

**Maple [C]** time = 1.322, size = 662, normalized size = 14.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(-a+c+(-b+d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2, x)
```

```
[Out] 2*I/(a*d-b*c)/(csgn(I*(b*x+a)/(d*x+c))*csgn(I*(b*x+a))*csgn(I/(d*x+c))*Pi-csgn(I*(b*x+a)/(d*x+c))^2*csgn(I*(b*x+a))*Pi-csgn(I*(b*x+a)/(d*x+c))^2*csgn(I*(b*x+a)/(d*x+c))
```

$$\begin{aligned} & I/(d*x+c))*\text{Pi}+\text{csgn}(I*(b*x+a)/(d*x+c))^3*\text{Pi}+2*I*\ln(b*x+a)-2*I*\ln(d*x+c))*\ln(b*x-d*x+a-c) \\ & -(-I*\text{csgn}(I*(b*x+a)/(d*x+c))*\text{csgn}(I*(b*x+a))*\text{csgn}(I/(d*x+c))*\text{Pi} \\ & +I*\text{csgn}(I*(b*x+a)/(d*x+c))^2*\text{csgn}(I*(b*x+a))*\text{Pi}+I*\text{csgn}(I*(b*x+a)/(d*x+c))^2 \\ & *\text{csgn}(I/(d*x+c))*\text{Pi}+I*\text{Pi}*\text{csgn}(I/(d*x+c))*\text{csgn}(I*(b*x-d*x+a-c))*\text{csgn}(I/(d*x+c) \\ & *(b*x-d*x+a-c))-I*\text{Pi}*\text{csgn}(I/(d*x+c))*\text{csgn}(I/(d*x+c)*(b*x-d*x+a-c))^2-I*\text{csgn}(I*(b*x+a)/(d*x+c))^3 \\ & *\text{Pi}+2*I*\text{Pi}*\text{csgn}(I/(d*x+c)*(b*x-d*x+a-c))^2-I*\text{Pi}*\text{csgn}(I*(b*x-d*x+a-c))*\text{csgn}(I/(d*x+c)*(b*x-d*x+a-c))^2 \\ & -I*\text{Pi}*\text{csgn}(I/(d*x+c)*(b*x-d*x+a-c))^3-2*I*\text{Pi}+2*\ln(b*x+a))/(a*d-b*c)/(-I*\text{csgn}(I*(b*x+a)/(d*x+c))^3*\text{Pi} \\ & +I*\text{csgn}(I*(b*x+a)/(d*x+c))^2*\text{csgn}(I*(b*x+a))*\text{Pi}+I*\text{csgn}(I*(b*x+a)/(d*x+c))^2*\text{csgn}(I/(d*x+c))*\text{Pi} \\ & -I*\text{csgn}(I*(b*x+a)/(d*x+c))*\text{csgn}(I*(b*x+a))*\text{csgn}(I/(d*x+c))*\text{Pi}+2*\ln(b*x+a)-2*\ln(d*x+c)) \end{aligned}$$

**Maxima [A]** time = 1.49692, size = 80, normalized size = 1.78

$$\frac{\log(-(b-d)x-a+c)-\log(bx+a)}{(bc-ad)\log(bx+a)-(bc-ad)\log(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(-a+c+(-b+d)\*x)/log((b\*x+a)/(d\*x+c))+log(1+(-b\*x-a)/(d\*x+c))/(b\*x+a)/(d\*x+c)/log((b\*x+a)/(d\*x+c))^2,x, algorithm="maxima")

[Out] -(log(-(b-d)\*x-a+c)-log(b\*x+a))/((b\*c-a\*d)\*log(b\*x+a)-(b\*c-a\*d)\*log(d\*x+c))

**Fricas [A]** time = 2.06832, size = 104, normalized size = 2.31

$$\frac{\log\left(-\frac{(b-d)x+a-c}{dx+c}\right)}{(bc-ad)\log\left(\frac{bx+a}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(-a+c+(-b+d)\*x)/log((b\*x+a)/(d\*x+c))+log(1+(-b\*x-a)/(d\*x+c))/(b\*x+a)/(d\*x+c)/log((b\*x+a)/(d\*x+c))^2,x, algorithm="fricas")

[Out] -log(-((b-d)\*x+a-c)/(d\*x+c))/((b\*c-a\*d)\*log((b\*x+a)/(d\*x+c)))

**Sympy [A]** time = 3.05976, size = 44, normalized size = 0.98

$$\frac{\log\left(\frac{-a-bx}{c+dx} + 1\right)}{ad \log\left(\frac{a+bx}{c+dx}\right) - bc \log\left(\frac{a+bx}{c+dx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(-a+c+(-b+d)\*x)/ln((b\*x+a)/(d\*x+c))+ln(1+(-b\*x-a)/(d\*x+c))/(b\*x+a)/(d\*x+c)/ln((b\*x+a)/(d\*x+c))\*\*2,x)

[Out] log((-a - b\*x)/(c + d\*x) + 1)/(a\*d\*log((a + b\*x)/(c + d\*x)) - b\*c\*log((a + b\*x)/(c + d\*x)))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{((b-d)x + a - c)(dx + c) \log\left(\frac{bx+a}{dx+c}\right)} + \frac{\log\left(-\frac{bx+a}{dx+c} + 1\right)}{(bx + a)(dx + c) \log\left(\frac{bx+a}{dx+c}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(-a+c+(-b+d)\*x)/log((b\*x+a)/(d\*x+c))+log(1+(-b\*x-a)/(d\*x+c))/(b\*x+a)/(d\*x+c)/log((b\*x+a)/(d\*x+c))^2,x, algorithm="giac")

[Out] integrate(-1/(((b - d)\*x + a - c)\*(d\*x + c)\*log((b\*x + a)/(d\*x + c))) + log(- (b\*x + a)/(d\*x + c) + 1)/((b\*x + a)\*(d\*x + c)\*log((b\*x + a)/(d\*x + c))^2), x)

$$3.75 \quad \int \left( -\frac{1}{(a+bx)(a-c+(b-d)x) \log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx$$

**Optimal.** Leaf size=45

$$\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad) \log\left(\frac{a+bx}{c+dx}\right)}$$

[Out] `-(Log[1 - (c + d*x)/(a + b*x)]/((b*c - a*d)*Log[(a + b*x)/(c + d*x]]))`

**Rubi [F]** time = 0.512104, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \left( -\frac{1}{(a+bx)(a-c+(b-d)x) \log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx$$

Verification is Not applicable to the result.

[In] `Int[-(1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x]])) + Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2), x]`

[Out] `-Defer[Int][1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x]]), x] + (b*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((c + d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)`

Rubi steps



$$\int \left( -\frac{1}{(a+bx)(a-c+(b-d)x) \log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1 - \frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx = -\int \frac{1}{(a+bx)(a-c+(b-d)x) \log\left(\frac{a+bx}{c+dx}\right)} dx$$

$$= -\int \frac{1}{(a+bx)(a-c+(b-d)x) \log\left(\frac{a+bx}{c+dx}\right)} dx$$

$$= \frac{b \int \frac{\log\left(1 - \frac{c+dx}{a+bx}\right)}{(a+bx) \log^2\left(\frac{a+bx}{c+dx}\right)} dx}{bc-ad} - \frac{d \int \frac{\log\left(1 - \frac{c+dx}{a+bx}\right)}{(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} dx}{bc-ad}$$

**Mathematica [A]** time = 0.0851547, size = 45, normalized size = 1.

$$-\frac{\log\left(1 - \frac{c+dx}{a+bx}\right)}{(bc-ad) \log\left(\frac{a+bx}{c+dx}\right)}$$

Antiderivative was successfully verified.

[In] Integrate[-(1/((a + b\*x)\*(a - c + (b - d)\*x)\*Log[(a + b\*x)/(c + d\*x])) + Log[1 - (c + d\*x)/(a + b\*x)]/((a + b\*x)\*(c + d\*x)\*Log[(a + b\*x)/(c + d\*x)]^2), x]

[Out] -(Log[1 - (c + d\*x)/(a + b\*x)]/((b\*c - a\*d)\*Log[(a + b\*x)/(c + d\*x)]))

**Maple [C]** time = 1.214, size = 503, normalized size = 11.2

$$\frac{2i \ln(bx - dx + a - c)}{ad - bc} \left( \operatorname{csgn}\left(\frac{i(bx + a)}{dx + c}\right) \operatorname{csgn}(i(bx + a)) \operatorname{csgn}\left(\frac{i}{dx + c}\right) \pi - \left( \operatorname{csgn}\left(\frac{i(bx + a)}{dx + c}\right) \right)^2 \operatorname{csgn}(i(bx + a)) \pi - \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(b\*x+a)/(a-c+(b-d)\*x)/ln((b\*x+a)/(d\*x+c))+ln(1+(-d\*x-c)/(b\*x+a))/(b\*x+a)/(d\*x+c)/ln((b\*x+a)/(d\*x+c))^2,x)

[Out] 2\*I/(a\*d-b\*c)/(csgn(I\*(b\*x+a)/(d\*x+c))\*csgn(I\*(b\*x+a))\*csgn(I/(d\*x+c))\*Pi-csgn(I\*(b\*x+a)/(d\*x+c))^2\*csgn(I\*(b\*x+a))\*Pi-csgn(I\*(b\*x+a)/(d\*x+c))^2\*csgn(I\*(b\*x+a)/(d\*x+c))

$$\frac{I/(d*x+c)*\text{Pi}+\text{csgn}(I*(b*x+a)/(d*x+c))^3*\text{Pi}+2*I*\ln(b*x+a)-2*I*\ln(d*x+c))*\ln(b*x-d*x+a-c)-(I*\text{Pi}*\text{csgn}(I*(b*x-d*x+a-c))*\text{csgn}(I/(b*x+a))*\text{csgn}(I/(b*x+a))*(b*x-d*x+a-c))-I*\text{Pi}*\text{csgn}(I*(b*x-d*x+a-c))*\text{csgn}(I/(b*x+a))*(b*x-d*x+a-c))^2-I*\text{Pi}*\text{csgn}(I/(b*x+a))*\text{csgn}(I/(b*x+a))*(b*x-d*x+a-c))^2+I*\text{Pi}*\text{csgn}(I/(b*x+a))*(b*x-d*x+a-c))^3+2*\ln(b*x+a))/(a*d-b*c)/(-I*\text{csgn}(I*(b*x+a)/(d*x+c))^3*\text{Pi}+I*\text{csgn}(I*(b*x+a)/(d*x+c))^2*\text{csgn}(I*(b*x+a))*\text{Pi}+I*\text{csgn}(I*(b*x+a)/(d*x+c))^2*\text{csgn}(I/(d*x+c))*\text{Pi}-I*\text{csgn}(I*(b*x+a)/(d*x+c))*\text{csgn}(I*(b*x+a))*\text{csgn}(I/(d*x+c))*\text{Pi}+2*\ln(b*x+a)-2*\ln(d*x+c))$$

**Maxima [A]** time = 1.67998, size = 78, normalized size = 1.73

$$\frac{\log((b-d)x+a-c) - \log(bx+a)}{(bc-ad)\log(bx+a) - (bc-ad)\log(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/(b\*x+a)/(a-c+(b-d)\*x)/log((b\*x+a)/(d\*x+c))+log(1+(-d\*x-c)/(b\*x+a))/(b\*x+a)/(d\*x+c)/log((b\*x+a)/(d\*x+c))^2,x, algorithm="maxima")

[Out] -(log((b-d)\*x+a-c) - log(b\*x+a))/((b\*c-a\*d)\*log(b\*x+a) - (b\*c-a\*d)\*log(d\*x+c))

**Fricas [A]** time = 1.99166, size = 103, normalized size = 2.29

$$\frac{\log\left(\frac{(b-d)x+a-c}{bx+a}\right)}{(bc-ad)\log\left(\frac{bx+a}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/(b\*x+a)/(a-c+(b-d)\*x)/log((b\*x+a)/(d\*x+c))+log(1+(-d\*x-c)/(b\*x+a))/(b\*x+a)/(d\*x+c)/log((b\*x+a)/(d\*x+c))^2,x, algorithm="fricas")

[Out] -log(((b-d)\*x+a-c)/(b\*x+a))/((b\*c-a\*d)\*log((b\*x+a)/(d\*x+c)))

**Sympy [A]** time = 3.32851, size = 44, normalized size = 0.98

$$\frac{\log\left(1 + \frac{-c-dx}{a+bx}\right)}{ad \log\left(\frac{a+bx}{c+dx}\right) - bc \log\left(\frac{a+bx}{c+dx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/(b\*x+a)/(a-c+(b-d)\*x)/ln((b\*x+a)/(d\*x+c))+ln(1+(-d\*x-c)/(b\*x+a))/(b\*x+a)/(d\*x+c)/ln((b\*x+a)/(d\*x+c))\*\*2,x)

[Out] log(1 + (-c - d\*x)/(a + b\*x))/(a\*d\*log((a + b\*x)/(c + d\*x)) - b\*c\*log((a + b\*x)/(c + d\*x)))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{((b-d)x+a-c)(bx+a)\log\left(\frac{bx+a}{dx+c}\right)} + \frac{\log\left(-\frac{dx+c}{bx+a}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/(b\*x+a)/(a-c+(b-d)\*x)/log((b\*x+a)/(d\*x+c))+log(1+(-d\*x-c)/(b\*x+a))/(b\*x+a)/(d\*x+c)/log((b\*x+a)/(d\*x+c))^2,x, algorithm="giac")

[Out] integrate(-1/(((b - d)\*x + a - c)\*(b\*x + a)\*log((b\*x + a)/(d\*x + c))) + log(-d\*x + c)/(b\*x + a) + 1)/((b\*x + a)\*(d\*x + c)\*log((b\*x + a)/(d\*x + c))^2), x)

$$3.76 \quad \int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx$$

**Optimal.** Leaf size=560

$$\frac{fnPolyLog\left(2, -\frac{\sqrt{g}(a+bx)}{b\sqrt{f-a}\sqrt{g}}\right)}{2g^2} - \frac{fnPolyLog\left(2, \frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^2} + \frac{fnPolyLog\left(2, -\frac{\sqrt{g}(c+dx)}{d\sqrt{f-c}\sqrt{g}}\right)}{2g^2} + \frac{fnPolyLog\left(2, \frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^2} + \frac{a^2}{2g^2}$$

[Out]  $-(a*n*x)/(2*b*g) + (c*n*x)/(2*d*g) + (a^2*n*Log[a + b*x])/(2*b^2*g) - (n*x^2*Log[a + b*x])/(2*g) - (c^2*n*Log[c + d*x])/(2*d^2*g) + (n*x^2*Log[c + d*x])/(2*g) + (x^2*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(2*g) - (f*n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^2) - (f*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])])/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/(2*g^2) + (f*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f - g*x^2])/(2*g^2) - (f*n*PolyLog[2, -((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*g^2) - (f*n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^2) + (f*n*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*g^2) + (f*n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^2)$

**Rubi [A]** time = 0.734765, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 9, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$ , Rules used = {2513, 266, 43, 2416, 2395, 260, 2394, 2393, 2391}

$$\frac{fnPolyLog\left(2, -\frac{\sqrt{g}(a+bx)}{b\sqrt{f-a}\sqrt{g}}\right)}{2g^2} - \frac{fnPolyLog\left(2, \frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^2} + \frac{fnPolyLog\left(2, -\frac{\sqrt{g}(c+dx)}{d\sqrt{f-c}\sqrt{g}}\right)}{2g^2} + \frac{fnPolyLog\left(2, \frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^2} + \frac{a^2}{2g^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f - g\*x^2), x]

[Out]  $-(a*n*x)/(2*b*g) + (c*n*x)/(2*d*g) + (a^2*n*Log[a + b*x])/(2*b^2*g) - (n*x^2*Log[a + b*x])/(2*g) - (c^2*n*Log[c + d*x])/(2*d^2*g) + (n*x^2*Log[c + d*x])/(2*g) + (x^2*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(2*g) - (f*n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^2) - (f*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])])/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/(2*g^2) + (f*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f - g*x^2])/(2*g^2) - (f*n*PolyLog[2, -((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*g^2) - (f*n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^2) + (f*n*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*g^2) + (f*n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^2)$

```
[g]*x))/(b*Sqrt[f] - a*Sqrt[g]))/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqrt[
f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g]))/(2*g^2) + (f*(n*Log[a + b*x] - L
og[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f - g*x^2))/(2*g^2) - (
f*n*PolyLog[2, -((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g])))]/(2*g^2) - (
f*n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])))]/(2*g^2) + (f*n
*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g])))]/(2*g^2) + (f*n
*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])))]/(2*g^2)
```

### Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFX_), x_Symbol] := Dist[p*r, Int[RFX*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFX*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFX, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFX, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFX, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n
]
```

### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a
+ b*Log[c*(d + e*x)^n]]^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e^n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

eQ[q, -1]

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 2394

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]\*(b\_))/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))]\*(b\_))/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx &= n \int \frac{x^3 \log(a+bx)}{f-gx^2} dx - n \int \frac{x^3 \log(c+dx)}{f-gx^2} dx - \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right) \\
&= n \int \left(-\frac{x \log(a+bx)}{g} + \frac{fx \log(a+bx)}{g(f-gx^2)}\right) dx - n \int \left(-\frac{x \log(c+dx)}{g} + \frac{fx \log(c+dx)}{g(f-gx^2)}\right) dx - \frac{1}{2} \\
&= -\frac{n \int x \log(a+bx) dx}{g} + \frac{n \int x \log(c+dx) dx}{g} + \frac{(fn) \int \frac{x \log(a+bx)}{f-gx^2} dx}{g} - \frac{(fn) \int \frac{x \log(c+dx)}{f-gx^2} dx}{g} \\
&= -\frac{nx^2 \log(a+bx)}{2g} + \frac{nx^2 \log(c+dx)}{2g} + \frac{x^2 \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{2g} \\
&= -\frac{nx^2 \log(a+bx)}{2g} + \frac{nx^2 \log(c+dx)}{2g} + \frac{x^2 \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{2g} \\
&= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2n \log(a+bx)}{2b^2g} - \frac{nx^2 \log(a+bx)}{2g} - \frac{c^2n \log(c+dx)}{2d^2g} + \frac{nx^2 \log(c+dx)}{2g} + \frac{x^2}{2g} \\
&= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2n \log(a+bx)}{2b^2g} - \frac{nx^2 \log(a+bx)}{2g} - \frac{c^2n \log(c+dx)}{2d^2g} + \frac{nx^2 \log(c+dx)}{2g} + \frac{x^2}{2g} \\
&= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2n \log(a+bx)}{2b^2g} - \frac{nx^2 \log(a+bx)}{2g} - \frac{c^2n \log(c+dx)}{2d^2g} + \frac{nx^2 \log(c+dx)}{2g} + \frac{x^2}{2g}
\end{aligned}$$

**Mathematica [A]** time = 0.387355, size = 461, normalized size = 0.82

$$fn \left( \text{PolyLog}\left(2, \frac{b(\sqrt{f}-\sqrt{gx})}{a\sqrt{g}+b\sqrt{f}}\right) - \text{PolyLog}\left(2, \frac{d(\sqrt{f}-\sqrt{gx})}{c\sqrt{g}+d\sqrt{f}}\right) + \log(\sqrt{f}-\sqrt{gx}) \left( \log\left(\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right) - \log\left(\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right) \right) \right) + fn \left( \text{Poly}
\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f - g\*x^2), x]

[Out]  $(-(g*x^2*\text{Log}[e*((a + b*x)/(c + d*x))^n]) + (g*n*(a^2*d^2*\text{Log}[a + b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*\text{Log}[c + d*x]))) / (b^2*d^2) - f*\text{Log}[e*((a + b*x)/(c + d*x))^n]*\text{Log}[\text{Sqrt}[f] - \text{Sqrt}[g]*x] - f*\text{Log}[e*((a + b*x)/(c + d*x))^n]*\text{Log}[\text{Sqrt}[f] + \text{Sqrt}[g]*x] + f*n*((\text{Log}[(\text{Sqrt}[g]*(a + b*x))/(b*\text{Sqrt}[f] + a*\text{Sqrt}[g]]$

g]]) - Log[(Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] + c\*Sqrt[g])]\*Log[Sqrt[f] - Sqrt[g]\*x] + PolyLog[2, (b\*(Sqrt[f] - Sqrt[g]\*x))/(b\*Sqrt[f] + a\*Sqrt[g])] - PolyLog[2, (d\*(Sqrt[f] - Sqrt[g]\*x))/(d\*Sqrt[f] + c\*Sqrt[g])] + f\*n\*(Log[-(Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] - a\*Sqrt[g])] - Log[-(Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] - c\*Sqrt[g])])\*Log[Sqrt[f] + Sqrt[g]\*x] + PolyLog[2, (b\*(Sqrt[f] + Sqrt[g]\*x))/(b\*Sqrt[f] - a\*Sqrt[g])] - PolyLog[2, (d\*(Sqrt[f] + Sqrt[g]\*x))/(d\*Sqrt[f] - c\*Sqrt[g])])]/(2\*g^2)

**Maple [F]** time = 0.493, size = 0, normalized size = 0.

$$\int \frac{x^3}{-gx^2 + f} \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*ln(e\*((b\*x+a)/(d\*x+c))^n)/(-g\*x^2+f),x)

[Out] int(x^3\*ln(e\*((b\*x+a)/(d\*x+c))^n)/(-g\*x^2+f),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{x^3 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{gx^2 - f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(e\*((b\*x+a)/(d\*x+c))^n)/(-g\*x^2+f),x, algorithm="maxima")

[Out] -integrate(x^3\*log(e\*((b\*x + a)/(d\*x + c))^n)/(g\*x^2 - f), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( - \frac{x^3 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{gx^2 - f}, x \right)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-x^3*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2 - f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(-x^3*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

$$3.77 \quad \int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx$$

**Optimal.** Leaf size=550

$$\frac{\sqrt{fn}\text{PolyLog}\left(2, -\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}} - \frac{\sqrt{fn}\text{PolyLog}\left(2, \frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^{3/2}} - \frac{\sqrt{fn}\text{PolyLog}\left(2, -\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}} + \frac{\sqrt{fn}\text{PolyLog}\left(2, \frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^{3/2}}$$

[Out]  $-\left(\frac{n(a+bx)\text{Log}[a+bx]}{b\sqrt{g}} + \frac{n(c+dx)\text{Log}[c+dx]}{d\sqrt{g}} + (x(n\text{Log}[a+bx] - \text{Log}[e((a+bx)/(c+dx))^n] - n\text{Log}[c+dx]))/g - (\text{Sqrt}[f]\text{ArcTanh}[(\text{Sqrt}[g]x)/\text{Sqrt}[f]](n\text{Log}[a+bx] - \text{Log}[e((a+bx)/(c+dx))^n] - n\text{Log}[c+dx]))/g^{3/2} - (\text{Sqrt}[f]n\text{Log}[a+bx]\text{Log}[(b(\text{Sqrt}[f] - \text{Sqrt}[g]x))/(b\text{Sqrt}[f] + a\text{Sqrt}[g])])/(2g^{3/2}) + (\text{Sqrt}[f]n\text{Log}[c+dx]\text{Log}[(d(\text{Sqrt}[f] - \text{Sqrt}[g]x))/(d\text{Sqrt}[f] + c\text{Sqrt}[g])])/(2g^{3/2}) + (\text{Sqrt}[f]n\text{Log}[a+bx]\text{Log}[(b(\text{Sqrt}[f] + \text{Sqrt}[g]x))/(b\text{Sqrt}[f] - a\text{Sqrt}[g])])/(2g^{3/2}) - (\text{Sqrt}[f]n\text{Log}[c+dx]\text{Log}[(d(\text{Sqrt}[f] + \text{Sqrt}[g]x))/(d\text{Sqrt}[f] - c\text{Sqrt}[g])])/(2g^{3/2}) + (\text{Sqrt}[f]n\text{PolyLog}[2, -((\text{Sqrt}[g](a+bx))/(b\text{Sqrt}[f] - a\text{Sqrt}[g]))])/(2g^{3/2}) - (\text{Sqrt}[f]n\text{PolyLog}[2, (\text{Sqrt}[g](a+bx))/(b\text{Sqrt}[f] + a\text{Sqrt}[g])])/(2g^{3/2}) - (\text{Sqrt}[f]n\text{PolyLog}[2, -((\text{Sqrt}[g](c+dx))/(d\text{Sqrt}[f] - c\text{Sqrt}[g]))])/(2g^{3/2}) + (\text{Sqrt}[f]n\text{PolyLog}[2, (\text{Sqrt}[g](c+dx))/(d\text{Sqrt}[f] + c\text{Sqrt}[g])])/(2g^{3/2})\right)$

**Rubi [A]** time = 0.565428, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 10, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {2513, 321, 208, 2416, 2389, 2295, 2409, 2394, 2393, 2391}

$$\frac{\sqrt{fn}\text{PolyLog}\left(2, -\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}} - \frac{\sqrt{fn}\text{PolyLog}\left(2, \frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^{3/2}} - \frac{\sqrt{fn}\text{PolyLog}\left(2, -\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}} + \frac{\sqrt{fn}\text{PolyLog}\left(2, \frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2\text{Log}[e((a+bx)/(c+dx))^n])/(f-gx^2), x]$

[Out]  $-\left(\frac{n(a+bx)\text{Log}[a+bx]}{b\sqrt{g}} + \frac{n(c+dx)\text{Log}[c+dx]}{d\sqrt{g}} + (x(n\text{Log}[a+bx] - \text{Log}[e((a+bx)/(c+dx))^n] - n\text{Log}[c+dx]))/g - (\text{Sqrt}[f]\text{ArcTanh}[(\text{Sqrt}[g]x)/\text{Sqrt}[f]](n\text{Log}[a+bx] - \text{Log}[e((a+bx)/(c+dx))^n] - n\text{Log}[c+dx]))/g^{3/2} - (\text{Sqrt}[f]n\text{Log}[a+bx]\text{Log}[(b(\text{Sqrt}[f] - \text{Sqrt}[g]x))/(b\text{Sqrt}[f] + a\text{Sqrt}[g])])/(2g^{3/2}) + (\text{Sqrt}[f]n\text{Log}[c+dx]\text{Log}[(d(\text{Sqrt}[f] - \text{Sqrt}[g]x))/(d\text{Sqrt}[f] + c\text{Sqrt}[g])])/(2g^{3/2})\right)$

$$+ (\text{Sqrt}[f]*n*\text{Log}[a + b*x]*\text{Log}[(b*(\text{Sqrt}[f] + \text{Sqrt}[g]*x))/(b*\text{Sqrt}[f] - a*\text{Sqrt}[g])])/(2*g^(3/2)) - (\text{Sqrt}[f]*n*\text{Log}[c + d*x]*\text{Log}[(d*(\text{Sqrt}[f] + \text{Sqrt}[g]*x))/(d*\text{Sqrt}[f] - c*\text{Sqrt}[g])])/(2*g^(3/2)) + (\text{Sqrt}[f]*n*\text{PolyLog}[2, -((\text{Sqrt}[g]*(a + b*x))/(b*\text{Sqrt}[f] - a*\text{Sqrt}[g]))])/(2*g^(3/2)) - (\text{Sqrt}[f]*n*\text{PolyLog}[2, (\text{Sqrt}[g]*(a + b*x))/(b*\text{Sqrt}[f] + a*\text{Sqrt}[g])])/(2*g^(3/2)) - (\text{Sqrt}[f]*n*\text{PolyLog}[2, -((\text{Sqrt}[g]*(c + d*x))/(d*\text{Sqrt}[f] - c*\text{Sqrt}[g]))])/(2*g^(3/2)) + (\text{Sqrt}[f]*n*\text{PolyLog}[2, (\text{Sqrt}[g]*(c + d*x))/(d*\text{Sqrt}[f] + c*\text{Sqrt}[g])])/(2*g^(3/2))$$

### Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))^(r_.)]*(RFX_), x_Symbol] := Dist[p*r, Int[RFX*Log[a + b*x], x], x] + (Dist[q*r, Int[RFX*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFX, x], x]) /; FreeQ[{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFX, x] && NeQ[b*c - a*d, 0] && !MatchQ[RFX, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.)] /; IntegersQ[m, n]
```

### Rule 321

```
Int[((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.))^(n_.))^(p_.), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

### Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.), x_Symbol] := Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x]
]; FreeQ[{c, n}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.)
)*(x_)^(r_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx &= n \int \frac{x^2 \log(a+bx)}{f-gx^2} dx - n \int \frac{x^2 \log(c+dx)}{f-gx^2} dx - \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right) \\
&= \frac{x\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{g} + n \int \left(-\frac{\log(a+bx)}{g} + \frac{f \log(a+bx)}{g(f-gx^2)}\right) dx \\
&= \frac{x\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{g} - \frac{\sqrt{f} \tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{g^{3/2}} \\
&= \frac{x\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{g} - \frac{\sqrt{f} \tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{g^{3/2}} \\
&= -\frac{n(a+bx) \log(a+bx)}{bg} + \frac{n(c+dx) \log(c+dx)}{dg} + \frac{x\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{g} \\
&= -\frac{n(a+bx) \log(a+bx)}{bg} + \frac{n(c+dx) \log(c+dx)}{dg} + \frac{x\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{g} \\
&= -\frac{n(a+bx) \log(a+bx)}{bg} + \frac{n(c+dx) \log(c+dx)}{dg} + \frac{x\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{g} \\
&= -\frac{n(a+bx) \log(a+bx)}{bg} + \frac{n(c+dx) \log(c+dx)}{dg} + \frac{x\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{g}
\end{aligned}$$

**Mathematica [A]** time = 0.24023, size = 467, normalized size = 0.85

$$\sqrt{f}n \left( \text{PolyLog}\left(2, \frac{b(\sqrt{f}-\sqrt{g}x)}{a\sqrt{g}+b\sqrt{f}}\right) - \text{PolyLog}\left(2, \frac{d(\sqrt{f}-\sqrt{g}x)}{c\sqrt{g}+d\sqrt{f}}\right) + \log(\sqrt{f}-\sqrt{g}x) \left( \log\left(\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right) - \log\left(\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right) \right) \right) - \sqrt{f}n \left( \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f - g\*x^2), x]

[Out] ((-2\*sqrt[g]\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/b + (2\*(b\*c - a\*d)\*sqrt[g]\*n\*Log[c + d\*x])/(b\*d) - sqrt[f]\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[S

```

qrt[f] - Sqrt[g]*x] + Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] +
Sqrt[g]*x] + Sqrt[f]*n*((Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])]) -
Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*Log[Sqrt[f] - Sqrt[g]*x]
+ PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - PolyLog[
2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - Sqrt[f]*n*((Log[-(
Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g])]) - Log[-((Sqrt[g]*(c + d*x))/(
d*Sqrt[f] - c*Sqrt[g])]))*Log[Sqrt[f] + Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f]
+ Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*
x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*g^(3/2))

```

**Maple [F]** time = 0.418, size = 0, normalized size = 0.

$$\int \frac{x^2}{-gx^2 + f} \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

```
[Out] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{x^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{gx^2 - f}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")`

[Out] `integral(-x^2*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2 - f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")`

[Out] `integrate(-x^2*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)`

$$3.78 \quad \int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx$$

**Optimal.** Leaf size=403

$$\frac{n \operatorname{PolyLog}\left(2, -\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g} - \frac{n \operatorname{PolyLog}\left(2, \frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g} + \frac{n \operatorname{PolyLog}\left(2, -\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g} + \frac{n \operatorname{PolyLog}\left(2, \frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g} + \frac{\log(f-gx^2)}{2g}$$

[Out]  $-(n \operatorname{Log}[a + b*x] * \operatorname{Log}[(b * (\operatorname{Sqrt}[f] - \operatorname{Sqrt}[g]*x)) / (b * \operatorname{Sqrt}[f] + a * \operatorname{Sqrt}[g])]) / (2 * g) + (n \operatorname{Log}[c + d*x] * \operatorname{Log}[(d * (\operatorname{Sqrt}[f] - \operatorname{Sqrt}[g]*x)) / (d * \operatorname{Sqrt}[f] + c * \operatorname{Sqrt}[g])]) / (2 * g) - (n \operatorname{Log}[a + b*x] * \operatorname{Log}[(b * (\operatorname{Sqrt}[f] + \operatorname{Sqrt}[g]*x)) / (b * \operatorname{Sqrt}[f] - a * \operatorname{Sqrt}[g])]) / (2 * g) + (n \operatorname{Log}[c + d*x] * \operatorname{Log}[(d * (\operatorname{Sqrt}[f] + \operatorname{Sqrt}[g]*x)) / (d * \operatorname{Sqrt}[f] - c * \operatorname{Sqrt}[g])]) / (2 * g) + ((n \operatorname{Log}[a + b*x] - \operatorname{Log}[e * ((a + b*x) / (c + d*x))^n] - n \operatorname{Log}[c + d*x]) * \operatorname{Log}[f - g*x^2]) / (2 * g) - (n \operatorname{PolyLog}[2, -((\operatorname{Sqrt}[g] * (a + b*x)) / (b * \operatorname{Sqrt}[f] - a * \operatorname{Sqrt}[g])])]) / (2 * g) - (n \operatorname{PolyLog}[2, (\operatorname{Sqrt}[g] * (a + b*x)) / (b * \operatorname{Sqrt}[f] + a * \operatorname{Sqrt}[g])]) / (2 * g) + (n \operatorname{PolyLog}[2, -((\operatorname{Sqrt}[g] * (c + d*x)) / (d * \operatorname{Sqrt}[f] - c * \operatorname{Sqrt}[g])])]) / (2 * g) + (n \operatorname{PolyLog}[2, (\operatorname{Sqrt}[g] * (c + d*x)) / (d * \operatorname{Sqrt}[f] + c * \operatorname{Sqrt}[g])]) / (2 * g)$

**Rubi [A]** time = 0.352342, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2513, 260, 2416, 2394, 2393, 2391}

$$\frac{n \operatorname{PolyLog}\left(2, -\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g} - \frac{n \operatorname{PolyLog}\left(2, \frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g} + \frac{n \operatorname{PolyLog}\left(2, -\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g} + \frac{n \operatorname{PolyLog}\left(2, \frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g} + \frac{\log(f-gx^2)}{2g}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x * \operatorname{Log}[e * ((a + b*x) / (c + d*x))^n]) / (f - g*x^2), x]$

[Out]  $-(n \operatorname{Log}[a + b*x] * \operatorname{Log}[(b * (\operatorname{Sqrt}[f] - \operatorname{Sqrt}[g]*x)) / (b * \operatorname{Sqrt}[f] + a * \operatorname{Sqrt}[g])]) / (2 * g) + (n \operatorname{Log}[c + d*x] * \operatorname{Log}[(d * (\operatorname{Sqrt}[f] - \operatorname{Sqrt}[g]*x)) / (d * \operatorname{Sqrt}[f] + c * \operatorname{Sqrt}[g])]) / (2 * g) - (n \operatorname{Log}[a + b*x] * \operatorname{Log}[(b * (\operatorname{Sqrt}[f] + \operatorname{Sqrt}[g]*x)) / (b * \operatorname{Sqrt}[f] - a * \operatorname{Sqrt}[g])]) / (2 * g) + (n \operatorname{Log}[c + d*x] * \operatorname{Log}[(d * (\operatorname{Sqrt}[f] + \operatorname{Sqrt}[g]*x)) / (d * \operatorname{Sqrt}[f] - c * \operatorname{Sqrt}[g])]) / (2 * g) + ((n \operatorname{Log}[a + b*x] - \operatorname{Log}[e * ((a + b*x) / (c + d*x))^n] - n \operatorname{Log}[c + d*x]) * \operatorname{Log}[f - g*x^2]) / (2 * g) - (n \operatorname{PolyLog}[2, -((\operatorname{Sqrt}[g] * (a + b*x)) / (b * \operatorname{Sqrt}[f] - a * \operatorname{Sqrt}[g])])]) / (2 * g) - (n \operatorname{PolyLog}[2, (\operatorname{Sqrt}[g] * (a + b*x)) / (b * \operatorname{Sqrt}[f] + a * \operatorname{Sqrt}[g])]) / (2 * g) + (n \operatorname{PolyLog}[2, -((\operatorname{Sqrt}[g] * (c + d*x)) / (d * \operatorname{Sqrt}[f] - c * \operatorname{Sqrt}[g])])]) / (2 * g) + (n \operatorname{PolyLog}[2, (\operatorname{Sqrt}[g] * (c + d*x)) / (d * \operatorname{Sqrt}[f] + c * \operatorname{Sqrt}[g])]) / (2 * g)$



t[g]])]/(2\*g)

### Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFX_), x_Symbol] := Dist[p*r, Int[RFX*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFX*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFX, x], x] /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFX, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFX, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n
]
```

### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a
+ b*Log[c*(d + e*x)^n]]^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{f-gx^2} dx &= n \int \frac{x \log(a+bx)}{f-gx^2} dx - n \int \frac{x \log(c+dx)}{f-gx^2} dx - \left( n \log(a+bx) - \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) - n \log(c+dx) \right) \log(f-gx^2) \\
&= \frac{\left( n \log(a+bx) - \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) - n \log(c+dx) \right) \log(f-gx^2)}{2g} + n \int \left( \frac{\log(a+bx)}{2\sqrt{g}(\sqrt{f}-\sqrt{gx})} - \frac{\log(c+dx)}{2\sqrt{g}(\sqrt{f}+\sqrt{gx})} \right) dx \\
&= \frac{\left( n \log(a+bx) - \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) - n \log(c+dx) \right) \log(f-gx^2)}{2g} + \frac{n \int \frac{\log(a+bx)}{\sqrt{f}-\sqrt{gx}} dx}{2\sqrt{g}} - \frac{n \int \frac{\log(c+dx)}{\sqrt{f}+\sqrt{gx}} dx}{2\sqrt{g}} \\
&= -\frac{n \log(a+bx) \log \left( \frac{b(\sqrt{f}-\sqrt{gx})}{b\sqrt{f}+a\sqrt{g}} \right)}{2g} + \frac{n \log(c+dx) \log \left( \frac{d(\sqrt{f}-\sqrt{gx})}{d\sqrt{f}+c\sqrt{g}} \right)}{2g} - \frac{n \log(a+bx) \log \left( \frac{b(\sqrt{f}+\sqrt{gx})}{b\sqrt{f}-a\sqrt{g}} \right)}{2g} \\
&= -\frac{n \log(a+bx) \log \left( \frac{b(\sqrt{f}-\sqrt{gx})}{b\sqrt{f}+a\sqrt{g}} \right)}{2g} + \frac{n \log(c+dx) \log \left( \frac{d(\sqrt{f}-\sqrt{gx})}{d\sqrt{f}+c\sqrt{g}} \right)}{2g} - \frac{n \log(a+bx) \log \left( \frac{b(\sqrt{f}+\sqrt{gx})}{b\sqrt{f}-a\sqrt{g}} \right)}{2g} \\
&= -\frac{n \log(a+bx) \log \left( \frac{b(\sqrt{f}-\sqrt{gx})}{b\sqrt{f}+a\sqrt{g}} \right)}{2g} + \frac{n \log(c+dx) \log \left( \frac{d(\sqrt{f}-\sqrt{gx})}{d\sqrt{f}+c\sqrt{g}} \right)}{2g} - \frac{n \log(a+bx) \log \left( \frac{b(\sqrt{f}+\sqrt{gx})}{b\sqrt{f}-a\sqrt{g}} \right)}{2g}
\end{aligned}$$

**Mathematica [A]** time = 0.121141, size = 413, normalized size = 1.02

$$-n \text{PolyLog} \left( 2, \frac{b(\sqrt{f}-\sqrt{gx})}{a\sqrt{g}+b\sqrt{f}} \right) - n \text{PolyLog} \left( 2, \frac{b(\sqrt{f}+\sqrt{gx})}{b\sqrt{f}-a\sqrt{g}} \right) + n \text{PolyLog} \left( 2, \frac{d(\sqrt{f}-\sqrt{gx})}{c\sqrt{g}+d\sqrt{f}} \right) + n \text{PolyLog} \left( 2, \frac{d(\sqrt{f}+\sqrt{gx})}{d\sqrt{f}-c\sqrt{g}} \right) + \log \left( \sqrt{f-gx^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f - g\*x^2), x]

[Out] -(-(n\*Log[(Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] + a\*Sqrt[g])]\*Log[Sqrt[f] - Sqrt[g]\*x]) + Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[Sqrt[f] - Sqrt[g]\*x] + n\*Log[(Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] + c\*Sqrt[g])]\*Log[Sqrt[f] - Sqrt[g]\*x] - n\*Log[-((Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] - a\*Sqrt[g]))]\*Log[Sqrt[f] + Sqrt[g]\*x] + Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[Sqrt[f] + Sqrt[g]\*x] + n\*Log[-((Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] - c\*Sqrt[g]))]\*Log[Sqrt[f] + Sqrt[g]\*x] - n\*PolyLog[2, (b\*(Sqrt[f] - Sqrt[g]\*x))/(b\*Sqrt[f] + a\*Sqrt[g])] + n\*PolyLog[2, (d\*(Sqrt[f] - Sqrt[g]\*x))/(d\*Sqrt[f] + c\*Sqrt[g])] - n\*PolyLog[2, (b\*(Sqrt[f] + Sqrt[g]\*x))/(b\*Sqrt[f] - a\*Sqrt[g])] + n\*PolyLog[2, (d\*(Sqrt[f] + Sqrt[g]\*x))/(d\*Sqrt[f] - c\*Sqrt[g])])

$/(d*\text{Sqrt}[f] - c*\text{Sqrt}[g]))/(2*g)$

---

**Maple [F]** time = 0.44, size = 0, normalized size = 0.

$$\int \frac{x}{-gx^2 + f} \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f), x)`

[Out] `int(x*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2 - f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f), x, algorithm="maxima")`

[Out] `-integrate(x*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2 - f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f), x, algorithm="fricas")`

[Out] `integral(-x*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x**2+f), x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{x \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2 - f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f), x, algorithm="giac")`

[Out] `integrate(-x*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)`

$$3.79 \quad \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx$$

**Optimal.** Leaf size=291

$$\frac{n\text{PolyLog}\left(2, \frac{(a+bx)(d\sqrt{f}-c\sqrt{g})}{(c+dx)(b\sqrt{f}-a\sqrt{g})}\right)}{2\sqrt{f}\sqrt{g}} - \frac{n\text{PolyLog}\left(2, \frac{(a+bx)(c\sqrt{g}+d\sqrt{f})}{(c+dx)(a\sqrt{g}+b\sqrt{f})}\right)}{2\sqrt{f}\sqrt{g}} + \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1 - \frac{(a+bx)(d\sqrt{f}-c\sqrt{g})}{(c+dx)(b\sqrt{f}-a\sqrt{g})}\right)}{2\sqrt{f}\sqrt{g}} - \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1 - \frac{(a+bx)(c\sqrt{g}+d\sqrt{f})}{(c+dx)(a\sqrt{g}+b\sqrt{f})}\right)}{2\sqrt{f}\sqrt{g}}$$

[Out] (Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[1 - ((d\*Sqrt[f] - c\*Sqrt[g])\*(a + b\*x))/((b\*Sqrt[f] - a\*Sqrt[g])\*(c + d\*x))])/(2\*Sqrt[f]\*Sqrt[g]) - (Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[1 - ((d\*Sqrt[f] + c\*Sqrt[g])\*(a + b\*x))/((b\*Sqrt[f] + a\*Sqrt[g])\*(c + d\*x))])/(2\*Sqrt[f]\*Sqrt[g]) + (n\*PolyLog[2, ((d\*Sqrt[f] - c\*Sqrt[g])\*(a + b\*x))/((b\*Sqrt[f] - a\*Sqrt[g])\*(c + d\*x))])/(2\*Sqrt[f]\*Sqrt[g]) - (n\*PolyLog[2, ((d\*Sqrt[f] + c\*Sqrt[g])\*(a + b\*x))/((b\*Sqrt[f] + a\*Sqrt[g])\*(c + d\*x))])/(2\*Sqrt[f]\*Sqrt[g])

**Rubi [A]** time = 0.317435, antiderivative size = 468, normalized size of antiderivative = 1.61, number of steps used = 18, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {2513, 2409, 2394, 2393, 2391, 208}

$$\frac{n\text{PolyLog}\left(2, -\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2\sqrt{f}\sqrt{g}} - \frac{n\text{PolyLog}\left(2, \frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}} - \frac{n\text{PolyLog}\left(2, -\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2\sqrt{f}\sqrt{g}} + \frac{n\text{PolyLog}\left(2, \frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}} - \frac{\tanh^{-1}\left(\frac{(a+bx)\sqrt{g}}{(c+dx)\sqrt{g}}\right)}{2\sqrt{f}\sqrt{g}}$$

Antiderivative was successfully verified.

[In] Int[Log[e\*((a + b\*x)/(c + d\*x))^n]/(f - g\*x^2), x]

[Out] -((ArcTanh[(Sqrt[g]\*x)/Sqrt[f]]\*(n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x]))/(Sqrt[f]\*Sqrt[g]) - (n\*Log[a + b\*x]\*Log[(b\*(Sqrt[f] - Sqrt[g]\*x))/(b\*Sqrt[f] + a\*Sqrt[g])])/(2\*Sqrt[f]\*Sqrt[g]) + (n\*Log[c + d\*x]\*Log[(d\*(Sqrt[f] - Sqrt[g]\*x))/(d\*Sqrt[f] + c\*Sqrt[g])])/(2\*Sqrt[f]\*Sqrt[g]) + (n\*Log[a + b\*x]\*Log[(b\*(Sqrt[f] + Sqrt[g]\*x))/(b\*Sqrt[f] - a\*Sqrt[g])])/(2\*Sqrt[f]\*Sqrt[g]) - (n\*Log[c + d\*x]\*Log[(d\*(Sqrt[f] + Sqrt[g]\*x))/(d\*Sqrt[f] - c\*Sqrt[g])])/(2\*Sqrt[f]\*Sqrt[g]) + (n\*PolyLog[2, -((Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] - a\*Sqrt[g]))])/(2\*Sqrt[f]\*Sqrt[g]) - (n\*PolyLog[2, (Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] + a\*Sqrt[g])])/(2\*Sqrt[f]\*Sqrt[g]) - (n\*PolyLog[2, -((Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] - c\*Sqrt[g]))])/(2\*Sqrt[f]\*Sqrt[g]) + (n\*PolyLog[2, (Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] + c\*Sqrt[g])])/(2\*Sqrt[f]\*Sqrt[g])

g])

### Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFX_), x_Symbol] := Dist[p*r, Int[RFX*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFX*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFX, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFX, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFX, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n
]
```

### Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx &= n \int \frac{\log(a+bx)}{f-gx^2} dx - n \int \frac{\log(c+dx)}{f-gx^2} dx - \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{\sqrt{f}\sqrt{g}} + n \int \left(\frac{\log(a+bx)}{2\sqrt{f}(\sqrt{f}-\sqrt{gx})} + \frac{\log(c+dx)}{2\sqrt{f}(\sqrt{f}+\sqrt{gx})}\right) dx \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{\sqrt{f}\sqrt{g}} + \frac{n \int \frac{\log(a+bx)}{\sqrt{f}-\sqrt{gx}} dx}{2\sqrt{f}} + \frac{n \int \frac{\log(c+dx)}{\sqrt{f}+\sqrt{gx}} dx}{2\sqrt{f}} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{\sqrt{f}\sqrt{g}} - \frac{n \log(a+bx) \log\left(\frac{b(\sqrt{f}-\sqrt{gx})}{b\sqrt{f}+a\sqrt{g}}\right)}{2\sqrt{f}\sqrt{g}} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{\sqrt{f}\sqrt{g}} - \frac{n \log(a+bx) \log\left(\frac{b(\sqrt{f}-\sqrt{gx})}{b\sqrt{f}+a\sqrt{g}}\right)}{2\sqrt{f}\sqrt{g}} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{\sqrt{f}\sqrt{g}} - \frac{n \log(a+bx) \log\left(\frac{b(\sqrt{f}-\sqrt{gx})}{b\sqrt{f}+a\sqrt{g}}\right)}{2\sqrt{f}\sqrt{g}}
\end{aligned}$$

**Mathematica [A]** time = 0.101591, size = 421, normalized size = 1.45

$$n \text{PolyLog}\left(2, \frac{b(\sqrt{f}-\sqrt{gx})}{a\sqrt{g}+b\sqrt{f}}\right) - n \text{PolyLog}\left(2, \frac{b(\sqrt{f}+\sqrt{gx})}{b\sqrt{f}-a\sqrt{g}}\right) - n \text{PolyLog}\left(2, \frac{d(\sqrt{f}-\sqrt{gx})}{c\sqrt{g}+d\sqrt{f}}\right) + n \text{PolyLog}\left(2, \frac{d(\sqrt{f}+\sqrt{gx})}{d\sqrt{f}-c\sqrt{g}}\right) - \log\left(\sqrt{f}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*((a + b\*x)/(c + d\*x))^n]/(f - g\*x^2), x]

[Out] (n\*Log[(Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] + a\*Sqrt[g])]\*Log[Sqrt[f] - Sqrt[g]\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[Sqrt[f] - Sqrt[g]\*x] - n\*Log[(Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] + c\*Sqrt[g])]\*Log[Sqrt[f] - Sqrt[g]\*x] - n\*Log[-((Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] - a\*Sqrt[g]))]\*Log[Sqrt[f] + Sqrt[g]\*x] + Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[Sqrt[f] + Sqrt[g]\*x] + n\*Log[-((Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] - c\*Sqrt[g]))]\*Log[Sqrt[f] + Sqrt[g]\*x] + n\*PolyLog[2, (b\*(Sqrt[f] - Sqrt[g]\*x))/(b\*Sqrt[f] + a\*Sqrt[g])] - n\*PolyLog[2, (d\*(Sqrt[f] - Sqrt[g]\*x))/(d\*Sqrt[f] + c\*Sqrt[g])] - n\*PolyLog[2, (b\*(Sqrt[f] + Sqrt[g]\*x))/(b\*Sqrt[f] - a\*Sqrt[g])] - n\*PolyLog[2, (d\*(Sqrt[f] + Sqrt[g]\*x))/(d\*Sqrt[f] - c\*Sqrt[g])]

$] * x) / (b * \text{Sqrt}[f] - a * \text{Sqrt}[g]) + n * \text{PolyLog}[2, (d * (\text{Sqrt}[f] + \text{Sqrt}[g] * x)) / (d * \text{Sqrt}[f] - c * \text{Sqrt}[g])] / (2 * \text{Sqrt}[f] * \text{Sqrt}[g])$

**Maple [F]** time = 0.474, size = 0, normalized size = 0.

$$\int \frac{1}{-gx^2 + f} \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)`

[Out] `int(ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{\log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{gx^2 - f}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")`

[Out] `integral(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)`



---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f), x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f), x, algorithm="giac")`

[Out] `integrate(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)`

$$3.80 \quad \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f-gx^2)} dx$$

**Optimal.** Leaf size=518

$$-\frac{n\text{PolyLog}\left(2, -\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f} - \frac{n\text{PolyLog}\left(2, \frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f} + \frac{n\text{PolyLog}\left(2, \frac{bx}{a} + 1\right)}{f} + \frac{n\text{PolyLog}\left(2, -\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f} + \frac{n\text{PolyLog}\left(2, \frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2f}$$

[Out] (n\*Log[-((b\*x)/a)]\*Log[a + b\*x])/f - (n\*Log[-((d\*x)/c)]\*Log[c + d\*x])/f - (Log[x]\*(n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x]))/f - (n\*Log[a + b\*x]\*Log[(b\*(Sqrt[f] - Sqrt[g]\*x))/(b\*Sqrt[f] + a\*Sqrt[g])])/(2\*f) + (n\*Log[c + d\*x]\*Log[(d\*(Sqrt[f] - Sqrt[g]\*x))/(d\*Sqrt[f] + c\*Sqrt[g])])/(2\*f) - (n\*Log[a + b\*x]\*Log[(b\*(Sqrt[f] + Sqrt[g]\*x))/(b\*Sqrt[f] - a\*Sqrt[g])])/(2\*f) + (n\*Log[c + d\*x]\*Log[(d\*(Sqrt[f] + Sqrt[g]\*x))/(d\*Sqrt[f] - c\*Sqrt[g])])/(2\*f) + ((n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x])\*Log[f - g\*x^2])/(2\*f) - (n\*PolyLog[2, -((Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] - a\*Sqrt[g]))])/(2\*f) - (n\*PolyLog[2, (Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] + a\*Sqrt[g])])/(2\*f) + (n\*PolyLog[2, 1 + (b\*x)/a])/f + (n\*PolyLog[2, -((Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] - c\*Sqrt[g]))])/(2\*f) + (n\*PolyLog[2, (Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] + c\*Sqrt[g])])/(2\*f) - (n\*PolyLog[2, 1 + (d\*x)/c])/f

**Rubi [A]** time = 0.602265, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {2513, 266, 36, 29, 31, 2416, 2394, 2315, 260, 2393, 2391}

$$-\frac{n\text{PolyLog}\left(2, -\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f} - \frac{n\text{PolyLog}\left(2, \frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f} + \frac{n\text{PolyLog}\left(2, \frac{bx}{a} + 1\right)}{f} + \frac{n\text{PolyLog}\left(2, -\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f} + \frac{n\text{PolyLog}\left(2, \frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2f}$$

Antiderivative was successfully verified.

[In] Int[Log[e\*((a + b\*x)/(c + d\*x))^n]/(x\*(f - g\*x^2)), x]

[Out] (n\*Log[-((b\*x)/a)]\*Log[a + b\*x])/f - (n\*Log[-((d\*x)/c)]\*Log[c + d\*x])/f - (Log[x]\*(n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x]))/f - (n\*Log[a + b\*x]\*Log[(b\*(Sqrt[f] - Sqrt[g]\*x))/(b\*Sqrt[f] + a\*Sqrt[g])])/(2\*f) + (n\*Log[c + d\*x]\*Log[(d\*(Sqrt[f] - Sqrt[g]\*x))/(d\*Sqrt[f] + c\*Sqrt[g])])/(2\*f) - (n\*Log[a + b\*x]\*Log[(b\*(Sqrt[f] + Sqrt[g]\*x))/(b\*Sqrt[f] - a\*Sqrt[g])])/(2\*f) + (n\*Log[c + d\*x]\*Log[(d\*(Sqrt[f] + Sqrt[g]\*x))/(d\*Sqrt[f] - c\*Sqrt[g])])/(2\*f) + ((n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x])\*Log[f - g\*x^2])/(2\*f) - (n\*PolyLog[2, -((Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] - a\*Sqrt[g]))])/(2\*f) - (n\*PolyLog[2, (Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] + a\*Sqrt[g])])/(2\*f) + (n\*PolyLog[2, 1 + (b\*x)/a])/f + (n\*PolyLog[2, -((Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] - c\*Sqrt[g]))])/(2\*f) + (n\*PolyLog[2, (Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] + c\*Sqrt[g])])/(2\*f) - (n\*PolyLog[2, 1 + (d\*x)/c])/f

```
Sqrt[g]])/(2*f) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f]
- c*Sqrt[g])])/(2*f) + ((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] -
n*Log[c + d*x]*Log[f - g*x^2])/(2*f) - (n*PolyLog[2, -((Sqrt[g]*(a + b*x)
)/(b*Sqrt[f] - a*Sqrt[g]))])/(2*f) - (n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*S
qrt[f] + a*Sqrt[g])])/(2*f) + (n*PolyLog[2, 1 + (b*x)/a])/f + (n*PolyLog[2,
-((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*f) + (n*PolyLog[2, (Sq
rt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f) - (n*PolyLog[2, 1 + (d*x)/
c])/f
```

### Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFX_), x_Symbol] := Dist[p*r, Int[RFX*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFX*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFX, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFX, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFX, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(p_.), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a
+ b*Log[c*(d + e*x)^n]]^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.)))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.)))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f-gx^2)} dx &= n \int \frac{\log(a+bx)}{x(f-gx^2)} dx - n \int \frac{\log(c+dx)}{x(f-gx^2)} dx - \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right) \\
&= n \int \left(\frac{\log(a+bx)}{fx} - \frac{gx \log(a+bx)}{f(-f+gx^2)}\right) dx - n \int \left(\frac{\log(c+dx)}{fx} - \frac{gx \log(c+dx)}{f(-f+gx^2)}\right) dx - \frac{1}{2} \left(n \log(a+bx) - n \log(c+dx)\right) \\
&= \frac{n \int \frac{\log(a+bx)}{x} dx}{f} - \frac{n \int \frac{\log(c+dx)}{x} dx}{f} - \frac{(gn) \int \frac{x \log(a+bx)}{-f+gx^2} dx}{f} + \frac{(gn) \int \frac{x \log(c+dx)}{-f+gx^2} dx}{f} - \frac{\left(n \log(a+bx) - n \log(c+dx)\right)}{f} \\
&= \frac{n \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{n \log\left(-\frac{dx}{c}\right) \log(c+dx)}{f} - \frac{\log(x) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{f} \\
&= \frac{n \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{n \log\left(-\frac{dx}{c}\right) \log(c+dx)}{f} - \frac{\log(x) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{f} \\
&= \frac{n \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{n \log\left(-\frac{dx}{c}\right) \log(c+dx)}{f} - \frac{\log(x) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{f} \\
&= \frac{n \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{n \log\left(-\frac{dx}{c}\right) \log(c+dx)}{f} - \frac{\log(x) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{f} \\
&= \frac{n \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{n \log\left(-\frac{dx}{c}\right) \log(c+dx)}{f} - \frac{\log(x) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{f} \\
&= \frac{n \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{n \log\left(-\frac{dx}{c}\right) \log(c+dx)}{f} - \frac{\log(x) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{f}
\end{aligned}$$

**Mathematica [A]** time = 0.186835, size = 487, normalized size = 0.94

$$-n \text{PolyLog}\left(2, \frac{b(\sqrt{f}-\sqrt{gx})}{a\sqrt{g}+b\sqrt{f}}\right) - n \text{PolyLog}\left(2, \frac{b(\sqrt{f}+\sqrt{gx})}{b\sqrt{f}-a\sqrt{g}}\right) + 2n \text{PolyLog}\left(2, -\frac{bx}{a}\right) + n \text{PolyLog}\left(2, \frac{d(\sqrt{f}-\sqrt{gx})}{c\sqrt{g}+d\sqrt{f}}\right) + n \text{PolyLog}\left(2, \frac{d(\sqrt{f}+\sqrt{gx})}{d\sqrt{f}-c\sqrt{g}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*((a + b\*x)/(c + d\*x))^n]/(x\*(f - g\*x^2)), x]

[Out] -(2\*n\*Log[x]\*Log[1 + (b\*x)/a] - 2\*Log[x]\*Log[e\*((a + b\*x)/(c + d\*x))^n] - 2\*n\*Log[x]\*Log[1 + (d\*x)/c] - n\*Log[(Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] + a\*Sqrt[g])]\*Log[Sqrt[f] - Sqrt[g]\*x] + Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[Sqrt[f]

- Sqrt[g]\*x] + n\*Log[(Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] + c\*Sqrt[g])]\*Log[Sqrt[f] - Sqrt[g]\*x] - n\*Log[-((Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] - a\*Sqrt[g]))]\*Log[Sqrt[f] + Sqrt[g]\*x] + Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[Sqrt[f] + Sqrt[g]\*x] + n\*Log[-((Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] - c\*Sqrt[g]))]\*Log[Sqrt[f] + Sqrt[g]\*x] + 2\*n\*PolyLog[2, -((b\*x)/a)] - 2\*n\*PolyLog[2, -((d\*x)/c)] - n\*PolyLog[2, (b\*(Sqrt[f] - Sqrt[g]\*x))/(b\*Sqrt[f] + a\*Sqrt[g])] + n\*PolyLog[2, (d\*(Sqrt[f] - Sqrt[g]\*x))/(d\*Sqrt[f] + c\*Sqrt[g])] - n\*PolyLog[2, (b\*(Sqrt[f] + Sqrt[g]\*x))/(b\*Sqrt[f] - a\*Sqrt[g])] + n\*PolyLog[2, (d\*(Sqrt[f] + Sqrt[g]\*x))/(d\*Sqrt[f] - c\*Sqrt[g])]/(2\*f)

**Maple [F]** time = 0.457, size = 0, normalized size = 0.

$$\int \frac{1}{x(-gx^2 + f)} \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*((b\*x+a)/(d\*x+c))^n)/x/(-g\*x^2+f),x)

[Out] int(ln(e\*((b\*x+a)/(d\*x+c))^n)/x/(-g\*x^2+f),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(gx^2 - f)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*((b\*x+a)/(d\*x+c))^n)/x/(-g\*x^2+f),x, algorithm="maxima")

[Out] -integrate(log(e\*((b\*x + a)/(d\*x + c))^n)/((g\*x^2 - f)\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^3 - fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="fricas")`

[Out] `integral(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^3 - f*x), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(e*((b*x+a)/(d*x+c))^n)/x/(-g*x**2+f),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(gx^2-f)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="giac")`

[Out] `integrate(-log(e*((b*x + a)/(d*x + c))^n)/((g*x^2 - f)*x), x)`

$$3.81 \quad \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)} dx$$

**Optimal.** Leaf size=596

$$\frac{\sqrt{gn}\text{PolyLog}\left(2, -\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}} - \frac{\sqrt{gn}\text{PolyLog}\left(2, \frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f^{3/2}} - \frac{\sqrt{gn}\text{PolyLog}\left(2, -\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}} + \frac{\sqrt{gn}\text{PolyLog}\left(2, \frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2f^{3/2}}$$

[Out] (b\*n\*Log[x])/(a\*f) - (d\*n\*Log[x])/(c\*f) - (b\*n\*Log[a + b\*x])/(a\*f) - (n\*Log[a + b\*x])/(f\*x) + (d\*n\*Log[c + d\*x])/(c\*f) + (n\*Log[c + d\*x])/(f\*x) + (n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x])/(f\*x) - (Sqrt[g]\*ArcTanh[(Sqrt[g]\*x)/Sqrt[f]]\*(n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x]))/f^(3/2) - (Sqrt[g]\*n\*Log[a + b\*x]\*Log[(b\*(Sqrt[f] - Sqrt[g]\*x))/(b\*Sqrt[f] + a\*Sqrt[g])])/(2\*f^(3/2)) + (Sqrt[g]\*n\*Log[c + d\*x]\*Log[(d\*(Sqrt[f] - Sqrt[g]\*x))/(d\*Sqrt[f] + c\*Sqrt[g])])/(2\*f^(3/2)) + (Sqrt[g]\*n\*Log[a + b\*x]\*Log[(b\*(Sqrt[f] + Sqrt[g]\*x))/(b\*Sqrt[f] - a\*Sqrt[g])])/(2\*f^(3/2)) - (Sqrt[g]\*n\*Log[c + d\*x]\*Log[(d\*(Sqrt[f] + Sqrt[g]\*x))/(d\*Sqrt[f] - c\*Sqrt[g])])/(2\*f^(3/2)) + (Sqrt[g]\*n\*PolyLog[2, -((Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] - a\*Sqrt[g]))])/(2\*f^(3/2)) - (Sqrt[g]\*n\*PolyLog[2, (Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] + a\*Sqrt[g])])/(2\*f^(3/2)) - (Sqrt[g]\*n\*PolyLog[2, -((Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] - c\*Sqrt[g]))])/(2\*f^(3/2)) + (Sqrt[g]\*n\*PolyLog[2, (Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] + c\*Sqrt[g])])/(2\*f^(3/2))

**Rubi [A]** time = 0.583434, antiderivative size = 596, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 12, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2513, 325, 208, 2416, 2395, 36, 29, 31, 2409, 2394, 2393, 2391}

$$\frac{\sqrt{gn}\text{PolyLog}\left(2, -\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}} - \frac{\sqrt{gn}\text{PolyLog}\left(2, \frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f^{3/2}} - \frac{\sqrt{gn}\text{PolyLog}\left(2, -\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}} + \frac{\sqrt{gn}\text{PolyLog}\left(2, \frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2f^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Log[e\*((a + b\*x)/(c + d\*x))^n]/(x^2\*(f - g\*x^2)), x]

[Out] (b\*n\*Log[x])/(a\*f) - (d\*n\*Log[x])/(c\*f) - (b\*n\*Log[a + b\*x])/(a\*f) - (n\*Log[a + b\*x])/(f\*x) + (d\*n\*Log[c + d\*x])/(c\*f) + (n\*Log[c + d\*x])/(f\*x) + (n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x])/(f\*x) - (Sqrt[g]\*ArcTanh[(Sqrt[g]\*x)/Sqrt[f]]\*(n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x]))/f^(3/2) - (Sqrt[g]\*n\*Log[a + b\*x]\*Log[(b\*(Sqrt[f] - Sqrt[g]\*x))/(b\*Sqrt[f] + a\*Sqrt[g])])/(2\*f^(3/2)) + (Sqrt[g]\*n\*Log[c + d\*x]\*Log[(d\*(Sqrt[f] - Sqrt[g]\*x))/(d\*Sqrt[f] + c\*Sqrt[g])])/(2\*f^(3/2)) + (Sqrt[g]\*n\*Log[a + b\*x]\*Log[(b\*(Sqrt[f] + Sqrt[g]\*x))/(b\*Sqrt[f] - a\*Sqrt[g])])/(2\*f^(3/2)) - (Sqrt[g]\*n\*Log[c + d\*x]\*Log[(d\*(Sqrt[f] + Sqrt[g]\*x))/(d\*Sqrt[f] - c\*Sqrt[g])])/(2\*f^(3/2)) + (Sqrt[g]\*n\*PolyLog[2, -((Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] - a\*Sqrt[g]))])/(2\*f^(3/2)) - (Sqrt[g]\*n\*PolyLog[2, (Sqrt[g]\*(a + b\*x))/(b\*Sqrt[f] + a\*Sqrt[g])])/(2\*f^(3/2)) - (Sqrt[g]\*n\*PolyLog[2, -((Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] - c\*Sqrt[g]))])/(2\*f^(3/2)) + (Sqrt[g]\*n\*PolyLog[2, (Sqrt[g]\*(c + d\*x))/(d\*Sqrt[f] + c\*Sqrt[g])])/(2\*f^(3/2))



```

*x))n] - n*Log[c + d*x])/f(3/2) - (Sqrt[g]*n*Log[a + b*x]*Log[(b*(Sqrt[f]
] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g]))/(2*f(3/2)) + (Sqrt[g]*n*Log[c +
d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g]))/(2*f(3/2)) +
(Sqrt[g]*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g]
)]))/(2*f(3/2)) - (Sqrt[g]*n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d
*Sqrt[f] - c*Sqrt[g]))/(2*f(3/2)) + (Sqrt[g]*n*PolyLog[2, -((Sqrt[g]*(a +
b*x))/(b*Sqrt[f] - a*Sqrt[g]))]/(2*f(3/2)) - (Sqrt[g]*n*PolyLog[2, (Sqrt
[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g]))]/(2*f(3/2)) - (Sqrt[g]*n*PolyLog[2
, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))]/(2*f(3/2)) + (Sqrt[g]*n
*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g]))]/(2*f(3/2))

```

### Rule 2513

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_)^(p_.))*((c_.) + (d_.)*(x_)^(q_.))
^(r_.)]*(RFX_), x_Symbol] := Dist[p*r, Int[RFX*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFX*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFX, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFX, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFX, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]

```

### Rule 325

```

Int[((c_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]

```

### Rule 208

```

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

### Rule 2416

```

Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.))*((h_.)*(x_)
^(m_.))*((f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a
+ b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

```

### Rule 2395

```

Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_
)^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])/

```

$(g*(q + 1)), x] - \text{Dist}[(b*e*n)/(g*(q + 1)), \text{Int}[(f + g*x)^(q + 1)/(d + e*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, q\}, x\} \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{NeQ}[q, -1]$

### Rule 36

$\text{Int}[1/(((a_.) + (b_.)*(x_)) * ((c_.) + (d_.)*(x_))), x\_Symbol] \rightarrow \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[b*c - a*d, 0]$

### Rule 29

$\text{Int}[(x_)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$

### Rule 31

$\text{Int}[(a_.) + (b_.)*(x_)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x\}$

### Rule 2409

$\text{Int}[(a_.) + \text{Log}[(c_.) * ((d_.) + (e_.)*(x_)^{(n_.)}) * (b_.)]^{(p_.)} * ((f_.) + (g_.)*(x_)^{(r_.)})^{(q_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{Log}[c*(d + e*x)^n])^p, (f + g*x^r)^q, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, r\}, x\} \&\& \text{IntegerQ}[p, 0] \&\& \text{IntegerQ}[q] \&\& (\text{GtQ}[q, 0] \|\| (\text{IntegerQ}[r] \&\& \text{NeQ}[r, 1]))$

### Rule 2394

$\text{Int}[(a_.) + \text{Log}[(c_.) * ((d_.) + (e_.)*(x_)^{(n_.)}) * (b_.)] / ((f_.) + (g_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)] * (a + b*\text{Log}[c*(d + e*x)^n]))/g, x] - \text{Dist}[(b*e*n)/g, \text{Int}[\text{Log}[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n\}, x\} \&\& \text{NeQ}[e*f - d*g, 0]$

### Rule 2393

$\text{Int}[(a_.) + \text{Log}[(c_.) * ((d_.) + (e_.)*(x_))] * (b_.) / ((f_.) + (g_.)*(x_)), x\_Symbol] \rightarrow \text{Dist}[1/g, \text{Subst}[\text{Int}[(a + b*\text{Log}[1 + (c*e*x)/g]]/x, x], x, f + g*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g\}, x\} \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{EqQ}[g + c*(e*f - d*g), 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c_.) * ((d_.) + (e_.)*(x_)^{(n_.)})] / (x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \&\& \text{EqQ}[c*d, 1]$

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)} dx &= n \int \frac{\log(a+bx)}{x^2(f-gx^2)} dx - n \int \frac{\log(c+dx)}{x^2(f-gx^2)} dx - \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right) \\
&= \frac{n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)}{fx} + n \int \left(\frac{\log(a+bx)}{fx^2} + \frac{g \log(a+bx)}{f(f-gx^2)}\right) dx - n \int \\
&= \frac{n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)}{fx} - \frac{\sqrt{g} \tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{f^{3/2}} \\
&= -\frac{n \log(a+bx)}{fx} + \frac{n \log(c+dx)}{fx} + \frac{n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)}{fx} - \frac{\sqrt{g} \tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{f^{3/2}} \\
&= -\frac{n \log(a+bx)}{fx} + \frac{n \log(c+dx)}{fx} + \frac{n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)}{fx} - \frac{\sqrt{g} \tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{f^{3/2}} \\
&= \frac{bn \log(x)}{af} - \frac{dn \log(x)}{cf} - \frac{bn \log(a+bx)}{af} - \frac{n \log(a+bx)}{fx} + \frac{dn \log(c+dx)}{cf} + \frac{n \log(c+dx)}{fx} + \frac{n \log(c+dx)}{fx} - \frac{\sqrt{g} \tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{f^{3/2}} \\
&= \frac{bn \log(x)}{af} - \frac{dn \log(x)}{cf} - \frac{bn \log(a+bx)}{af} - \frac{n \log(a+bx)}{fx} + \frac{dn \log(c+dx)}{cf} + \frac{n \log(c+dx)}{fx} + \frac{n \log(c+dx)}{fx} + \frac{n \log(c+dx)}{fx} - \frac{\sqrt{g} \tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{f^{3/2}} \\
&= \frac{bn \log(x)}{af} - \frac{dn \log(x)}{cf} - \frac{bn \log(a+bx)}{af} - \frac{n \log(a+bx)}{fx} + \frac{dn \log(c+dx)}{cf} + \frac{n \log(c+dx)}{fx} + \frac{n \log(c+dx)}{fx} + \frac{n \log(c+dx)}{fx} - \frac{\sqrt{g} \tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{f^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.30863, size = 479, normalized size = 0.8

$$\frac{\sqrt{g}n \left( \text{PolyLog}\left(2, \frac{b(\sqrt{f}-\sqrt{g}x)}{a\sqrt{g}+b\sqrt{f}}\right) - \text{PolyLog}\left(2, \frac{d(\sqrt{f}-\sqrt{g}x)}{c\sqrt{g}+d\sqrt{f}}\right) + \log(\sqrt{f}-\sqrt{g}x) \left( \log\left(\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right) - \log\left(\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right) \right) \right) - \sqrt{g}n \left( \text{PolyLog}\left(2, \frac{b(\sqrt{f}-\sqrt{g}x)}{a\sqrt{g}+b\sqrt{f}}\right) - \text{PolyLog}\left(2, \frac{d(\sqrt{f}-\sqrt{g}x)}{c\sqrt{g}+d\sqrt{f}}\right) + \log(\sqrt{f}-\sqrt{g}x) \left( \log\left(\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right) - \log\left(\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right) \right) \right)}{f^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*((a + b\*x)/(c + d\*x))^n]/(x^2\*(f - g\*x^2)), x]

```
[Out] ((-2*Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n])/x + (2*Sqrt[f]*n*((b*c - a*d)*
Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) - Sqrt[g]*Log[e*((a +
b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] + Sqrt[g]*Log[e*((a + b*x)/(c +
d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + Sqrt[g]*n*((Log[(Sqrt[g]*(a + b*x))/(b
*Sqrt[f] + a*Sqrt[g])] - Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*)
Log[Sqrt[f] - Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f]
+ a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g]
)]) - Sqrt[g]*n*((Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log
[-((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x]
+ PolyLog[2, (b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2
, (d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*f^(3/2))
```

**Maple [F]** time = 0.455, size = 0, normalized size = 0.

$$\int \frac{1}{x^2(-gx^2 + f)} \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{\log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{gx^4 - fx^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*((b\*x+a)/(d\*x+c))^n)/x^2/(-g\*x^2+f),x, algorithm="fricas")

[Out] integral(-log(e\*((b\*x + a)/(d\*x + c))^n)/(g\*x^4 - f\*x^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*((b\*x+a)/(d\*x+c))^n)/x\*\*2/(-g\*x\*\*2+f),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(gx^2-f)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*((b\*x+a)/(d\*x+c))^n)/x^2/(-g\*x^2+f),x, algorithm="giac")

[Out] integrate(-log(e\*((b\*x + a)/(d\*x + c))^n)/((g\*x^2 - f)\*x^2), x)

$$3.82 \quad \int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} dx$$

**Optimal.** Leaf size=1046

result too large to display

```
[Out] (a*n*x)/(2*b*h) - (c*n*x)/(2*d*h) - (a^2*n*Log[a + b*x])/(2*b^2*h) + (n*x^2
*Log[a + b*x])/(2*h) - (g*n*(a + b*x)*Log[a + b*x])/(b*h^2) + (c^2*n*Log[c
+ d*x])/(2*d^2*h) - (n*x^2*Log[c + d*x])/(2*h) + (g*n*(c + d*x)*Log[c + d*x
])/(d*h^2) + (g*x*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[
c + d*x]))/h^2 - (x^2*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*
Log[c + d*x]))/(2*h) - (g*(g^2 - 3*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*
h]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h^
3*Sqrt[g^2 - 4*f*h]) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n
*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqr
t[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*
f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(
g - Sqrt[g^2 - 4*f*h])))])/(2*h^3) + ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g
^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*
h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h + (g*(g^2 - 3*f*h))
)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x)
)/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h)*(n*Log[a +
b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2
])/(2*h^3) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2
, (2*h*(a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))]/(2*h^3) + ((g^2 - f
*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a
*h - b*(g + Sqrt[g^2 - 4*f*h])))]/(2*h^3) - ((g^2 - f*h - (g*(g^2 - 3*f*h))
)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 -
4*f*h])))]/(2*h^3) - ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*
PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))]/(2*h^3)
```

**Rubi [A]** time = 1.71701, antiderivative size = 1046, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 14, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {2513, 2418, 2389, 2295, 2395, 43, 2394, 2393, 2391, 701, 634, 618, 206, 628}

$$-\frac{n \log(a+bx)a^2}{2b^2h} + \frac{nxa}{2bh} - \frac{cnx}{2dh} + \frac{nx^2 \log(a+bx)}{2h} - \frac{gn(a+bx) \log(a+bx)}{bh^2} - \frac{nx^2 \log(c+dx)}{2h} + \frac{c^2n \log(c+dx)}{2d^2h} + \frac{gn(c+dx)}{2dh}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (a*n*x)/(2*b*h) - (c*n*x)/(2*d*h) - (a^2*n*Log[a + b*x])/(2*b^2*h) + (n*x^2*Log[a + b*x])/(2*h) - (g*n*(a + b*x)*Log[a + b*x])/(b*h^2) + (c^2*n*Log[c + d*x])/(2*d^2*h) - (n*x^2*Log[c + d*x])/(2*h) + (g*n*(c + d*x)*Log[c + d*x])/(d*h^2) + (g*x*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/h^2 - (x^2*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(2*h) - (g*(g^2 - 3*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h^3*Sqrt[g^2 - 4*f*h]) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))])/(2*h^3) + ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h)*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2])/(2*h^3) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^3) + ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*h^3) - ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^3) - ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*h^3)
```

### Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]*(RFx_), x_Symbol] := Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dist[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFx, x], x]) /; FreeQ[{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0] && !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^ (q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
]^n))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```



Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
:> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} dx &= n \int \frac{x^3 \log(a+bx)}{f+gx+hx^2} dx - n \int \frac{x^3 \log(c+dx)}{f+gx+hx^2} dx - \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right) \int \frac{1}{f+gx+hx^2} dx \\
&= n \int \left(-\frac{g \log(a+bx)}{h^2} + \frac{x \log(a+bx)}{h} + \frac{(fg + (g^2 - fh)x) \log(a+bx)}{h^2(f+gx+hx^2)}\right) dx - n \int \left(-\frac{g \log(c+dx)}{h^2} + \frac{x \log(c+dx)}{h} + \frac{(fg + (g^2 - fh)x) \log(c+dx)}{h^2(f+gx+hx^2)}\right) dx \\
&= \frac{gx \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{h^2} - \frac{x^2 \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{2h} \\
&= \frac{nx^2 \log(a+bx)}{2h} - \frac{nx^2 \log(c+dx)}{2h} + \frac{gx \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{h^2} - \frac{x^2 \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{2h} \\
&= \frac{nx^2 \log(a+bx)}{2h} - \frac{gn(a+bx) \log(a+bx)}{bh^2} - \frac{nx^2 \log(c+dx)}{2h} + \frac{gn(c+dx) \log(c+dx)}{dh^2} + \frac{gx \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{h^2} \\
&= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2 n \log(a+bx)}{2b^2 h} + \frac{nx^2 \log(a+bx)}{2h} - \frac{gn(a+bx) \log(a+bx)}{bh^2} + \frac{c^2 n \log(c+dx)}{2d^2 h} \\
&= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2 n \log(a+bx)}{2b^2 h} + \frac{nx^2 \log(a+bx)}{2h} - \frac{gn(a+bx) \log(a+bx)}{bh^2} + \frac{c^2 n \log(c+dx)}{2d^2 h} \\
&= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2 n \log(a+bx)}{2b^2 h} + \frac{nx^2 \log(a+bx)}{2h} - \frac{gn(a+bx) \log(a+bx)}{bh^2} + \frac{c^2 n \log(c+dx)}{2d^2 h}
\end{aligned}$$

**Mathematica [A]** time = 1.37733, size = 1240, normalized size = 1.19

$$\frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) h^2 + \frac{n(b(b \log(c+dx)c^2 + d(ad-bc)x) - a^2 d^2 \log(a+bx)) h^2}{b^2 d^2} - \frac{2g(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) h}{b} + \frac{2(bc-ad)gn \log(c+dx) h}{bd} + \frac{2fg \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c^2 d^2}}{h^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g\*x + h\*x^2),x]

[Out]  $(h^2 x^2 \text{Log}[e((a + b x)/(c + d x))^n] - (2 g h (a + b x) \text{Log}[e((a + b x)/(c + d x))^n])/b + (2 (b c - a d) g h n \text{Log}[c + d x])/(b d) + (h^2 n (-a^2 d^2 \text{Log}[a + b x]) + b (d (-b c) + a d) x + b c^2 \text{Log}[c + d x]))/(b^2 d^2) + (2 f g h \text{Log}[e((a + b x)/(c + d x))^n] \text{Log}[g - \text{Sqrt}[g^2 - 4 f h] + 2 h x])/ \text{Sqrt}[g^2 - 4 f h] + (g^2 - f h) (1 - g/\text{Sqrt}[g^2 - 4 f h]) \text{Log}[e((a + b x)/(c + d x))^n] \text{Log}[g - \text{Sqrt}[g^2 - 4 f h] + 2 h x] - (2 f g h \text{Log}[e((a + b x)/(c + d x))^n] \text{Log}[g + \text{Sqrt}[g^2 - 4 f h] + 2 h x])/ \text{Sqrt}[g^2 - 4 f h] + (g^2 - f h) (1 + g/\text{Sqrt}[g^2 - 4 f h]) \text{Log}[e((a + b x)/(c + d x))^n] \text{Log}[g + \text{Sqrt}[g^2 - 4 f h] + 2 h x] - (2 f g h n ((\text{Log}[(2 h (a + b x))/(-b g) + 2 a h + b \text{Sqrt}[g^2 - 4 f h]]) - \text{Log}[(2 h (c + d x))/(-d g) + 2 c h + d \text{Sqrt}[g^2 - 4 f h]])) \text{Log}[g - \text{Sqrt}[g^2 - 4 f h] + 2 h x] + \text{PolyLog}[2, (b(-g + \text{Sqrt}[g^2 - 4 f h] - 2 h x))/(-b g) + 2 a h + b \text{Sqrt}[g^2 - 4 f h]]) - \text{PolyLog}[2, (d(-g + \text{Sqrt}[g^2 - 4 f h] - 2 h x))/(2 c h + d(-g + \text{Sqrt}[g^2 - 4 f h]))])]) / \text{Sqrt}[g^2 - 4 f h] - ((g^2 - f h) (-g + \text{Sqrt}[g^2 - 4 f h]) n ((\text{Log}[(2 h (a + b x))/(-b g) + 2 a h + b \text{Sqrt}[g^2 - 4 f h]]) - \text{Log}[(2 h (c + d x))/(-d g) + 2 c h + d \text{Sqrt}[g^2 - 4 f h]])) \text{Log}[g - \text{Sqrt}[g^2 - 4 f h] + 2 h x] + \text{PolyLog}[2, (b(-g + \text{Sqrt}[g^2 - 4 f h] - 2 h x))/(-b g) + 2 a h + b \text{Sqrt}[g^2 - 4 f h]]) - \text{PolyLog}[2, (d(-g + \text{Sqrt}[g^2 - 4 f h] - 2 h x))/(2 c h + d(-g + \text{Sqrt}[g^2 - 4 f h]))])]) / \text{Sqrt}[g^2 - 4 f h] + (2 f g h n ((\text{Log}[(2 h (a + b x))/(2 a h - b(g + \text{Sqrt}[g^2 - 4 f h]))]) - \text{Log}[(2 h (c + d x))/(2 c h - d(g + \text{Sqrt}[g^2 - 4 f h]))]) \text{Log}[g + \text{Sqrt}[g^2 - 4 f h] + 2 h x] + \text{PolyLog}[2, (b(g + \text{Sqrt}[g^2 - 4 f h] + 2 h x))/(-2 a h + b(g + \text{Sqrt}[g^2 - 4 f h]))]) - \text{PolyLog}[2, (d(g + \text{Sqrt}[g^2 - 4 f h] + 2 h x))/(-2 c h + d(g + \text{Sqrt}[g^2 - 4 f h]))])]) / \text{Sqrt}[g^2 - 4 f h] - ((g^2 - f h) (g + \text{Sqrt}[g^2 - 4 f h]) n ((\text{Log}[(2 h (a + b x))/(2 a h - b(g + \text{Sqrt}[g^2 - 4 f h]))]) - \text{Log}[(2 h (c + d x))/(2 c h - d(g + \text{Sqrt}[g^2 - 4 f h]))]) \text{Log}[g + \text{Sqrt}[g^2 - 4 f h] + 2 h x] + \text{PolyLog}[2, (b(g + \text{Sqrt}[g^2 - 4 f h] + 2 h x))/(-2 a h + b(g + \text{Sqrt}[g^2 - 4 f h]))]) - \text{PolyLog}[2, (d(g + \text{Sqrt}[g^2 - 4 f h] + 2 h x))/(-2 c h + d(g + \text{Sqrt}[g^2 - 4 f h]))])]) / \text{Sqrt}[g^2 - 4 f h]) / (2 h^3)$

**Maple [F]** time = 1.511, size = 0, normalized size = 0.

$$\int \frac{x^3}{hx^2 + gx + f} \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*ln(e\*((b\*x+a)/(d\*x+c))^n)/(h\*x^2+g\*x+f),x)

[Out] `int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)`

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{x^3 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{hx^2 + gx + f}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas")`

[Out] `integral(x^3*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2 + gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

$$3.83 \quad \int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} dx$$

**Optimal.** Leaf size=831

$$\frac{n(a+bx)\log(a+bx)}{bh} - \frac{\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n \log\left(-\frac{b(g+2hx-\sqrt{g^2-4fh})}{2ah-b(g-\sqrt{g^2-4fh})}\right)\log(a+bx)}{2h^2} - \frac{\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n \log\left(-\frac{b(g+2hx+\sqrt{g^2-4fh})}{2ah-b(g+\sqrt{g^2-4fh})}\right)\log(a+bx)}{2h^2}$$

[Out] (n\*(a + b\*x)\*Log[a + b\*x])/(b\*h) - (n\*(c + d\*x)\*Log[c + d\*x])/(d\*h) - (x\*(n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x]))/h + ((g^2 - 2\*f\*h)\*ArcTanh[(g + 2\*h\*x)/Sqrt[g^2 - 4\*f\*h]]\*(n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x]))/(h^2\*Sqrt[g^2 - 4\*f\*h]) - ((g - (g^2 - 2\*f\*h)/Sqrt[g^2 - 4\*f\*h])\*n\*Log[a + b\*x]\*Log[-((b\*(g - Sqrt[g^2 - 4\*f\*h] + 2\*h\*x))/(2\*a\*h - b\*(g - Sqrt[g^2 - 4\*f\*h])))])/(2\*h^2) + ((g - (g^2 - 2\*f\*h)/Sqrt[g^2 - 4\*f\*h])\*n\*Log[c + d\*x]\*Log[-((d\*(g - Sqrt[g^2 - 4\*f\*h] + 2\*h\*x))/(2\*c\*h - d\*(g - Sqrt[g^2 - 4\*f\*h])))])/(2\*h^2) - ((g + (g^2 - 2\*f\*h)/Sqrt[g^2 - 4\*f\*h])\*n\*Log[a + b\*x]\*Log[-((b\*(g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x))/(2\*a\*h - b\*(g + Sqrt[g^2 - 4\*f\*h])))])/(2\*h^2) + ((g + (g^2 - 2\*f\*h)/Sqrt[g^2 - 4\*f\*h])\*n\*Log[c + d\*x]\*Log[-((d\*(g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x))/(2\*c\*h - d\*(g + Sqrt[g^2 - 4\*f\*h])))])/(2\*h^2) + (g\*(n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x])\*Log[f + g\*x + h\*x^2])/(2\*h^2) - ((g - (g^2 - 2\*f\*h)/Sqrt[g^2 - 4\*f\*h])\*n\*PolyLog[2, (2\*h\*(a + b\*x))/(2\*a\*h - b\*(g - Sqrt[g^2 - 4\*f\*h]))])/(2\*h^2) - ((g + (g^2 - 2\*f\*h)/Sqrt[g^2 - 4\*f\*h])\*n\*PolyLog[2, (2\*h\*(a + b\*x))/(2\*a\*h - b\*(g + Sqrt[g^2 - 4\*f\*h]))])/(2\*h^2) + ((g - (g^2 - 2\*f\*h)/Sqrt[g^2 - 4\*f\*h])\*n\*PolyLog[2, (2\*h\*(c + d\*x))/(2\*c\*h - d\*(g - Sqrt[g^2 - 4\*f\*h]))])/(2\*h^2) + ((g + (g^2 - 2\*f\*h)/Sqrt[g^2 - 4\*f\*h])\*n\*PolyLog[2, (2\*h\*(c + d\*x))/(2\*c\*h - d\*(g + Sqrt[g^2 - 4\*f\*h]))])/(2\*h^2)

**Rubi [A]** time = 1.06817, antiderivative size = 831, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 12, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {2513, 2418, 2389, 2295, 2394, 2393, 2391, 703, 634, 618, 206, 628}

$$\frac{n(a+bx)\log(a+bx)}{bh} - \frac{\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n \log\left(-\frac{b(g+2hx-\sqrt{g^2-4fh})}{2ah-b(g-\sqrt{g^2-4fh})}\right)\log(a+bx)}{2h^2} - \frac{\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n \log\left(-\frac{b(g+2hx+\sqrt{g^2-4fh})}{2ah-b(g+\sqrt{g^2-4fh})}\right)\log(a+bx)}{2h^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g\*x + h\*x^2), x]

```
[Out] (n*(a + b*x)*Log[a + b*x])/(b*h) - (n*(c + d*x)*Log[c + d*x])/(d*h) - (x*(n
*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/h + ((g^2
- 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((
a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h^2*Sqrt[g^2 - 4*f*h]) - ((g - (
g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f
*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))]/(2*h^2) + ((g - (g^2 -
2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] +
2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))]/(2*h^2) - ((g + (g^2 - 2*f*
h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*
x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))]/(2*h^2) + ((g + (g^2 - 2*f*h)/Sq
rt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(
2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))]/(2*h^2) + (g*(n*Log[a + b*x] - Log[e*
((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2])/(2*h^2) -
((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h
- b*(g - Sqrt[g^2 - 4*f*h])))]/(2*h^2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f
*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))]/(2*
h^2) + ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/
(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))]/(2*h^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^
2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]
)))]/(2*h^2)
```

### Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFX_), x_Symbol] := Dist[p*r, Int[RFX*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFX*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFX, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFX, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFX, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n
]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFX_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFX, x]},
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFX, x] && IntegerQ[p]
```

### Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 703

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(e\*(d + e\*x)^(m - 1))/(c\*(m - 1)), x] + Dist[1/c, Int[((d + e\*x)^(m - 2)\*Simp[c\*d^2 - a\*e^2 + e\*(2\*c\*d - b\*e)\*x, x])/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[2\*c\*d - b\*e, 0] && GtQ[m, 1]

Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]



Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} dx &= n \int \frac{x^2 \log(a+bx)}{f+gx+hx^2} dx - n \int \frac{x^2 \log(c+dx)}{f+gx+hx^2} dx - \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right) \\
&= -\frac{x\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{h} + n \int \left(\frac{\log(a+bx)}{h} - \frac{(f+gx)\log(a+bx)}{h(f+gx+hx^2)}\right) dx \\
&= -\frac{x\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{h} + \frac{n \int \log(a+bx) dx}{h} - \frac{n \int \frac{(f+gx)\log(a+bx)}{f+gx+hx^2} dx}{h} \\
&= -\frac{x\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{h} + \frac{g\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{2h^2} \\
&= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{h} \\
&= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{h} \\
&= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{h} \\
&= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{h}
\end{aligned}$$

**Mathematica [A]** time = 5.47177, size = 1105, normalized size = 1.33

$$2dh\sqrt{g^2-4fh}(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - 2bdfh\log(g+2hx-\sqrt{g^2-4fh})\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + bdg(g-\sqrt{g^2-4fh})\log(g-\sqrt{g^2-4fh})$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g\*x + h\*x^2),x]

[Out] (2\*d\*h\*Sqrt[g^2 - 4\*f\*h]\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n] - 2\*(b\*c - a\*d)\*h\*Sqrt[g^2 - 4\*f\*h]\*n\*Log[c + d\*x] - 2\*b\*d\*f\*h\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[g - Sqrt[g^2 - 4\*f\*h] + 2\*h\*x] + b\*d\*g\*(g - Sqrt[g^2 - 4\*f\*h])\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[g - Sqrt[g^2 - 4\*f\*h] + 2\*h\*x] + 2\*b\*d\*f\*h\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x] - b\*d\*g\*(g + Sqrt[g^2 - 4\*f\*h])\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x] + 2\*b\*d\*f\*h\*n\*((Log[(2\*h\*(a + b\*x))/(-(b\*g) + 2\*a\*h + b\*Sqrt[g^2 - 4\*f\*h])]) - Log[(2\*h\*(c + d\*x))/(-(d\*g) + 2\*c\*h + d\*Sqrt[g^2 - 4\*f\*h])])]\*Log[g - Sqrt[g^2 - 4\*f\*h] + 2\*h\*x] + PolyLog[2, (b\*(-g + Sqrt[g^2 - 4\*f\*h] - 2\*h\*x))/(-(b\*g) + 2\*a\*h + b\*Sqrt[g^2 - 4\*f\*h])] - PolyLog[2, (d\*(-g + Sqrt[g^2 - 4\*f\*h] - 2\*h\*x))/(2\*c\*h + d\*(-g + Sqrt[g^2 - 4\*f\*h]))] - b\*d\*g\*(g - Sqrt[g^2 - 4\*f\*h])\*n\*((Log[(2\*h\*(a + b\*x))/(-(b\*g) + 2\*a\*h + b\*Sqrt[g^2 - 4\*f\*h])]) - Log[(2\*h\*(c + d\*x))/(-(d\*g) + 2\*c\*h + d\*Sqrt[g^2 - 4\*f\*h])])]\*Log[g - Sqrt[g^2 - 4\*f\*h] + 2\*h\*x] + PolyLog[2, (b\*(-g + Sqrt[g^2 - 4\*f\*h] - 2\*h\*x))/(-(b\*g) + 2\*a\*h + b\*Sqrt[g^2 - 4\*f\*h])] - PolyLog[2, (d\*(-g + Sqrt[g^2 - 4\*f\*h] - 2\*h\*x))/(2\*c\*h + d\*(-g + Sqrt[g^2 - 4\*f\*h]))] - 2\*b\*d\*f\*h\*n\*((Log[(2\*h\*(a + b\*x))/(2\*a\*h - b\*(g + Sqrt[g^2 - 4\*f\*h]))] - Log[(2\*h\*(c + d\*x))/(2\*c\*h - d\*(g + Sqrt[g^2 - 4\*f\*h]))])]\*Log[g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x] + PolyLog[2, (b\*(g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x))/(-2\*a\*h + b\*(g + Sqrt[g^2 - 4\*f\*h]))] - PolyLog[2, (d\*(g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x))/(-2\*c\*h + d\*(g + Sqrt[g^2 - 4\*f\*h]))] + b\*d\*g\*(g + Sqrt[g^2 - 4\*f\*h])\*n\*((Log[(2\*h\*(a + b\*x))/(2\*a\*h - b\*(g + Sqrt[g^2 - 4\*f\*h]))] - Log[(2\*h\*(c + d\*x))/(2\*c\*h - d\*(g + Sqrt[g^2 - 4\*f\*h]))])]\*Log[g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x] + PolyLog[2, (b\*(g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x))/(-2\*a\*h + b\*(g + Sqrt[g^2 - 4\*f\*h]))] - PolyLog[2, (d\*(g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x))/(-2\*c\*h + d\*(g + Sqrt[g^2 - 4\*f\*h]))])]/(2\*b\*d\*h^2\*Sqrt[g^2 - 4\*f\*h])

**Maple [F]** time = 1.393, size = 0, normalized size = 0.

$$\int \frac{x^2}{hx^2 + gx + f} \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*ln(e\*((b\*x+a)/(d\*x+c))^n)/(h\*x^2+g\*x+f),x)

[Out] int(x^2\*ln(e\*((b\*x+a)/(d\*x+c))^n)/(h\*x^2+g\*x+f),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(e\*((b\*x+a)/(d\*x+c))^n)/(h\*x^2+g\*x+f),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{x^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{hx^2 + gx + f}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(e\*((b\*x+a)/(d\*x+c))^n)/(h\*x^2+g\*x+f),x, algorithm="fricas")

[Out] integral(x^2\*log(e\*((b\*x + a)/(d\*x + c))^n)/(h\*x^2 + g\*x + f), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n)/(h\*x\*\*2+g\*x+f),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2 + gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

$$3.84 \quad \int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} dx$$

**Optimal.** Leaf size=685

$$\frac{n\left(1 - \frac{g}{\sqrt{g^2-4fh}}\right) \text{PolyLog}\left(2, \frac{2h(a+bx)}{2ah-b(g-\sqrt{g^2-4fh})}\right)}{2h} + \frac{n\left(\frac{g}{\sqrt{g^2-4fh}} + 1\right) \text{PolyLog}\left(2, \frac{2h(a+bx)}{2ah-b(\sqrt{g^2-4fh}+g)}\right)}{2h} - \frac{n\left(1 - \frac{g}{\sqrt{g^2-4fh}}\right) \text{PolyLog}\left(2, \frac{2h(a+bx)}{2ah-b(\sqrt{g^2-4fh}+g)}\right)}{2h}$$

[Out]  $-\left(\frac{g \operatorname{ArcTanh}\left[\frac{g+2hx}{\sqrt{g^2-4fh}}\right] \left(n \log\left[\frac{a+bx}{c+dx}\right] - \log\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] - n \log[c+dx]\right)}{h \sqrt{g^2-4fh}}\right) + \left(\frac{\left(1 - \frac{g}{\sqrt{g^2-4fh}}\right) n \log[a+bx] \log\left[-\frac{b(g-\sqrt{g^2-4fh}+2hx)}{2ah-b(g-\sqrt{g^2-4fh})}\right]}{2h} - \frac{\left(1 - \frac{g}{\sqrt{g^2-4fh}}\right) n \log[c+dx] \log\left[-\frac{d(g-\sqrt{g^2-4fh}+2hx)}{2ch-d(g-\sqrt{g^2-4fh})}\right]}{2h} + \frac{\left(1 + \frac{g}{\sqrt{g^2-4fh}}\right) n \log[a+bx] \log\left[-\frac{b(g+\sqrt{g^2-4fh}+2hx)}{2ah-b(g+\sqrt{g^2-4fh})}\right]}{2h} - \frac{\left(1 + \frac{g}{\sqrt{g^2-4fh}}\right) n \log[c+dx] \log\left[-\frac{d(g+\sqrt{g^2-4fh}+2hx)}{2ch-d(g+\sqrt{g^2-4fh})}\right]}{2h} - \frac{\left(n \log[a+bx] - \log\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] - n \log[c+dx]\right) \log[f+gx+hx^2]}{2h} + \frac{\left(1 - \frac{g}{\sqrt{g^2-4fh}}\right) n \operatorname{PolyLog}\left[2, \frac{2h(a+bx)}{2ah-b(g-\sqrt{g^2-4fh})}\right]}{2h} + \frac{\left(1 + \frac{g}{\sqrt{g^2-4fh}}\right) n \operatorname{PolyLog}\left[2, \frac{2h(a+bx)}{2ah-b(g+\sqrt{g^2-4fh})}\right]}{2h} - \frac{\left(1 - \frac{g}{\sqrt{g^2-4fh}}\right) n \operatorname{PolyLog}\left[2, \frac{2h(c+dx)}{2ch-d(g-\sqrt{g^2-4fh})}\right]}{2h} - \frac{\left(1 + \frac{g}{\sqrt{g^2-4fh}}\right) n \operatorname{PolyLog}\left[2, \frac{2h(c+dx)}{2ch-d(g+\sqrt{g^2-4fh})}\right]}{2h}\right)$

**Rubi [A]** time = 0.63553, antiderivative size = 685, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$ , Rules used = {2513, 2418, 2394, 2393, 2391, 634, 618, 206, 628}

$$\frac{n\left(1 - \frac{g}{\sqrt{g^2-4fh}}\right) \text{PolyLog}\left(2, \frac{2h(a+bx)}{2ah-b(g-\sqrt{g^2-4fh})}\right)}{2h} + \frac{n\left(\frac{g}{\sqrt{g^2-4fh}} + 1\right) \text{PolyLog}\left(2, \frac{2h(a+bx)}{2ah-b(\sqrt{g^2-4fh}+g)}\right)}{2h} - \frac{n\left(1 - \frac{g}{\sqrt{g^2-4fh}}\right) \text{PolyLog}\left(2, \frac{2h(a+bx)}{2ah-b(\sqrt{g^2-4fh}+g)}\right)}{2h}$$

Antiderivative was successfully verified.

[In]  $\text{Int}\left[\frac{x \log\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}{f+gx+hx^2}, x\right]$

[Out]  $-\left(\frac{g \operatorname{ArcTanh}\left[\frac{g+2hx}{\sqrt{g^2-4fh}}\right] \left(n \log[a+bx] - \log\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] - n \log[c+dx]\right)}{h \sqrt{g^2-4fh}}\right) + \left(\frac{\left(1 - \frac{g}{\sqrt{g^2-4fh}}\right) n \log[a+bx] \log\left[-\frac{b(g-\sqrt{g^2-4fh}+2hx)}{2ah-b(g-\sqrt{g^2-4fh})}\right]}{2h} - \frac{\left(1 - \frac{g}{\sqrt{g^2-4fh}}\right) n \log[c+dx] \log\left[-\frac{d(g-\sqrt{g^2-4fh}+2hx)}{2ch-d(g-\sqrt{g^2-4fh})}\right]}{2h} + \frac{\left(1 + \frac{g}{\sqrt{g^2-4fh}}\right) n \log[a+bx] \log\left[-\frac{b(g+\sqrt{g^2-4fh}+2hx)}{2ah-b(g+\sqrt{g^2-4fh})}\right]}{2h} - \frac{\left(1 + \frac{g}{\sqrt{g^2-4fh}}\right) n \log[c+dx] \log\left[-\frac{d(g+\sqrt{g^2-4fh}+2hx)}{2ch-d(g+\sqrt{g^2-4fh})}\right]}{2h} - \frac{\left(n \log[a+bx] - \log\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] - n \log[c+dx]\right) \log[f+gx+hx^2]}{2h} + \frac{\left(1 - \frac{g}{\sqrt{g^2-4fh}}\right) n \operatorname{PolyLog}\left[2, \frac{2h(a+bx)}{2ah-b(g-\sqrt{g^2-4fh})}\right]}{2h} + \frac{\left(1 + \frac{g}{\sqrt{g^2-4fh}}\right) n \operatorname{PolyLog}\left[2, \frac{2h(a+bx)}{2ah-b(g+\sqrt{g^2-4fh})}\right]}{2h} - \frac{\left(1 - \frac{g}{\sqrt{g^2-4fh}}\right) n \operatorname{PolyLog}\left[2, \frac{2h(c+dx)}{2ch-d(g-\sqrt{g^2-4fh})}\right]}{2h} - \frac{\left(1 + \frac{g}{\sqrt{g^2-4fh}}\right) n \operatorname{PolyLog}\left[2, \frac{2h(c+dx)}{2ch-d(g+\sqrt{g^2-4fh})}\right]}{2h}\right)$

$$\begin{aligned}
& g^2 - 4fh) * n * \text{Log}[a + bx] * \text{Log}[-((b(g - \sqrt{g^2 - 4fh}) + 2hx))/(2ah - b(g - \sqrt{g^2 - 4fh})))] / (2h) - ((1 - g/\sqrt{g^2 - 4fh}) * n * \text{Log}[c + dx] * \text{Log}[-((d(g - \sqrt{g^2 - 4fh}) + 2hx))/(2ch - d(g - \sqrt{g^2 - 4fh})))] / (2h) + ((1 + g/\sqrt{g^2 - 4fh}) * n * \text{Log}[a + bx] * \text{Log}[-((b(g + \sqrt{g^2 - 4fh}) + 2hx))/(2ah - b(g + \sqrt{g^2 - 4fh})))] / (2h) - ((1 + g/\sqrt{g^2 - 4fh}) * n * \text{Log}[c + dx] * \text{Log}[-((d(g + \sqrt{g^2 - 4fh}) + 2hx))/(2ch - d(g + \sqrt{g^2 - 4fh})))] / (2h) - ((n * \text{Log}[a + bx] - \text{Log}[e((a + bx)/(c + dx))^n] - n * \text{Log}[c + dx]) * \text{Log}[f + gx + hx^2]) / (2h) + ((1 - g/\sqrt{g^2 - 4fh}) * n * \text{PolyLog}[2, (2h(a + bx))/(2ah - b(g - \sqrt{g^2 - 4fh}))]) / (2h) + ((1 + g/\sqrt{g^2 - 4fh}) * n * \text{PolyLog}[2, (2h(a + bx))/(2ah - b(g + \sqrt{g^2 - 4fh}))]) / (2h) - ((1 - g/\sqrt{g^2 - 4fh}) * n * \text{PolyLog}[2, (2h(c + dx))/(2ch - d(g - \sqrt{g^2 - 4fh}))]) / (2h) - ((1 + g/\sqrt{g^2 - 4fh}) * n * \text{PolyLog}[2, (2h(c + dx))/(2ch - d(g + \sqrt{g^2 - 4fh}))]) / (2h)
\end{aligned}$$

### Rule 2513

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]*(RFx_), x_Symbol] := Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dist[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFx, x], x]) /; FreeQ[{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0] && !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.)] /; IntegersQ[m, n]

```

### Rule 2418

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

```

### Rule 2394

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]

```

### Rule 2393

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*

```

$(e*f - d*g), 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c_.)*(d_) + (e_.)*(x_)^{(n_.)}]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$

### Rule 634

$\text{Int}[(d_.) + (e_.)*(x_)]/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x\_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

### Rule 618

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{-1}, x\_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 206

$\text{Int}[(a_) + (b_.)*(x_)^2]^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 628

$\text{Int}[(d_) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x\_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

### Rubi steps



$$\begin{aligned}
\int \frac{x \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{f+gx+hx^2} dx &= n \int \frac{x \log(a+bx)}{f+gx+hx^2} dx - n \int \frac{x \log(c+dx)}{f+gx+hx^2} dx - \left( n \log(a+bx) - \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) - n \log(c+dx) \right) \\
&= n \int \left( \frac{\left( 1 - \frac{g}{\sqrt{g^2-4fh}} \right) \log(a+bx)}{g - \sqrt{g^2-4fh} + 2hx} + \frac{\left( 1 + \frac{g}{\sqrt{g^2-4fh}} \right) \log(a+bx)}{g + \sqrt{g^2-4fh} + 2hx} \right) dx - n \int \left( \frac{\left( 1 - \frac{g}{\sqrt{g^2-4fh}} \right) \log(c+dx)}{g - \sqrt{g^2-4fh} + 2hx} + \frac{\left( 1 + \frac{g}{\sqrt{g^2-4fh}} \right) \log(c+dx)}{g + \sqrt{g^2-4fh} + 2hx} \right) dx \\
&= - \frac{\left( n \log(a+bx) - \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) - n \log(c+dx) \right) \log(f+gx+hx^2)}{2h} + \left( \left( 1 - \frac{g}{\sqrt{g^2-4fh}} \right) n \log(a+bx) - \left( 1 + \frac{g}{\sqrt{g^2-4fh}} \right) n \log(c+dx) \right) \\
&= - \frac{g \tanh^{-1} \left( \frac{g+2hx}{\sqrt{g^2-4fh}} \right) \left( n \log(a+bx) - \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) - n \log(c+dx) \right)}{h\sqrt{g^2-4fh}} + \frac{\left( 1 - \frac{g}{\sqrt{g^2-4fh}} \right) n \log(a+bx) - \left( 1 + \frac{g}{\sqrt{g^2-4fh}} \right) n \log(c+dx)}{h\sqrt{g^2-4fh}} \\
&= - \frac{g \tanh^{-1} \left( \frac{g+2hx}{\sqrt{g^2-4fh}} \right) \left( n \log(a+bx) - \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) - n \log(c+dx) \right)}{h\sqrt{g^2-4fh}} + \frac{\left( 1 - \frac{g}{\sqrt{g^2-4fh}} \right) n \log(a+bx) - \left( 1 + \frac{g}{\sqrt{g^2-4fh}} \right) n \log(c+dx)}{h\sqrt{g^2-4fh}} \\
&= - \frac{g \tanh^{-1} \left( \frac{g+2hx}{\sqrt{g^2-4fh}} \right) \left( n \log(a+bx) - \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) - n \log(c+dx) \right)}{h\sqrt{g^2-4fh}} + \frac{\left( 1 - \frac{g}{\sqrt{g^2-4fh}} \right) n \log(a+bx) - \left( 1 + \frac{g}{\sqrt{g^2-4fh}} \right) n \log(c+dx)}{h\sqrt{g^2-4fh}}
\end{aligned}$$

**Mathematica [A]** time = 0.684548, size = 539, normalized size = 0.79

$$\frac{n \left( g - \sqrt{g^2 - 4fh} \right) \left( \text{PolyLog} \left( 2, \frac{b(\sqrt{g^2-4fh}-g-2hx)}{2ah+b\sqrt{g^2-4fh}+b(-g)} \right) - \text{PolyLog} \left( 2, \frac{d(\sqrt{g^2-4fh}-g-2hx)}{2ch+d(\sqrt{g^2-4fh}-g)} \right) \right) + \log \left( -\sqrt{g^2 - 4fh} + g + 2hx \right) \log \left( \frac{a+bx}{c+dx} \right)}{h\sqrt{g^2-4fh}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g\*x + h\*x^2), x]

[Out] ((-g + Sqrt[g^2 - 4\*f\*h])\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[g - Sqrt[g^2 - 4\*f\*h] + 2\*h\*x] + (g + Sqrt[g^2 - 4\*f\*h])\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x] + (g - Sqrt[g^2 - 4\*f\*h])\*n\*((Log[(2\*h\*(a + b\*x))/(-(b\*g) + 2\*a\*h + b\*Sqrt[g^2 - 4\*f\*h])]) - Log[(2\*h\*(c + d\*x))/(-(d

$$\begin{aligned}
 & *g) + 2*c*h + d*\text{Sqrt}[g^2 - 4*f*h])]) * \text{Log}[g - \text{Sqrt}[g^2 - 4*f*h] + 2*h*x] + \text{PolyLog}[2, (b*(-g + \text{Sqrt}[g^2 - 4*f*h] - 2*h*x))/(-b*g) + 2*a*h + b*\text{Sqrt}[g^2 - 4*f*h])] - \text{PolyLog}[2, (d*(-g + \text{Sqrt}[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + \text{Sqrt}[g^2 - 4*f*h]))] - (g + \text{Sqrt}[g^2 - 4*f*h])*n*((\text{Log}[(2*h*(a + b*x))/(2*a*h - b*(g + \text{Sqrt}[g^2 - 4*f*h]))] - \text{Log}[(2*h*(c + d*x))/(2*c*h - d*(g + \text{Sqrt}[g^2 - 4*f*h]))]) * \text{Log}[g + \text{Sqrt}[g^2 - 4*f*h] + 2*h*x] + \text{PolyLog}[2, (b*(g + \text{Sqrt}[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + \text{Sqrt}[g^2 - 4*f*h]))] - \text{PolyLog}[2, (d*(g + \text{Sqrt}[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(g + \text{Sqrt}[g^2 - 4*f*h]))]))/ (2*h*\text{Sqrt}[g^2 - 4*f*h])
 \end{aligned}$$

**Maple [F]** time = 1.349, size = 0, normalized size = 0.

$$\int \frac{x}{hx^2 + gx + f} \ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*ln(e\*((b\*x+a)/(d\*x+c))^n)/(h\*x^2+g\*x+f),x)

[Out] int(x\*ln(e\*((b\*x+a)/(d\*x+c))^n)/(h\*x^2+g\*x+f),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(e\*((b\*x+a)/(d\*x+c))^n)/(h\*x^2+g\*x+f),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2 + gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas")`

[Out] `integral(x*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*ln(e*((b*x+a)/(d*x+c))^n)/(h*x**2+g*x+f),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{hx^2 + gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")`

[Out] `integrate(x*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)`

$$3.85 \quad \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} dx$$

**Optimal.** Leaf size=401

$$\frac{n \operatorname{PolyLog}\left(2, \frac{2(a+bx)(c^2h-cdg+d^2f)}{(c+dx)(-\sqrt{g^2-4fh}(bc-ad)+2ach-adg-bcg+2bdf)}\right)}{\sqrt{g^2-4fh}} + \frac{n \operatorname{PolyLog}\left(2, \frac{2(a+bx)(c^2h-cdg+d^2f)}{(c+dx)(\sqrt{g^2-4fh}(bc-ad)+2ach-adg-bcg+2bdf)}\right)}{\sqrt{g^2-4fh}} - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)$$

[Out]  $-\left(\frac{\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] \operatorname{Log}\left[1 - \left(2(d^2f - c*d*g + c^2h)\right)(a+bx)\right]}{\left((2*b*d*f - b*c*g - a*d*g + 2*a*c*h - (b*c - a*d)*\operatorname{Sqrt}[g^2 - 4*f*h])\right)(c+dx)}\right)}{\operatorname{Sqrt}[g^2 - 4*f*h]} + \frac{\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] \operatorname{Log}\left[1 - \left(2(d^2f - c*d*g + c^2h)\right)(a+bx)\right]}{\left((2*b*d*f - b*c*g - a*d*g + 2*a*c*h + (b*c - a*d)*\operatorname{Sqrt}[g^2 - 4*f*h])\right)(c+dx)}\right)}{\operatorname{Sqrt}[g^2 - 4*f*h]} - \frac{n \operatorname{PolyLog}\left[2, \left(2(d^2f - c*d*g + c^2h)\right)(a+bx)\right]}{\left((2*b*d*f - b*c*g - a*d*g + 2*a*c*h - (b*c - a*d)*\operatorname{Sqrt}[g^2 - 4*f*h])\right)(c+dx)}\right)}{\operatorname{Sqrt}[g^2 - 4*f*h]} + \frac{n \operatorname{PolyLog}\left[2, \left(2(d^2f - c*d*g + c^2h)\right)(a+bx)\right]}{\left((2*b*d*f - b*c*g - a*d*g + 2*a*c*h + (b*c - a*d)*\operatorname{Sqrt}[g^2 - 4*f*h])\right)(c+dx)}\right)}{\operatorname{Sqrt}[g^2 - 4*f*h]}$

**Rubi [A]** time = 0.51787, antiderivative size = 545, normalized size of antiderivative = 1.36, number of steps used = 19, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {2513, 2418, 2394, 2393, 2391, 618, 206}

$$\frac{n \operatorname{PolyLog}\left(2, \frac{2h(a+bx)}{2ah-b(g-\sqrt{g^2-4fh})}\right)}{\sqrt{g^2-4fh}} - \frac{n \operatorname{PolyLog}\left(2, \frac{2h(a+bx)}{2ah-b(\sqrt{g^2-4fh}+g)}\right)}{\sqrt{g^2-4fh}} - \frac{n \operatorname{PolyLog}\left(2, \frac{2h(c+dx)}{2ch-d(g-\sqrt{g^2-4fh})}\right)}{\sqrt{g^2-4fh}} + \frac{n \operatorname{PolyLog}\left(2, \frac{2h(c+dx)}{2ch-d(\sqrt{g^2-4fh}+g)}\right)}{\sqrt{g^2-4fh}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]/(f+gx+hx^2), x\right]$

[Out]  $\left(2 \operatorname{ArcTanh}\left[\frac{g+2hx}{\operatorname{Sqrt}[g^2-4fh]}\right] \left(n \operatorname{Log}[a+bx] - \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] - n \operatorname{Log}[c+dx]\right) + \frac{n \operatorname{Log}[a+bx] \operatorname{Log}\left[-\frac{b(g-\operatorname{Sqrt}[g^2-4fh]+2hx)}{2ah-b(g-\operatorname{Sqrt}[g^2-4fh])}\right]}{\operatorname{Sqrt}[g^2-4fh]} - \frac{n \operatorname{Log}[c+dx] \operatorname{Log}\left[-\frac{d(g-\operatorname{Sqrt}[g^2-4fh]+2hx)}{2ch-d(g-\operatorname{Sqrt}[g^2-4fh])}\right]}{\operatorname{Sqrt}[g^2-4fh]} - \frac{n \operatorname{Log}[a+bx] \operatorname{Log}\left[-\frac{b(g+\operatorname{Sqrt}[g^2-4fh]+2hx)}{2ah-b(g+\operatorname{Sqrt}[g^2-4fh])}\right]}{\operatorname{Sqrt}[g^2-4fh]} + \frac{n \operatorname{Log}[c+dx] \operatorname{Log}\left[-\frac{d(g+\operatorname{Sqrt}[g^2-4fh]+2hx)}{2ch-d(g+\operatorname{Sqrt}[g^2-4fh])}\right]}{\operatorname{Sqrt}[g^2-4fh]} + n \operatorname{PolyLog}\left[2, \frac{2h(a+bx)}{2ah-b(g-\operatorname{Sqrt}[g^2-4fh])}\right] - n \operatorname{PolyLog}\left[2, \frac{2h(a+bx)}{2ah-b(\sqrt{g^2-4fh}+g)}\right] - n \operatorname{PolyLog}\left[2, \frac{2h(c+dx)}{2ch-d(g-\operatorname{Sqrt}[g^2-4fh])}\right] + n \operatorname{PolyLog}\left[2, \frac{2h(c+dx)}{2ch-d(\sqrt{g^2-4fh}+g)}\right]\right) / \operatorname{Sqrt}[g^2-4fh]$

$$- 4*f*h] - (n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])]) / Sqrt[g^2 - 4*f*h] - (n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])]) / Sqrt[g^2 - 4*f*h] + (n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])]) / Sqrt[g^2 - 4*f*h]$$

### Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_), x_Symbol] := Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r, Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n
]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e^n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} dx &= n \int \frac{\log(a+bx)}{f+gx+hx^2} dx - n \int \frac{\log(c+dx)}{f+gx+hx^2} dx - \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right) \\
 &= n \int \left(\frac{2h \log(a+bx)}{\sqrt{g^2-4fh}(g-\sqrt{g^2-4fh}+2hx)} - \frac{2h \log(a+bx)}{\sqrt{g^2-4fh}(g+\sqrt{g^2-4fh}+2hx)}\right) dx - n \int \left(\frac{2 \tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{(2hn) \int \frac{\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx} dx}{\sqrt{g^2-4fh}}\right) \\
 &= \frac{2 \tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{n \log(a+bx) \log\left(-\frac{b(g-\sqrt{g^2-4fh})}{2ah-b}\right)}{\sqrt{g^2-4fh}} \\
 &= \frac{2 \tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{n \log(a+bx) \log\left(-\frac{b(g-\sqrt{g^2-4fh})}{2ah-b}\right)}{\sqrt{g^2-4fh}} \\
 &= \frac{2 \tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{n \log(a+bx) \log\left(-\frac{b(g-\sqrt{g^2-4fh})}{2ah-b}\right)}{\sqrt{g^2-4fh}}
 \end{aligned}$$

**Mathematica [A]** time = 0.307974, size = 515, normalized size = 1.28

$$\frac{-n \text{PolyLog}\left(2, \frac{b(\sqrt{g^2-4fh}-g-2hx)}{2ah+b(\sqrt{g^2-4fh}-g)}\right) + n \text{PolyLog}\left(2, \frac{b(\sqrt{g^2-4fh}+g+2hx)}{b(\sqrt{g^2-4fh}+g)-2ah}\right) + n \text{PolyLog}\left(2, \frac{d(\sqrt{g^2-4fh}-g-2hx)}{2ch+d\sqrt{g^2-4fh}+d(-g)}\right) - n \text{PolyLog}\left(2, \frac{d(\sqrt{g^2-4fh}+g+2hx)}{2ch+d\sqrt{g^2-4fh}+d(-g)}\right)}{\sqrt{g^2-4fh}}$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*((a + b\*x)/(c + d\*x))^n]/(f + g\*x + h\*x^2),x]

[Out]  $(-n \operatorname{Log}[(2h(a + bx))/(-bg + 2ah + b\sqrt{g^2 - 4fh})]) \operatorname{Log}[g - \sqrt{g^2 - 4fh} + 2hx] + \operatorname{Log}[e((a + bx)/(c + dx))^n] \operatorname{Log}[g - \sqrt{g^2 - 4fh} + 2hx] + n \operatorname{Log}[(2h(c + dx))/(-dg + 2ch + d\sqrt{g^2 - 4fh})] \operatorname{Log}[g - \sqrt{g^2 - 4fh} + 2hx] + n \operatorname{Log}[(2h(a + bx))/(2ah - b(g + \sqrt{g^2 - 4fh}))] \operatorname{Log}[g + \sqrt{g^2 - 4fh} + 2hx] - \operatorname{Log}[e((a + bx)/(c + dx))^n] \operatorname{Log}[g + \sqrt{g^2 - 4fh} + 2hx] - n \operatorname{Log}[(2h(c + dx))/(2ch - d(g + \sqrt{g^2 - 4fh}))] \operatorname{Log}[g + \sqrt{g^2 - 4fh} + 2hx] + n \operatorname{PolyLog}[2, (d(-g + \sqrt{g^2 - 4fh}) - 2hx)/(-dg + 2ch + d\sqrt{g^2 - 4fh})] - n \operatorname{PolyLog}[2, (b(-g + \sqrt{g^2 - 4fh}) - 2hx)/(2ah + b(-g + \sqrt{g^2 - 4fh}))] + n \operatorname{PolyLog}[2, (b(g + \sqrt{g^2 - 4fh} + 2hx))/(-2ah + b(g + \sqrt{g^2 - 4fh}))] - n \operatorname{PolyLog}[2, (d(g + \sqrt{g^2 - 4fh} + 2hx))/(-2ch + d(g + \sqrt{g^2 - 4fh}))])/\sqrt{g^2 - 4fh}$

**Maple [F]** time = 1.362, size = 0, normalized size = 0.

$$\int \frac{1}{hx^2 + gx + f} \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*((b\*x+a)/(d\*x+c))^n)/(h\*x^2+g\*x+f),x)

[Out] int(ln(e\*((b\*x+a)/(d\*x+c))^n)/(h\*x^2+g\*x+f),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*((b\*x+a)/(d\*x+c))^n)/(h\*x^2+g\*x+f),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{hx^2 + gx + f}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*((b\*x+a)/(d\*x+c))^n)/(h\*x^2+g\*x+f),x, algorithm="fricas")

[Out] integral(log(e\*((b\*x + a)/(d\*x + c))^n)/(h\*x^2 + g\*x + f), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*((b\*x+a)/(d\*x+c))\*\*n)/(h\*x\*\*2+g\*x+f),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{hx^2 + gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*((b\*x+a)/(d\*x+c))^n)/(h\*x^2+g\*x+f),x, algorithm="giac")

[Out] integrate(log(e\*((b\*x + a)/(d\*x + c))^n)/(h\*x^2 + g\*x + f), x)



$$3.86 \quad \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f+gx+hx^2)} dx$$

**Optimal.** Leaf size=800

$$\frac{n \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{\left(\frac{g}{\sqrt{g^2-4fh}} + 1\right) n \log\left(-\frac{b(g+2hx-\sqrt{g^2-4fh})}{2ah-b(g-\sqrt{g^2-4fh})}\right) \log(a+bx)}{2f} - \frac{\left(1 - \frac{g}{\sqrt{g^2-4fh}}\right) n \log\left(-\frac{b(g+2hx+\sqrt{g^2-4fh})}{2ah-b(g+\sqrt{g^2-4fh})}\right)}{2f}$$

[Out] (n\*Log[-((b\*x)/a)]\*Log[a + b\*x])/f - (n\*Log[-((d\*x)/c)]\*Log[c + d\*x])/f - (g\*ArcTanh[(g + 2\*h\*x)/Sqrt[g^2 - 4\*f\*h]]\*(n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x]))/(f\*Sqrt[g^2 - 4\*f\*h]) - (Log[x]\*(n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x]))/f - ((1 + g/Sqrt[g^2 - 4\*f\*h])\*n\*Log[a + b\*x]\*Log[-((b\*(g - Sqrt[g^2 - 4\*f\*h] + 2\*h\*x))/(2\*a\*h - b\*(g - Sqrt[g^2 - 4\*f\*h])))])/(2\*f) + ((1 + g/Sqrt[g^2 - 4\*f\*h])\*n\*Log[c + d\*x]\*Log[-((d\*(g - Sqrt[g^2 - 4\*f\*h] + 2\*h\*x))/(2\*c\*h - d\*(g - Sqrt[g^2 - 4\*f\*h])))])/(2\*f) - ((1 - g/Sqrt[g^2 - 4\*f\*h])\*n\*Log[a + b\*x]\*Log[-((b\*(g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x))/(2\*a\*h - b\*(g + Sqrt[g^2 - 4\*f\*h])))])/(2\*f) + ((1 - g/Sqrt[g^2 - 4\*f\*h])\*n\*Log[c + d\*x]\*Log[-((d\*(g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x))/(2\*c\*h - d\*(g + Sqrt[g^2 - 4\*f\*h])))])/(2\*f) + ((n\*Log[a + b\*x] - Log[e\*((a + b\*x)/(c + d\*x))^n] - n\*Log[c + d\*x])\*Log[f + g\*x + h\*x^2])/f - ((1 + g/Sqrt[g^2 - 4\*f\*h])\*n\*PolyLog[2, (2\*h\*(a + b\*x))/(2\*a\*h - b\*(g - Sqrt[g^2 - 4\*f\*h]))]/(2\*f) - ((1 - g/Sqrt[g^2 - 4\*f\*h])\*n\*PolyLog[2, (2\*h\*(a + b\*x))/(2\*a\*h - b\*(g + Sqrt[g^2 - 4\*f\*h]))]/(2\*f) + (n\*PolyLog[2, 1 + (b\*x)/a])/f + ((1 + g/Sqrt[g^2 - 4\*f\*h])\*n\*PolyLog[2, (2\*h\*(c + d\*x))/(2\*c\*h - d\*(g - Sqrt[g^2 - 4\*f\*h]))]/(2\*f) + ((1 - g/Sqrt[g^2 - 4\*f\*h])\*n\*PolyLog[2, (2\*h\*(c + d\*x))/(2\*c\*h - d\*(g + Sqrt[g^2 - 4\*f\*h]))]/(2\*f) - (n\*PolyLog[2, 1 + (d\*x)/c])/f

**Rubi [A]** time = 0.980036, antiderivative size = 800, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 12, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {2513, 2418, 2394, 2315, 2393, 2391, 705, 29, 634, 618, 206, 628}

$$\frac{n \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{\left(\frac{g}{\sqrt{g^2-4fh}} + 1\right) n \log\left(-\frac{b(g+2hx-\sqrt{g^2-4fh})}{2ah-b(g-\sqrt{g^2-4fh})}\right) \log(a+bx)}{2f} - \frac{\left(1 - \frac{g}{\sqrt{g^2-4fh}}\right) n \log\left(-\frac{b(g+2hx+\sqrt{g^2-4fh})}{2ah-b(g+\sqrt{g^2-4fh})}\right)}{2f}$$

Antiderivative was successfully verified.

[In] Int[Log[e\*((a + b\*x)/(c + d\*x))^n]/(x\*(f + g\*x + h\*x^2)), x]

```
[Out] (n*Log[-((b*x)/a)]*Log[a + b*x])/f - (n*Log[-((d*x)/c)]*Log[c + d*x])/f - (
g*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b*x)
/(c + d*x))^n] - n*Log[c + d*x]))/(f*Sqrt[g^2 - 4*f*h]) - (Log[x]*(n*Log[a
+ b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/f - ((1 + g/Sqrt
[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*
a*h - b*(g - Sqrt[g^2 - 4*f*h])))]/(2*f) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*Lo
g[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g
^2 - 4*f*h])))]/(2*f) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b
*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))]/(2
*f) + ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*
f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))]/(2*f) + ((n*Log[a + b
*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2]
)/(2*f) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h -
b*(g - Sqrt[g^2 - 4*f*h])))]/(2*f) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2
, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))]/(2*f) + (n*PolyLog[
2, 1 + (b*x)/a])/f + ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x)
)/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))]/(2*f) + ((1 - g/Sqrt[g^2 - 4*f*h])*
n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))]/(2*f) -
(n*PolyLog[2, 1 + (d*x)/c])/f
```

### Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_), x_Symbol] := Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n
]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
]^n))/g, x] - Dist[(b*e^n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 705

Int[1/(((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)), x\_Symbol] := Dist[e^2/(c\*d^2 - b\*d\*e + a\*e^2), Int[1/(d + e\*x), x], x] + Dist[1/(c\*d^2 - b\*d\*e + a\*e^2), Int[(c\*d - b\*e - c\*e\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[2\*c\*d - b\*e, 0]

Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f+gx+hx^2)} dx &= n \int \frac{\log(a+bx)}{x(f+gx+hx^2)} dx - n \int \frac{\log(c+dx)}{x(f+gx+hx^2)} dx - \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\right) \\
&= n \int \left(\frac{\log(a+bx)}{fx} + \frac{(-g-hx)\log(a+bx)}{f(f+gx+hx^2)}\right) dx - n \int \left(\frac{\log(c+dx)}{fx} + \frac{(-g-hx)\log(c+dx)}{f(f+gx+hx^2)}\right) dx \\
&= -\frac{\log(x)\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{f} + \frac{n \int \frac{\log(a+bx)}{x} dx}{f} + \frac{n \int \frac{(-g-hx)\log(a+bx)}{f+gx+hx^2} dx}{f} \\
&= \frac{n \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{n \log\left(-\frac{dx}{c}\right) \log(c+dx)}{f} - \frac{\log(x)\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\right)}{f} \\
&= \frac{n \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{n \log\left(-\frac{dx}{c}\right) \log(c+dx)}{f} - \frac{g \tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\right)}{f\sqrt{g^2-4fh}} \\
&= \frac{n \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{n \log\left(-\frac{dx}{c}\right) \log(c+dx)}{f} - \frac{g \tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\right)}{f\sqrt{g^2-4fh}} \\
&= \frac{n \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{n \log\left(-\frac{dx}{c}\right) \log(c+dx)}{f} - \frac{g \tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\right)}{f\sqrt{g^2-4fh}} \\
&= \frac{n \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{n \log\left(-\frac{dx}{c}\right) \log(c+dx)}{f} - \frac{g \tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\right)}{f\sqrt{g^2-4fh}} \\
&= \frac{n \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{n \log\left(-\frac{dx}{c}\right) \log(c+dx)}{f} - \frac{g \tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right) \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\right)}{f\sqrt{g^2-4fh}}
\end{aligned}$$

**Mathematica [A]** time = 0.940115, size = 625, normalized size = 0.78

$$\frac{n(\sqrt{g^2-4fh+g}) \left( \text{PolyLog}\left(2, \frac{b(\sqrt{g^2-4fh-g-2hx})}{2ah+b\sqrt{g^2-4fh+b(-g)}}\right) - \text{PolyLog}\left(2, \frac{d(\sqrt{g^2-4fh-g-2hx})}{2ch+d(\sqrt{g^2-4fh-g})}\right) + \log(-\sqrt{g^2-4fh+g+2hx}) \left( \log\left(\frac{2h(a+bx)}{2ah+b\sqrt{g^2-4fh+b(-g)}}\right) - \log\left(\frac{2h(c+dx)}{2ch+d\sqrt{g^2-4fh+b(-g)}}\right) \right) \right)}{\sqrt{g^2-4fh}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f + g*x + h*x^2)),x]
```

```
[Out] (2*Log[x]*Log[e*((a + b*x)/(c + d*x))^n] - (1 + g/Sqrt[g^2 - 4*f*h])*Log[e*
((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] - (1 - g/Sqrt[g
^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h
*x] - 2*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x
)/a)] - PolyLog[2, -((d*x)/c)]) + ((g + Sqrt[g^2 - 4*f*h])*n*(Log[(2*h*(a
+ b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*
g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + Po
lyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2
- 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g
+ Sqrt[g^2 - 4*f*h]))])/Sqrt[g^2 - 4*f*h] + ((-g + Sqrt[g^2 - 4*f*h])*n*(
(Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])]) - Log[(2*h*(c + d
*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*
x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g
^2 - 4*f*h])]) - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d
*(g + Sqrt[g^2 - 4*f*h])]))])/Sqrt[g^2 - 4*f*h])/(2*f)
```

**Maple [F]** time = 1.414, size = 0, normalized size = 0.

$$\int \frac{1}{x(hx^2 + gx + f)} \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{hx^3 + gx^2 + fx}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="fricas")`

[Out] `integral(log(e*((b*x + a)/(d*x + c))^n)/(h*x^3 + g*x^2 + f*x), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(e*((b*x+a)/(d*x+c))**n)/x/(h*x**2+g*x+f),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{(hx^2 + gx + f)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="giac")`

[Out] `integrate(log(e*((b*x + a)/(d*x + c))^n)/((h*x^2 + g*x + f)*x), x)`

$$3.87 \quad \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)} dx$$

**Optimal.** Leaf size=995

result too large to display

```
[Out] (b*n*Log[x])/(a*f) - (d*n*Log[x])/(c*f) - (b*n*Log[a + b*x])/(a*f) - (n*Log[a + b*x])/(f*x) - (g*n*Log[-((b*x)/a)]*Log[a + b*x])/f^2 + (d*n*Log[c + d*x])/(c*f) + (n*Log[c + d*x])/(f*x) + (g*n*Log[-((d*x)/c)]*Log[c + d*x])/f^2 + (n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])/(f*x) + ((g^2 - 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(f^2*Sqrt[g^2 - 4*f*h]) + (g*Log[x]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/f^2 + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*f^2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))])/(2*f^2) + ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*f^2) - ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*f^2) - (g*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2])/(2*f^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f^2) + ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*f^2) - (g*n*PolyLog[2, 1 + (b*x)/a])/f^2 - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*f^2) - ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*f^2) + (g*n*PolyLog[2, 1 + (d*x)/c])/f^2
```

---

**Rubi [A]** time = 1.29131, antiderivative size = 995, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 16, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {2513, 2418, 2395, 36, 29, 31, 2394, 2315, 2393, 2391, 709, 800, 634, 618, 206, 628}

$$\frac{bn \log(x)}{af} - \frac{dn \log(x)}{cf} + \frac{g \left( n \log(a + bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c + dx) \right) \log(x)}{f^2} - \frac{bn \log(a + bx)}{af} - \frac{gn \log\left(-\frac{bx}{a}\right) \log(a)}{f^2}$$



Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f + g*x + h*x^2)),x]
```

```
[Out] (b*n*Log[x])/(a*f) - (d*n*Log[x])/(c*f) - (b*n*Log[a + b*x])/(a*f) - (n*Log[a + b*x])/(f*x) - (g*n*Log[-((b*x)/a)]*Log[a + b*x])/f^2 + (d*n*Log[c + d*x])/(c*f) + (n*Log[c + d*x])/(f*x) + (g*n*Log[-((d*x)/c)]*Log[c + d*x])/f^2 + (n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])/(f*x) + ((g^2 - 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(f^2*Sqrt[g^2 - 4*f*h]) + (g*Log[x]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/f^2 + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*f^2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))])/(2*f^2) + ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*f^2) - ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*f^2) - (g*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2])/(2*f^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f^2) + ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*f^2) - (g*n*PolyLog[2, 1 + (b*x)/a])/f^2 - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*f^2) - ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*f^2) + (g*n*PolyLog[2, 1 + (d*x)/c])/f^2
```

### Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]*(RFX_), x_Symbol] := Dist[p*r, Int[RFX*Log[a + b*x], x], x] + (Dist[q*r, Int[RFX*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFX, x], x]) /; FreeQ[{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFX, x] && NeQ[b*c - a*d, 0] && !MatchQ[RFX, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFX_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n]^p, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFX, x] && IntegerQ[p]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_))^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 709

```
Int[((d_.) + (e_.)*(x_)^(m_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]
```

### Rule 800

```
Int[(((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)} dx &= n \int \frac{\log(a+bx)}{x^2(f+gx+hx^2)} dx - n \int \frac{\log(c+dx)}{x^2(f+gx+hx^2)} dx - \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) - \\
&= \frac{n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)}{fx} + n \int \left(\frac{\log(a+bx)}{fx^2} - \frac{g \log(a+bx)}{f^2x} + \frac{(g^2-f)}{f^2}\right) dx \\
&= \frac{n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)}{fx} + \frac{n \int \frac{(g^2-fh+ghx) \log(a+bx)}{f+gx+hx^2} dx}{f^2} - \frac{n \int \frac{(g^2-fh+ghx)}{f+gx+hx^2} dx}{f^2} \\
&= -\frac{n \log(a+bx)}{fx} - \frac{gn \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f^2} + \frac{n \log(c+dx)}{fx} + \frac{gn \log\left(-\frac{dx}{c}\right) \log(c+dx)}{f^2} + \frac{n \log\left(\frac{g^2-fh+ghx}{f+gx+hx^2}\right)}{f^2} \\
&= -\frac{n \log(a+bx)}{fx} - \frac{gn \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f^2} + \frac{n \log(c+dx)}{fx} + \frac{gn \log\left(-\frac{dx}{c}\right) \log(c+dx)}{f^2} + \frac{n \log\left(\frac{g^2-fh+ghx}{f+gx+hx^2}\right)}{f^2} \\
&= \frac{bn \log(x)}{af} - \frac{dn \log(x)}{cf} - \frac{bn \log(a+bx)}{af} - \frac{n \log(a+bx)}{fx} - \frac{gn \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f^2} + \frac{dn \log\left(-\frac{dx}{c}\right) \log(c+dx)}{cf} \\
&= \frac{bn \log(x)}{af} - \frac{dn \log(x)}{cf} - \frac{bn \log(a+bx)}{af} - \frac{n \log(a+bx)}{fx} - \frac{gn \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f^2} + \frac{dn \log\left(-\frac{dx}{c}\right) \log(c+dx)}{cf} \\
&= \frac{bn \log(x)}{af} - \frac{dn \log(x)}{cf} - \frac{bn \log(a+bx)}{af} - \frac{n \log(a+bx)}{fx} - \frac{gn \log\left(-\frac{bx}{a}\right) \log(a+bx)}{f^2} + \frac{dn \log\left(-\frac{dx}{c}\right) \log(c+dx)}{cf}
\end{aligned}$$

**Mathematica [A]** time = 0.8248, size = 721, normalized size = 0.72

$$\frac{n(g\sqrt{g^2-4fh-2fh+g^2}) \left( \text{PolyLog}\left(2, \frac{b(\sqrt{g^2-4fh-g-2hx})}{2ah+b\sqrt{g^2-4fh+b(-g)}}\right) - \text{PolyLog}\left(2, \frac{d(\sqrt{g^2-4fh-g-2hx})}{2ch+d(\sqrt{g^2-4fh-g})}\right) + \log(-\sqrt{g^2-4fh+g+2hx}) \left( \log\left(\frac{2h(a+bx)}{2ah+b\sqrt{g^2-4fh+b(-g)}}\right) - \log\left(\frac{2h(c+dx)}{2ch+d\sqrt{g^2-4fh+d}}\right) \right) \right)}{\sqrt{g^2-4fh}}$$

Antiderivative was successfully verified.

[In] Integrate[Log[e\*((a + b\*x)/(c + d\*x))^n]/(x^2\*(f + g\*x + h\*x^2)),x]

[Out] ((-2\*f\*Log[e\*((a + b\*x)/(c + d\*x))^n])/x - 2\*g\*Log[x]\*Log[e\*((a + b\*x)/(c + d\*x))^n] + (2\*f\*n\*((b\*c - a\*d)\*Log[x] - b\*c\*Log[a + b\*x] + a\*d\*Log[c + d\*x]))/(a\*c) + (g + (g^2 - 2\*f\*h)/Sqrt[g^2 - 4\*f\*h])\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[g - Sqrt[g^2 - 4\*f\*h] + 2\*h\*x] + (g + (-g^2 + 2\*f\*h)/Sqrt[g^2 - 4\*f\*h])\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x] + 2\*g\*n\*(Log[x]\*(Log[1 + (b\*x)/a] - Log[1 + (d\*x)/c]) + PolyLog[2, -((b\*x)/a)] - PolyLog[2, -((d\*x)/c)]) - ((g^2 - 2\*f\*h + g\*Sqrt[g^2 - 4\*f\*h])\*n\*((Log[(2\*h\*(a + b\*x))/(-b\*g) + 2\*a\*h + b\*Sqrt[g^2 - 4\*f\*h]]) - Log[(2\*h\*(c + d\*x))/(-d\*g) + 2\*c\*h + d\*Sqrt[g^2 - 4\*f\*h]]))\*Log[g - Sqrt[g^2 - 4\*f\*h] + 2\*h\*x] + PolyLog[2, (b\*(-g + Sqrt[g^2 - 4\*f\*h] - 2\*h\*x))/(-b\*g) + 2\*a\*h + b\*Sqrt[g^2 - 4\*f\*h]] - PolyLog[2, (d\*(-g + Sqrt[g^2 - 4\*f\*h] - 2\*h\*x))/(2\*c\*h + d\*(-g + Sqrt[g^2 - 4\*f\*h]))])/Sqrt[g^2 - 4\*f\*h] + ((g^2 - 2\*f\*h - g\*Sqrt[g^2 - 4\*f\*h])\*n\*((Log[(2\*h\*(a + b\*x))/(2\*a\*h - b\*(g + Sqrt[g^2 - 4\*f\*h]))] - Log[(2\*h\*(c + d\*x))/(2\*c\*h - d\*(g + Sqrt[g^2 - 4\*f\*h]))])\*Log[g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x] + PolyLog[2, (b\*(g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x))/(-2\*a\*h + b\*(g + Sqrt[g^2 - 4\*f\*h]))] - PolyLog[2, (d\*(g + Sqrt[g^2 - 4\*f\*h] + 2\*h\*x))/(-2\*c\*h + d\*(g + Sqrt[g^2 - 4\*f\*h]))])/Sqrt[g^2 - 4\*f\*h])/(2\*f^2)

**Maple [F]** time = 1.398, size = 0, normalized size = 0.

$$\int \frac{1}{x^2(hx^2 + gx + f)} \ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*((b\*x+a)/(d\*x+c))^n)/x^2/(h\*x^2+g\*x+f),x)

[Out] int(ln(e\*((b\*x+a)/(d\*x+c))^n)/x^2/(h\*x^2+g\*x+f),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{hx^4 + gx^3 + fx^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="fricas")
```

```
[Out] integral(log(e*((b*x + a)/(d*x + c))^n)/(h*x^4 + g*x^3 + f*x^2), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x**2/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{(hx^2 + gx + f)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)/((h*x^2 + g*x + f)*x^2), x)
```

$$3.88 \quad \int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx} dx$$

**Optimal.** Leaf size=46

$$-\frac{\text{PolyLog}\left(2, 1 - \frac{a}{a+bx}\right)}{b} - \frac{\log\left(\frac{a}{a+bx}\right) \log\left(\frac{cx}{a+bx}\right)}{b}$$

[Out]  $-\left(\frac{\text{Log}\left[\frac{a}{a+bx}\right] \text{Log}\left[\frac{cx}{a+bx}\right]}{b}\right) - \text{PolyLog}\left[2, 1 - \frac{a}{a+bx}\right]/b$

**Rubi [A]** time = 0.158601, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {2488, 2411, 2343, 2333, 2315}

$$-\frac{\text{PolyLog}\left(2, 1 - \frac{a}{a+bx}\right)}{b} - \frac{\log\left(\frac{a}{a+bx}\right) \log\left(\frac{cx}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}\left[\frac{\text{Log}\left[\frac{cx}{a+bx}\right]}{a+bx}, x\right]$

[Out]  $-\left(\frac{\text{Log}\left[\frac{a}{a+bx}\right] \text{Log}\left[\frac{cx}{a+bx}\right]}{b}\right) - \text{PolyLog}\left[2, 1 - \frac{a}{a+bx}\right]/b$

### Rule 2488

$\text{Int}\left[\text{Log}\left[\left(e_{.}\right) \cdot \left(\left(f_{.}\right) \cdot \left(a_{.}\right) + \left(b_{.}\right) \cdot \left(x_{.}\right)\right)^{\left(p_{.}\right)} \cdot \left(\left(c_{.}\right) + \left(d_{.}\right) \cdot \left(x_{.}\right)\right)^{\left(q_{.}\right)}\right]^{\left(r_{.}\right)} \cdot \left(s_{.}\right) / \left(\left(g_{.}\right) + \left(h_{.}\right) \cdot \left(x_{.}\right)\right), x\_Symbol] \rightarrow -\text{Simp}\left[\frac{\text{Log}\left[-\left(\frac{b \cdot c - a \cdot d}{d \cdot (a + b \cdot x)}\right)\right] \cdot \text{Log}\left[e \cdot (f \cdot (a + b \cdot x))^p \cdot (c + d \cdot x)^q \cdot r^s\right]}{h}, x\right] + \text{Dist}\left[\frac{p \cdot r \cdot s \cdot (b \cdot c - a \cdot d)}{h}, \text{Int}\left[\frac{\text{Log}\left[-\left(\frac{b \cdot c - a \cdot d}{d \cdot (a + b \cdot x)}\right)\right] \cdot \text{Log}\left[e \cdot (f \cdot (a + b \cdot x))^p \cdot (c + d \cdot x)^q \cdot r^s\right]}{(a + b \cdot x) \cdot (c + d \cdot x)}, x\right], x\right] /;$  FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

### Rule 2411

$\text{Int}\left[\left(a_{.}\right) + \text{Log}\left[\left(c_{.}\right) \cdot \left(d_{.}\right) + \left(e_{.}\right) \cdot \left(x_{.}\right)\right]^{\left(n_{.}\right)} \cdot \left(b_{.}\right)\right]^{\left(p_{.}\right)} \cdot \left(\left(f_{.}\right) + \left(g_{.}\right) \cdot \left(x_{.}\right)\right)^{\left(q_{.}\right)} \cdot \left(\left(h_{.}\right) + \left(i_{.}\right) \cdot \left(x_{.}\right)\right)^{\left(r_{.}\right)}, x\_Symbol] \rightarrow \text{Dist}\left[1/e, \text{Subst}\left[\text{Int}\left[\frac{\left(g \cdot x\right)/e^q \cdot \left(e \cdot h - d \cdot i\right)/e + \left(i \cdot x\right)/e}{r \cdot (a + b \cdot \text{Log}\left[c \cdot x^n\right])^p}, x\right], x, d + e \cdot x\right] /;$  FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d



\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

### Rule 2343

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/((x\_)\*((d\_) + (e\_.)\*(x\_)^(r\_.))),  
x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*Log[c\*x])/(x\*(d + e\*x^(r/n))), x],  
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]

### Rule 2333

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)/(x\_))^(q\_.)\*(  
x\_)^(m\_.), x\_Symbol] := Int[(e + d\*x)^q\*(a + b\*Log[c\*x^n])^p, x] /; FreeQ[{  
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 -  
c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx} dx &= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b} + \frac{a \int \frac{\log\left(\frac{a}{a+bx}\right)}{x(a+bx)} dx}{b} \\
 &= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b} + \frac{a \operatorname{Subst}\left(\int \frac{\log\left(\frac{a}{x}\right)}{x\left(-\frac{a}{b} + \frac{x}{b}\right)} dx, x, a+bx\right)}{b^2} \\
 &= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b} - \frac{a \operatorname{Subst}\left(\int \frac{\log(ax)}{\left(-\frac{a}{b} + \frac{1}{bx}\right)x} dx, x, \frac{1}{a+bx}\right)}{b^2} \\
 &= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b} - \frac{a \operatorname{Subst}\left(\int \frac{\log(ax)}{\frac{1}{b} - \frac{ax}{b}} dx, x, \frac{1}{a+bx}\right)}{b^2} \\
 &= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b} - \frac{\operatorname{Li}_2\left(\frac{bx}{a+bx}\right)}{b}
 \end{aligned}$$

**Mathematica [A]** time = 0.0167248, size = 84, normalized size = 1.83

$$-\frac{\operatorname{PolyLog}\left(2, \frac{a+bx}{a}\right)}{b} - \frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b} + \frac{\log^2\left(\frac{a}{a+bx}\right)}{2b} + \frac{\log\left(-\frac{bx}{a}\right)\log\left(\frac{a}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Log[(c\*x)/(a + b\*x)]/(a + b\*x),x]

[Out] (Log[-((b\*x)/a)]\*Log[a/(a + b\*x)])/b + Log[a/(a + b\*x)]^2/(2\*b) - (Log[a/(a + b\*x)]\*Log[(c\*x)/(a + b\*x)])/b - PolyLog[2, (a + b\*x)/a]/b

**Maple [B]** time = 0.098, size = 97, normalized size = 2.1

$$-\frac{1}{b} \operatorname{dilog} \left( -\frac{1}{c} \left( b \left( \frac{c}{b} - \frac{ac}{b(bx+a)} \right) - c \right) \right) - \frac{1}{b} \ln \left( \frac{c}{b} - \frac{ac}{b(bx+a)} \right) \ln \left( -\frac{1}{c} \left( b \left( \frac{c}{b} - \frac{ac}{b(bx+a)} \right) - c \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x/(b\*x+a))/(b\*x+a),x)

[Out] -dilog(-(b\*(c/b-a\*c/b/(b\*x+a))-c)/c)/b-ln(c/b-a\*c/b/(b\*x+a))\*ln(-(b\*(c/b-a\*c/b/(b\*x+a))-c)/c)/b

**Maxima [B]** time = 1.12936, size = 128, normalized size = 2.78

$$\frac{\log(bx+a) \log\left(\frac{cx}{bx+a}\right)}{b} - \frac{c \log(bx+a)^2}{b} - \frac{2 \left( \log\left(\frac{bx}{a}+1\right) \log(x) + \operatorname{Li}_2\left(-\frac{bx}{a}\right) \right) c}{2c} + \frac{(c \log(bx+a) - c \log(x)) \log(bx+a)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x/(b\*x+a))/(b\*x+a),x, algorithm="maxima")

[Out] log(b\*x + a)\*log(c\*x/(b\*x + a))/b - 1/2\*(c\*log(b\*x + a)^2/b - 2\*(log(b\*x/a + 1)\*log(x) + dilog(-b\*x/a))\*c/b)/c + (c\*log(b\*x + a) - c\*log(x))\*log(b\*x + a)/(b\*c)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\log\left(\frac{cx}{bx+a}\right)}{bx+a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="fricas")`

[Out] `integral(log(c*x/(b*x + a))/(b*x + a), x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*x/(b*x+a))/(b*x+a),x)`

[Out] `Integral(log(c*x/(a + b*x))/(a + b*x), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{cx}{bx+a}\right)}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="giac")`

[Out] `integrate(log(c*x/(b*x + a))/(b*x + a), x)`

$$3.89 \quad \int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)} dx$$

**Optimal.** Leaf size=20

$$\frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

[Out] Log[(c\*x)/(a + b\*x)]^3/(3\*a)

**Rubi [A]** time = 0.0579114, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$ , Rules used = {2505}

$$\frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

Antiderivative was successfully verified.

[In] Int[Log[(c\*x)/(a + b\*x)]^2/(x\*(a + b\*x)),x]

[Out] Log[(c\*x)/(a + b\*x)]^3/(3\*a)

### Rule 2505

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{h = Simplify[u*(a + b*x)*(c + d*x)]},
  Simp[(h*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^(s + 1))/(p*r*(s + 1)*(b*c -
  a*d)), x] /; FreeQ[h, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[
  b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[s, -1]
```

### Rubi steps

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)} dx = \frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

**Mathematica [A]** time = 0.0961977, size = 20, normalized size = 1.

$$\frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

Antiderivative was successfully verified.

[In] Integrate[Log[(c\*x)/(a + b\*x)]^2/(x\*(a + b\*x)),x]

[Out] Log[(c\*x)/(a + b\*x)]^3/(3\*a)

**Maple [A]** time = 0.059, size = 29, normalized size = 1.5

$$\frac{1}{3a} \left( \ln \left( \frac{c}{b} - \frac{ac}{b(bx+a)} \right) \right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x/(b\*x+a))^2/x/(b\*x+a),x)

[Out] 1/3/a\*ln(c/b-a\*c/b/(b\*x+a))^3

**Maxima [B]** time = 1.19479, size = 190, normalized size = 9.5

$$-\left(\frac{\log(bx+a)}{a} - \frac{\log(x)}{a}\right) \log\left(\frac{cx}{bx+a}\right)^2 - \frac{(c \log(bx+a)^2 - 2c \log(bx+a) \log(x) + c \log(x)^2) \log\left(\frac{cx}{bx+a}\right)}{ac} - \frac{c^2 \log(bx+a)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x/(b\*x+a))^2/x/(b\*x+a),x, algorithm="maxima")

[Out] -(log(b\*x + a)/a - log(x)/a)\*log(c\*x/(b\*x + a))^2 - (c\*log(b\*x + a)^2 - 2\*c\*log(b\*x + a)\*log(x) + c\*log(x)^2)\*log(c\*x/(b\*x + a))/(a\*c) - 1/3\*(c^2\*log(b\*x + a)^3 - 3\*c^2\*log(b\*x + a)^2\*log(x) + 3\*c^2\*log(b\*x + a)\*log(x)^2 - c^2\*log(x)^3)/(a\*c^2)

**Fricas [A]** time = 1.98203, size = 38, normalized size = 1.9

$$\frac{\log\left(\frac{cx}{bx+a}\right)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x/(b*x + a))^3/a
```

**Sympy [A]** time = 0.390677, size = 14, normalized size = 0.7

$$\frac{\log\left(\frac{cx}{a+bx}\right)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x/(b*x+a))**2/x/(b*x+a),x)
```

```
[Out] log(c*x/(a + b*x))**3/(3*a)
```

**Giac [A]** time = 1.15435, size = 24, normalized size = 1.2

$$\frac{\log\left(\frac{cx}{bx+a}\right)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*log(c*x/(b*x + a))^3/a
```

$$3.90 \quad \int \frac{\log\left(\frac{a}{a+bx}\right) \log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)} dx$$

**Optimal.** Leaf size=82

$$-\frac{\text{PolyLog}\left(2, 1 - \frac{a}{a+bx}\right) \log^2\left(\frac{cx}{a+bx}\right)}{a} + \frac{2\text{PolyLog}\left(3, 1 - \frac{a}{a+bx}\right) \log\left(\frac{cx}{a+bx}\right)}{a} - \frac{2\text{PolyLog}\left(4, 1 - \frac{a}{a+bx}\right)}{a}$$

[Out]  $-\left(\frac{\text{Log}[(c*x)/(a + b*x)]^2 * \text{PolyLog}[2, 1 - a/(a + b*x)]}{a}\right) + \left(\frac{2 * \text{Log}[(c*x)/(a + b*x)] * \text{PolyLog}[3, 1 - a/(a + b*x)]}{a}\right) - \left(\frac{2 * \text{PolyLog}[4, 1 - a/(a + b*x)]}{a}\right)$

**Rubi [A]** time = 0.168258, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$ , Rules used = {2506, 2508, 6610}

$$-\frac{\text{PolyLog}\left(2, 1 - \frac{a}{a+bx}\right) \log^2\left(\frac{cx}{a+bx}\right)}{a} + \frac{2\text{PolyLog}\left(3, 1 - \frac{a}{a+bx}\right) \log\left(\frac{cx}{a+bx}\right)}{a} - \frac{2\text{PolyLog}\left(4, 1 - \frac{a}{a+bx}\right)}{a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Log}[a/(a + b*x)] * \text{Log}[(c*x)/(a + b*x)]^2)/(x*(a + b*x)), x]$

[Out]  $-\left(\frac{\text{Log}[(c*x)/(a + b*x)]^2 * \text{PolyLog}[2, 1 - a/(a + b*x)]}{a}\right) + \left(\frac{2 * \text{Log}[(c*x)/(a + b*x)] * \text{PolyLog}[3, 1 - a/(a + b*x)]}{a}\right) - \left(\frac{2 * \text{PolyLog}[4, 1 - a/(a + b*x)]}{a}\right)$

### Rule 2506

$\text{Int}[\text{Log}[v_*] * \text{Log}[(e_*) * ((f_*) * (a_*) + (b_*) * (x_*))^{(p_*)} * ((c_*) + (d_*) * (x_*))^{(q_*)}]^{(r_*)}]^{(s_*)} * (u_*) , x\_Symbol] := \text{With}[\{g = \text{Simplify}[(v - 1) * (c + d * x) / (a + b * x)], h = \text{Simplify}[u * (a + b * x) * (c + d * x)]\}, -\text{Simp}[(h * \text{PolyLog}[2, 1 - v] * \text{Log}[e * (f * (a + b * x))^p * (c + d * x)^q]^r]^s / (b * c - a * d), x] + \text{Dist}[h * p * r * s, \text{Int}[(\text{PolyLog}[2, 1 - v] * \text{Log}[e * (f * (a + b * x))^p * (c + d * x)^q]^r]^s / ((a + b * x) * (c + d * x)), x], x] /; \text{FreeQ}[\{g, h\}, x] /; \text{FreeQ}[\{a, b, c, d, e, f, p, q, r, s\}, x] \&\& \text{NeQ}[b * c - a * d, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{EqQ}[p + q, 0]$

### Rule 2508

$\text{Int}[\text{Log}[(e_*) * ((f_*) * (a_*) + (b_*) * (x_*))^{(p_*)} * ((c_*) + (d_*) * (x_*))^{(q_*)}]^{(r_*)}]^{(s_*)} * (u_*) * \text{PolyLog}[n_*, v_*] , x\_Symbol] := \text{With}[\{g = \text{Simplify}[(v * (c + d * x)) / (a + b * x)], h = \text{Simplify}[u * (a + b * x) * (c + d * x)]\}, \text{Simp}[(h * \text{PolyLog}[n + 1, v] * \text{Log}[e * (f * (a + b * x))^p * (c + d * x)^q]^r]^s / (b * c - a * d), x] - \text{Dist}[h * p * r * s, \text{Int}[(\text{PolyLog}[n + 1, v] * \text{Log}[e * (f * (a + b * x))^p * (c + d * x)^q]^r]^s / ((a + b * x) * (c + d * x)), x], x]$

$(a + b*x)*(c + d*x), x], x] /; \text{FreeQ}[\{g, h\}, x]] /; \text{FreeQ}[\{a, b, c, d, e, f, n, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{EqQ}[p + q, 0]$

### Rule 6610

$\text{Int}[(u\_)*\text{PolyLog}[n\_ , v\_], x\_Symbol] :> \text{With}[\{w = \text{DerivativeDivides}[v, u*v, x]\}, \text{Simp}[w*\text{PolyLog}[n + 1, v], x] /; \text{!FalseQ}[w]] /; \text{FreeQ}[n, x]$

### Rubi steps

$$\begin{aligned} \int \frac{\log\left(\frac{a}{a+bx}\right) \log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)} dx &= -\frac{\log^2\left(\frac{cx}{a+bx}\right) \text{Li}_2\left(1 - \frac{a}{a+bx}\right)}{a} + 2 \int \frac{\log\left(\frac{cx}{a+bx}\right) \text{Li}_2\left(1 - \frac{a}{a+bx}\right)}{x(a+bx)} dx \\ &= -\frac{\log^2\left(\frac{cx}{a+bx}\right) \text{Li}_2\left(1 - \frac{a}{a+bx}\right)}{a} + \frac{2 \log\left(\frac{cx}{a+bx}\right) \text{Li}_3\left(1 - \frac{a}{a+bx}\right)}{a} - 2 \int \frac{\text{Li}_3\left(1 - \frac{a}{a+bx}\right)}{x(a+bx)} dx \\ &= -\frac{\log^2\left(\frac{cx}{a+bx}\right) \text{Li}_2\left(1 - \frac{a}{a+bx}\right)}{a} + \frac{2 \log\left(\frac{cx}{a+bx}\right) \text{Li}_3\left(1 - \frac{a}{a+bx}\right)}{a} - \frac{2 \text{Li}_4\left(1 - \frac{a}{a+bx}\right)}{a} \end{aligned}$$

**Mathematica [A]** time = 0.013578, size = 76, normalized size = 0.93

$$-\frac{\text{PolyLog}\left(2, \frac{bx}{a+bx}\right) \log^2\left(\frac{cx}{a+bx}\right)}{a} + \frac{2 \text{PolyLog}\left(3, \frac{bx}{a+bx}\right) \log\left(\frac{cx}{a+bx}\right)}{a} - \frac{2 \text{PolyLog}\left(4, \frac{bx}{a+bx}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(Log[a/(a + b\*x)]\*Log[(c\*x)/(a + b\*x)]^2)/(x\*(a + b\*x)),x]

[Out] -((Log[(c\*x)/(a + b\*x)]^2\*PolyLog[2, (b\*x)/(a + b\*x)]/a) + (2\*Log[(c\*x)/(a + b\*x)]\*PolyLog[3, (b\*x)/(a + b\*x)]/a - (2\*PolyLog[4, (b\*x)/(a + b\*x)]/a

**Maple [F]** time = 0.531, size = 0, normalized size = 0.

$$\int \frac{1}{x(bx+a)} \ln\left(\frac{a}{bx+a}\right) \left(\ln\left(\frac{cx}{bx+a}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a/(b\*x+a))\*ln(c\*x/(b\*x+a))^2/x/(b\*x+a),x)



[Out] `int(ln(a/(b*x+a))*ln(c*x/(b*x+a))^2/x/(b*x+a),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\log(bx+a)^4 - 4 \log(bx+a)^3 \log(x)}{4a} + \int \frac{a \log(a) \log(c)^2 + 2a \log(a) \log(c) \log(x) + a \log(a) \log(x)^2 + (a(\log(a) + 2\log(c)) + (3bx + 2a)\log(x))\log(bx+a)^2 - (2a(\log(a) + \log(c))\log(x) + a\log(x)^2 + (2\log(a)\log(c) + \log(c)^2)a)\log(bx+a)}{a^2bx^2 + a^2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="maxima")`

[Out] `1/4*(log(b*x + a)^4 - 4*log(b*x + a)^3*log(x))/a + integrate((a*log(a)*log(c)^2 + 2*a*log(a)*log(c)*log(x) + a*log(a)*log(x)^2 + (a*(log(a) + 2*log(c)) + (3*b*x + 2*a)*log(x))*log(b*x + a)^2 - (2*a*(log(a) + log(c))*log(x) + a*log(x)^2 + (2*log(a)*log(c) + log(c)^2)*a)*log(b*x + a))/(a*b*x^2 + a^2*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\frac{cx}{bx+a}\right)^2 \log\left(\frac{a}{bx+a}\right)}{bx^2 + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="fricas")`

[Out] `integral(log(c*x/(b*x + a))^2*log(a/(b*x + a))/(b*x^2 + a*x), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{a}{a+bx}\right) \log\left(\frac{cx}{a+bx}\right)^2}{x(a+bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a/(b*x+a))*ln(c*x/(b*x+a))**2/x/(b*x+a),x)
```

```
[Out] Integral(log(a/(a + b*x))*log(c*x/(a + b*x))**2/(x*(a + b*x)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{cx}{bx+a}\right)^2 \log\left(\frac{a}{bx+a}\right)}{(bx+a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(c*x/(b*x + a))^2*log(a/(b*x + a))/((b*x + a)*x), x)
```

$$3.91 \quad \int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)} dx$$

**Optimal.** Leaf size=150

$$\frac{\text{PolyLog}\left(2, 1 - \frac{bc-ad}{b(c+dx)}\right) \log^2\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)} + \frac{2\text{PolyLog}\left(3, 1 - \frac{bc-ad}{b(c+dx)}\right) \log\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)} - \frac{2\text{PolyLog}\left(4, 1 - \frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

[Out] -((Log[(e\*(a + b\*x))/(c + d\*x)]^2\*PolyLog[2, 1 - (b\*c - a\*d)/(b\*(c + d\*x))]) / ((b\*c - a\*d)\*g)) + (2\*Log[(e\*(a + b\*x))/(c + d\*x)]\*PolyLog[3, 1 - (b\*c - a\*d)/(b\*(c + d\*x))]) / ((b\*c - a\*d)\*g) - (2\*PolyLog[4, 1 - (b\*c - a\*d)/(b\*(c + d\*x))]) / ((b\*c - a\*d)\*g)

**Rubi [A]** time = 0.247639, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 55,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.055$ , Rules used = {2506, 2508, 6610}

$$\frac{\text{PolyLog}\left(2, 1 - \frac{bc-ad}{b(c+dx)}\right) \log^2\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)} + \frac{2\text{PolyLog}\left(3, 1 - \frac{bc-ad}{b(c+dx)}\right) \log\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)} - \frac{2\text{PolyLog}\left(4, 1 - \frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(Log[(b\*c - a\*d)/(b\*(c + d\*x))]\*Log[(e\*(a + b\*x))/(c + d\*x)]^2)/((c + d\*x)\*(a\*g + b\*g\*x)), x]

[Out] -((Log[(e\*(a + b\*x))/(c + d\*x)]^2\*PolyLog[2, 1 - (b\*c - a\*d)/(b\*(c + d\*x))]) / ((b\*c - a\*d)\*g)) + (2\*Log[(e\*(a + b\*x))/(c + d\*x)]\*PolyLog[3, 1 - (b\*c - a\*d)/(b\*(c + d\*x))]) / ((b\*c - a\*d)\*g) - (2\*PolyLog[4, 1 - (b\*c - a\*d)/(b\*(c + d\*x))]) / ((b\*c - a\*d)\*g)

**Rule 2506**

Int[Log[v\_]\*Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]^(s\_.)\*(u\_), x\_Symbol] := With[{g = Simplify[((v - 1)\*(c + d\*x))/(a + b\*x)], h = Simplify[u\*(a + b\*x)\*(c + d\*x)]}, -Simp[h\*PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(b\*c - a\*d), x] + Dist[h\*p\*r\*s, Int[(PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]

Rule 2508

```
Int[Log[(e._)*((f._)*((a._) + (b._)*(x._))^(p._))*((c._) + (d._)*(x._))^(q._)]
^(r._)]^(s._)*(u._)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[(v*(c +
d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)]/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u._)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)} dx &= -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right) \text{Li}_2\left(1 - \frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2 \int \frac{\log\left(\frac{e(a+bx)}{c+dx}\right) \text{Li}_2\left(1 - \frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)} dx}{g} \\ &= -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right) \text{Li}_2\left(1 - \frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2 \log\left(\frac{e(a+bx)}{c+dx}\right) \text{Li}_3\left(1 - \frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} - \frac{2 \int \frac{\text{Li}_3\left(1 - \frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)} dx}{g} \\ &= -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right) \text{Li}_2\left(1 - \frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2 \log\left(\frac{e(a+bx)}{c+dx}\right) \text{Li}_3\left(1 - \frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} - \frac{2 \text{Li}_4\left(1 - \frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} \end{aligned}$$

**Mathematica [A]** time = 0.0376798, size = 110, normalized size = 0.73

$$\frac{-\text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right) \log^2\left(\frac{e(a+bx)}{c+dx}\right) + 2\text{PolyLog}\left(3, \frac{d(a+bx)}{b(c+dx)}\right) \log\left(\frac{e(a+bx)}{c+dx}\right) - 2\text{PolyLog}\left(4, \frac{d(a+bx)}{b(c+dx)}\right)}{g(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x))/(c + d*x)]^2)/((
(c + d*x)*(a*g + b*g*x)), x]
```

```
[Out] (-Log[(e*(a + b*x))/(c + d*x)]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))]
+ 2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] -
2*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))]/((b*c - a*d)*g)
```

---

**Maple [F]** time = 1.102, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)(bgx+ag)} \ln\left(\frac{-ad+bc}{b(dx+c)}\right) \left(\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln((-a*d+b*c)/b/(d*x+c))*ln(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x+a*g),x)`

[Out] `int(ln((-a*d+b*c)/b/(d*x+c))*ln(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x+a*g),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{4 \log(bx+a) \log(dx+c)^3 - \log(dx+c)^4}{4(bcg-adg)} - \int \frac{((d \log(bc-ad) - d \log(b))a - (c \log(bc-ad) - c \log(b))b) \log(bx+a)}{4(bcg-adg)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x+a*g),x, algorithm="maxima")`

[Out] `-1/4*(4*log(b*x + a)*log(d*x + c)^3 - log(d*x + c)^4)/(b*c*g - a*d*g) - integrate((((d*log(b*c - a*d) - d*log(b))*a - (c*log(b*c - a*d) - c*log(b))*b)*log(b*x + a)^2 + ((d*log(b*c - a*d) - d*log(b) + 2*d*log(e))*a - (c*(log(b*c - a*d) + 2*log(e)) - c*log(b))*b - (3*b*d*x + 2*b*c + a*d)*log(b*x + a))*log(d*x + c)^2 + (d*log(b*c - a*d)*log(e)^2 - d*log(b)*log(e)^2)*a - (c*log(b*c - a*d)*log(e)^2 - c*log(b)*log(e)^2)*b + 2*((d*log(b*c - a*d)*log(e) - d*log(b)*log(e))*a - (c*log(b*c - a*d)*log(e) - c*log(b)*log(e))*b)*log(b*x + a) + ((b*c - a*d)*log(b*x + a)^2 - (2*d*log(b*c - a*d)*log(e) - 2*d*log(b)*log(e) + d*log(e)^2)*a - (2*c*log(b)*log(e) - (2*log(b*c - a*d)*log(e) + log(e)^2)*c)*b - 2*((d*log(b*c - a*d) - d*log(b) + d*log(e))*a - (c*(log(b*c - a*d) + log(e)) - c*log(b))*b)*log(b*x + a))*log(d*x + c))/(a*b*c^2*g - a^2*c*d*g + (b^2*c*d*g - a*b*d^2*g)*x^2 + (b^2*c^2*g - a^2*d^2*g)*x), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \frac{bc-ad}{bdx+bc} \right) \log \left( \frac{bex+ae}{dx+c} \right)^2}{bdgx^2 + acg + (bc + ad)gx}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((-a\*d+b\*c)/b/(d\*x+c))\*log(e\*(b\*x+a)/(d\*x+c))^2/(d\*x+c)/(b\*g\*x+a\*g),x, algorithm="fricas")

[Out] integral(log((b\*c - a\*d)/(b\*d\*x + b\*c))\*log((b\*e\*x + a\*e)/(d\*x + c))^2/(b\*d\*g\*x^2 + a\*c\*g + (b\*c + a\*d)\*g\*x), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln((-a\*d+b\*c)/b/(d\*x+c))\*ln(e\*(b\*x+a)/(d\*x+c))^2/(d\*x+c)/(b\*g\*x+a\*g),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( \frac{(bx+a)e}{dx+c} \right)^2 \log \left( \frac{bc-ad}{(dx+c)b} \right)}{(bgx + ag)(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((-a\*d+b\*c)/b/(d\*x+c))\*log(e\*(b\*x+a)/(d\*x+c))^2/(d\*x+c)/(b\*g\*x+a\*g),x, algorithm="giac")

[Out] integrate(log((b\*x + a)\*e/(d\*x + c))^2\*log((b\*c - a\*d)/((d\*x + c)\*b))/((b\*g\*x + a\*g)\*(d\*x + c)), x)

$$3.92 \quad \int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)} dx$$

**Optimal.** Leaf size=160

$$\frac{\text{PolyLog}\left(2, 1 - \frac{bc-ad}{b(c+dx)}\right) \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)} + \frac{2n \text{PolyLog}\left(3, 1 - \frac{bc-ad}{b(c+dx)}\right) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)} - \frac{2n^2 \text{PolyLog}\left(4, 1 - \frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

[Out]  $-\left(\frac{\text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]^2 \text{PolyLog}\left[2, 1 - \frac{b^*c - a*d}{b^*(c + d*x)}\right]\right)}{(b^*c - a*d)*g)} + \frac{(2*n*\text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]*\text{PolyLog}\left[3, 1 - \frac{b^*c - a*d}{b^*(c + d*x)}\right])}{(b^*c - a*d)*g} - \frac{(2*n^2*\text{PolyLog}\left[4, 1 - \frac{b^*c - a*d}{b^*(c + d*x)}\right])}{(b^*c - a*d)*g}$

**Rubi [A]** time = 0.250151, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 58,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.052$ , Rules used = {2506, 2508, 6610}

$$\frac{\text{PolyLog}\left(2, 1 - \frac{bc-ad}{b(c+dx)}\right) \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)} + \frac{2n \text{PolyLog}\left(3, 1 - \frac{bc-ad}{b(c+dx)}\right) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)} - \frac{2n^2 \text{PolyLog}\left(4, 1 - \frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}\left[\frac{\text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]^2 \text{Log}\left[\frac{b^*c - a*d}{b^*(c + d*x)}\right]\right]}{(c + d*x)*(a*g + b*g*x)}, x\right]$

[Out]  $-\left(\frac{\text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]^2 \text{PolyLog}\left[2, 1 - \frac{b^*c - a*d}{b^*(c + d*x)}\right]\right)}{(b^*c - a*d)*g)} + \frac{(2*n*\text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]*\text{PolyLog}\left[3, 1 - \frac{b^*c - a*d}{b^*(c + d*x)}\right])}{(b^*c - a*d)*g} - \frac{(2*n^2*\text{PolyLog}\left[4, 1 - \frac{b^*c - a*d}{b^*(c + d*x)}\right])}{(b^*c - a*d)*g}$

### Rule 2506

$\text{Int}\left[\text{Log}[v_*] \text{Log}\left[\left(e_{*}\right) \left(\left(f_{*}\right) \left(\left(a_{*}\right) + \left(b_{*}\right) \left(x_{*}\right)\right)^{\left(p_{*}\right)} \left(\left(c_{*}\right) + \left(d_{*}\right) \left(x_{*}\right)\right)^{\left(q_{*}\right)} \left(r_{*}\right)^{\left(s_{*}\right)} \left(u_{*}\right), x_{\text{Symbol}}\right] := \text{With}\left[\left\{g = \text{Simplify}\left[\frac{(v-1)(c+dx)}{(a+bx)}\right], h = \text{Simplify}\left[u*(a+bx)*(c+dx)\right]\right\}, -\text{Simp}\left[\frac{h*\text{PolyLog}\left[2, 1-v\right]*\text{Log}\left[e*(f*(a+bx)^p*(c+dx)^q\right]^r\right]}{(b*c-a*d)}, x\right] + \text{Dist}\left[h*p*r*s, \text{Int}\left[\frac{\text{PolyLog}\left[2, 1-v\right]*\text{Log}\left[e*(f*(a+bx)^p*(c+dx)^q\right]^r\right]}{(s-1)}\right]}{(a+bx)*(c+dx)}, x\right] /;$  FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f

, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]

### Rule 2508

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*(u\_)\*PolyLog[n\_, v\_], x\_Symbol] :> With[{g = Simplify[(v\*(c + d\*x))/(a + b\*x)], h = Simplify[u\*(a + b\*x)\*(c + d\*x)]}, Simp[(h\*PolyLog[n + 1, v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(b\*c - a\*d), x] - Dist[h\*p\*r\*s, Int[(PolyLog[n + 1, v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(a + b\*x)\*(c + d\*x), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, n, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] :> With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]

### Rubi steps

$$\begin{aligned} \int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)} dx &= -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{Li}_2\left(1 - \frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{(2n) \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{Li}_2\left(1 - \frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)} dx}{g} \\ &= -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{Li}_2\left(1 - \frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2n \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{Li}_3\left(1 - \frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} - \frac{(2n^2) \int}{(bc-ad)g} \\ &= -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{Li}_2\left(1 - \frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2n \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \text{Li}_3\left(1 - \frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} - \frac{2n^2 \text{Li}_4}{(bc-ad)g} \end{aligned}$$

**Mathematica [B]** time = 0.455601, size = 559, normalized size = 3.49

$$3n \left( -2 \text{PolyLog}\left(3, \frac{d(a+bx)}{b(c+dx)}\right) + 2 \log\left(\frac{a+bx}{c+dx}\right) \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right) + \log\left(\frac{bc-ad}{bc+bdx}\right) \log^2\left(\frac{a+bx}{c+dx}\right) \right) \left( n \log\left(\frac{a+bx}{c+dx}\right) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Log[e\*((a + b\*x)/(c + d\*x))^n]^2\*Log[(b\*c - a\*d)/(b\*(c + d\*x))]) / ((c + d\*x)\*(a\*g + b\*g\*x)), x]



```
[Out] (Log[(a + b*x)/(c + d*x)]*(3*Log[e*((a + b*x)/(c + d*x))^n]^2 - 3*n*Log[e*(
(a + b*x)/(c + d*x))^n]*Log[(a + b*x)/(c + d*x)] + n^2*Log[(a + b*x)/(c + d
*x)]^2)*Log[(b*c - a*d)/(b*c + b*d*x)] + (3*(Log[e*((a + b*x)/(c + d*x))^n]
- n*Log[(a + b*x)/(c + d*x)])^2*(-Log[c/d + x]^2 - 2*Log[a/b + x]*Log[c +
d*x] + 2*Log[c/d + x]*Log[c + d*x] + 2*Log[(a + b*x)/(c + d*x)]*Log[c + d*x
] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + 2*PolyLog[2, (d*(a + b*
x))/(-b*c + a*d)]))/2 + 3*n*(-Log[e*((a + b*x)/(c + d*x))^n] + n*Log[(a +
b*x)/(c + d*x)])*(Log[(a + b*x)/(c + d*x)]^2*Log[(b*c - a*d)/(b*c + b*d*x)
] + 2*Log[(a + b*x)/(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*
PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] - n^2*(Log[(a + b*x)/(c + d*x)]^3*
Log[(b*c - a*d)/(b*c + b*d*x)] + 3*Log[(a + b*x)/(c + d*x)]^2*PolyLog[2, (d
*(a + b*x))/(b*(c + d*x))] - 6*Log[(a + b*x)/(c + d*x)]*PolyLog[3, (d*(a +
b*x))/(b*(c + d*x))] + 6*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/3*(b*c
- a*d)*g)
```

**Maple [F]** time = 5.443, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)(bgx+ag)} \left( \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)^2 \ln \left( \frac{-ad+bc}{b(dx+c)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b*g*x+a*g
),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b*g*x+a*g
),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 \log \left( \frac{bc-ad}{(dx+c)b} \right)}{(bgx+ag)(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x, algorithm="maxima")
```

[Out] integrate(log(e\*((b\*x + a)/(d\*x + c))^n)^2\*log((b\*c - a\*d)/((d\*x + c)\*b))/(b\*g\*x + a\*g)\*(d\*x + c), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 \log \left( \frac{bc-ad}{bdx+bc} \right)}{bdgx^2 + acg + (bc + ad)gx}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e\*((b\*x+a)/(d\*x+c))^n)^2\*log((-a\*d+b\*c)/b/(d\*x+c))/(d\*x+c)/(b\*g\*x+a\*g),x, algorithm="fricas")

[Out] integral(log(e\*((b\*x + a)/(d\*x + c))^n)^2\*log((b\*c - a\*d)/(b\*d\*x + b\*c))/(b\*d\*g\*x^2 + a\*c\*g + (b\*c + a\*d)\*g\*x), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(e\*((b\*x+a)/(d\*x+c))^n)\*\*2\*ln((-a\*d+b\*c)/b/(d\*x+c))/(d\*x+c)/(b\*g\*x+a\*g),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 \log \left( \frac{bc-ad}{(dx+c)b} \right)}{(bgx + ag)(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b  
*g*x+a*g),x, algorithm="giac")
```

```
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)^2*log((b*c - a*d)/((d*x + c)*b))/(  
(b*g*x + a*g)*(d*x + c)), x)
```

### 3.93 $\int \log\left(\frac{c(b+ax)}{x}\right) dx$

**Optimal.** Leaf size=25

$$x \log\left(ac + \frac{bc}{x}\right) + \frac{b \log(ax + b)}{a}$$

[Out] x\*Log[a\*c + (b\*c)/x] + (b\*Log[b + a\*x])/a

**Rubi [A]** time = 0.0115837, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {2453, 2448, 263, 31}

$$x \log\left(ac + \frac{bc}{x}\right) + \frac{b \log(ax + b)}{a}$$

Antiderivative was successfully verified.

[In] Int[Log[(c\*(b + a\*x))/x], x]

[Out] x\*Log[a\*c + (b\*c)/x] + (b\*Log[b + a\*x])/a

#### Rule 2453

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] := Int[(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v, x] && !BinomialMatchQ[v, x]
```

#### Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)], x_Symbol] := Simp[x*Log[c*(d + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]
```

#### Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rubi steps

$$\begin{aligned} \int \log\left(\frac{c(b+ax)}{x}\right) dx &= \int \log\left(ac + \frac{bc}{x}\right) dx \\ &= x \log\left(ac + \frac{bc}{x}\right) + (bc) \int \frac{1}{\left(ac + \frac{bc}{x}\right)x} dx \\ &= x \log\left(ac + \frac{bc}{x}\right) + (bc) \int \frac{1}{bc + acx} dx \\ &= x \log\left(ac + \frac{bc}{x}\right) + \frac{b \log(b+ax)}{a} \end{aligned}$$

**Mathematica [A]** time = 0.0048467, size = 28, normalized size = 1.12

$$\frac{(ax+b) \log\left(\frac{c(ax+b)}{x}\right)}{a} + \frac{b \log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x))/x], x]
```

```
[Out] (b*Log[x])/a + ((b + a*x)*Log[(c*(b + a*x))/x])/a
```

**Maple [A]** time = 0.188, size = 44, normalized size = 1.8

$$-\frac{b}{a} \ln\left(\frac{bc}{x}\right) + x \ln\left(ac + \frac{bc}{x}\right) + \frac{b}{a} \ln\left(ac + \frac{bc}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)/x), x)
```

```
[Out] -b/a*ln(b*c/x)+x*ln(a*c+b*c/x)+b*ln(a*c+b*c/x)/a
```

---

**Maxima [A]** time = 1.17715, size = 34, normalized size = 1.36

$$x \log\left(\frac{(ax+b)c}{x}\right) + \frac{b \log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*(a\*x+b)/x),x, algorithm="maxima")

[Out] x\*log((a\*x + b)\*c/x) + b\*log(a\*x + b)/a

---

**Fricas [A]** time = 1.93737, size = 63, normalized size = 2.52

$$\frac{ax \log\left(\frac{acx+bc}{x}\right) + b \log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*(a\*x+b)/x),x, algorithm="fricas")

[Out] (a\*x\*log((a\*c\*x + b\*c)/x) + b\*log(a\*x + b))/a

---

**Sympy [A]** time = 0.341546, size = 20, normalized size = 0.8

$$x \log\left(\frac{c(ax+b)}{x}\right) + \frac{b \log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*(a\*x+b)/x),x)

[Out] x\*log(c\*(a\*x + b)/x) + b\*log(a\*x + b)/a

---

**Giac [A]** time = 1.25673, size = 35, normalized size = 1.4

$$x \log\left(\frac{(ax+b)c}{x}\right) + \frac{b \log(|ax+b|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x),x, algorithm="giac")
```

```
[Out] x*log((a*x + b)*c/x) + b*log(abs(a*x + b))/a
```

### 3.94 $\int \log^2 \left( \frac{c(b+ax)}{x} \right) dx$

**Optimal.** Leaf size=67

$$-\frac{2b \operatorname{PolyLog}\left(2, \frac{b}{ax} + 1\right)}{a} + \frac{(ax+b) \log^2\left(ac + \frac{bc}{x}\right)}{a} - \frac{2b \log\left(-\frac{b}{ax}\right) \log\left(c\left(a + \frac{b}{x}\right)\right)}{a}$$

[Out] ((b + a\*x)\*Log[a\*c + (b\*c)/x]^2)/a - (2\*b\*Log[c\*(a + b/x)]\*Log[-(b/(a\*x))])/a - (2\*b\*PolyLog[2, 1 + b/(a\*x)])/a

**Rubi [A]** time = 0.0737622, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2453, 2449, 2454, 2394, 2315}

$$-\frac{2b \operatorname{PolyLog}\left(2, \frac{b}{ax} + 1\right)}{a} + \frac{(ax+b) \log^2\left(ac + \frac{bc}{x}\right)}{a} - \frac{2b \log\left(-\frac{b}{ax}\right) \log\left(c\left(a + \frac{b}{x}\right)\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[Log[(c\*(b + a\*x))/x]^2,x]

[Out] ((b + a\*x)\*Log[a\*c + (b\*c)/x]^2)/a - (2\*b\*Log[c\*(a + b/x)]\*Log[-(b/(a\*x))])/a - (2\*b\*PolyLog[2, 1 + b/(a\*x)])/a

#### Rule 2453

Int[((a\_.) + Log[(c\_.)\*(v\_)^(p\_.)]\*(b\_.))^(q\_.), x\_Symbol] :> Int[(a + b\*Log[c\*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v, x] && !BinomialMatchQ[v, x]

#### Rule 2449

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)/(x\_))^(p\_.)]\*(b\_.))^(q\_), x\_Symbol] :> Simp[((e + d\*x)\*(a + b\*Log[c\*(d + e/x)^p])^q)/d, x] + Dist[(b\*e\*p\*q)/d, Int[(a + b\*Log[c\*(d + e/x)^p])^(q-1)/x, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && IGtQ[q, 0]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Lo



```
g[c*(d + e*x)^p]^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
]^n))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rubi steps

$$\begin{aligned}
\int \log^2\left(\frac{c(b+ax)}{x}\right) dx &= \int \log^2\left(ac + \frac{bc}{x}\right) dx \\
&= \frac{(b+ax) \log^2\left(ac + \frac{bc}{x}\right)}{a} + \frac{(2b) \int \frac{\log\left(ac + \frac{bc}{x}\right)}{x} dx}{a} \\
&= \frac{(b+ax) \log^2\left(ac + \frac{bc}{x}\right)}{a} - \frac{(2b) \operatorname{Subst}\left(\int \frac{\log(ac+bcx)}{x} dx, x, \frac{1}{x}\right)}{a} \\
&= \frac{(b+ax) \log^2\left(ac + \frac{bc}{x}\right)}{a} - \frac{2b \log\left(c\left(a + \frac{b}{x}\right)\right) \log\left(-\frac{b}{ax}\right)}{a} + \frac{(2b^2c) \operatorname{Subst}\left(\int \frac{\log\left(-\frac{bx}{ac+bcx}\right)}{ac+bcx} dx, x, \frac{1}{x}\right)}{a} \\
&= \frac{(b+ax) \log^2\left(ac + \frac{bc}{x}\right)}{a} - \frac{2b \log\left(c\left(a + \frac{b}{x}\right)\right) \log\left(-\frac{b}{ax}\right)}{a} - \frac{2b \operatorname{Li}_2\left(1 + \frac{b}{ax}\right)}{a}
\end{aligned}$$

**Mathematica [A]** time = 0.0135973, size = 63, normalized size = 0.94

$$\frac{\log\left(\frac{c(ax+b)}{x}\right) \left( (ax+b) \log\left(\frac{c(ax+b)}{x}\right) - 2b \log\left(-\frac{b}{ax}\right) \right) - 2b \operatorname{PolyLog}\left(2, \frac{b}{ax} + 1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x))/x]^2,x]
```

[Out]  $(\text{Log}[(c*(b + a*x))/x]*(-2*b*\text{Log}[-(b/(a*x))]) + (b + a*x)*\text{Log}[(c*(b + a*x))/x]) - 2*b*\text{PolyLog}[2, 1 + b/(a*x)]/a$

**Maple [F]** time = 0.655, size = 0, normalized size = 0.

$$\int \left( \ln \left( \frac{c(ax+b)}{x} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(c*(a*x+b)/x)^2,x)`

[Out] `int(ln(c*(a*x+b)/x)^2,x)`

**Maxima [A]** time = 1.17904, size = 153, normalized size = 2.28

$$x \log \left( \frac{(ax+b)c}{x} \right)^2 + \frac{2b \log(ax+b) \log \left( \frac{(ax+b)c}{x} \right)}{a} + \frac{\left( \frac{c \log(ax+b)^2}{a} - \frac{2(\log(\frac{ax}{b}+1) \log(x) + \text{Li}_2(-\frac{ax}{b}))c}{a} \right) b}{c} - \frac{2(c \log(ax+b) - c \log(x)) b \log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*(a*x+b)/x)^2,x, algorithm="maxima")`

[Out] `x*log((a*x + b)*c/x)^2 + 2*b*log(a*x + b)*log((a*x + b)*c/x)/a + ((c*log(a*x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2*(c*log(a*x + b) - c*log(x))*b*log(a*x + b)/a)/c`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \log \left( \frac{acx + bc}{x} \right)^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*(a*x+b)/x)^2,x, algorithm="fricas")`

[Out] `integral(log((a*c*x + b*c)/x)^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$2b \int \frac{\log\left(ac + \frac{bc}{x}\right)}{ax + b} dx + x \log\left(\frac{c(ax + b)}{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*(a*x+b)/x)**2,x)`

[Out] `2*b*Integral(log(a*c + b*c/x)/(a*x + b), x) + x*log(c*(a*x + b)/x)**2`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \log\left(\frac{(ax + b)c}{x}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*(a*x+b)/x)^2,x, algorithm="giac")`

[Out] `integrate(log((a*x + b)*c/x)^2, x)`

### 3.95 $\int \log^3 \left( \frac{c(b+ax)}{x} \right) dx$

**Optimal.** Leaf size=97

$$-\frac{6b \operatorname{PolyLog}\left(2, \frac{b}{ax} + 1\right) \log\left(c\left(a + \frac{b}{x}\right)\right)}{a} + \frac{6b \operatorname{PolyLog}\left(3, \frac{b}{ax} + 1\right)}{a} + \frac{(ax + b) \log^3\left(ac + \frac{bc}{x}\right)}{a} - \frac{3b \log\left(-\frac{b}{ax}\right) \log^2\left(c\left(a + \frac{b}{x}\right)\right)}{a}$$

[Out] ((b + a\*x)\*Log[a\*c + (b\*c)/x]^3)/a - (3\*b\*Log[c\*(a + b/x)]^2\*Log[-(b/(a\*x))])/a - (6\*b\*Log[c\*(a + b/x)]\*PolyLog[2, 1 + b/(a\*x)])/a + (6\*b\*PolyLog[3, 1 + b/(a\*x)])/a

**Rubi [A]** time = 0.110328, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$ , Rules used = {2453, 2449, 2454, 2396, 2433, 2374, 6589}

$$-\frac{6b \operatorname{PolyLog}\left(2, \frac{b}{ax} + 1\right) \log\left(c\left(a + \frac{b}{x}\right)\right)}{a} + \frac{6b \operatorname{PolyLog}\left(3, \frac{b}{ax} + 1\right)}{a} + \frac{(ax + b) \log^3\left(ac + \frac{bc}{x}\right)}{a} - \frac{3b \log\left(-\frac{b}{ax}\right) \log^2\left(c\left(a + \frac{b}{x}\right)\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[Log[(c\*(b + a\*x))/x]^3,x]

[Out] ((b + a\*x)\*Log[a\*c + (b\*c)/x]^3)/a - (3\*b\*Log[c\*(a + b/x)]^2\*Log[-(b/(a\*x))])/a - (6\*b\*Log[c\*(a + b/x)]\*PolyLog[2, 1 + b/(a\*x)])/a + (6\*b\*PolyLog[3, 1 + b/(a\*x)])/a

#### Rule 2453

Int[((a\_.) + Log[(c\_.)\*(v\_)^(p\_.)]\*(b\_.))^(q\_.), x\_Symbol] :> Int[(a + b\*Log[c\*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v, x] && !BinomialMatchQ[v, x]

#### Rule 2449

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)/(x\_))^(p\_.)]\*(b\_.))^(q\_.), x\_Symbol] :> Simp[((e + d\*x)\*(a + b\*Log[c\*(d + e/x)^p])^q)/d, x] + Dist[(b\*e\*p\*q)/d, Int[(a + b\*Log[c\*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && IGtQ[q, 0]

#### Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.))*(b_.))^(q_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.))/(f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

### Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.))*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_)^(m_.))^(r_.)]*(g_.))*((k_.) + (l_.)*(x_)^(r_.)), x_Symbol] := Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

### Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]^(p_.))/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d*e, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \log^3\left(\frac{c(b+ax)}{x}\right) dx &= \int \log^3\left(ac + \frac{bc}{x}\right) dx \\
&= \frac{(b+ax)\log^3\left(ac + \frac{bc}{x}\right)}{a} + \frac{(3b)\int \frac{\log^2\left(ac + \frac{bc}{x}\right)}{x} dx}{a} \\
&= \frac{(b+ax)\log^3\left(ac + \frac{bc}{x}\right)}{a} - \frac{(3b)\text{Subst}\left(\int \frac{\log^2(ac+bcx)}{x} dx, x, \frac{1}{x}\right)}{a} \\
&= \frac{(b+ax)\log^3\left(ac + \frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a + \frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} + \frac{(6b^2c)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)\log(ac+bcx)}{ac+bcx} dx, x, \frac{1}{x}\right)}{a} \\
&= \frac{(b+ax)\log^3\left(ac + \frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a + \frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} + \frac{(6b)\text{Subst}\left(\int \frac{\log(x)\log\left(-\frac{b\left(-\frac{a}{b} + \frac{x}{bc}\right)}{a}\right)}{x} dx, x, \frac{1}{x}\right)}{a} \\
&= \frac{(b+ax)\log^3\left(ac + \frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a + \frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} - \frac{6b\log\left(c\left(a + \frac{b}{x}\right)\right)\text{Li}_2\left(1 + \frac{b}{ax}\right)}{a} + \frac{6b\text{Li}_2\left(1 + \frac{b}{ax}\right)}{a}
\end{aligned}$$

**Mathematica [A]** time = 0.0253013, size = 91, normalized size = 0.94

$$\frac{-6b\text{PolyLog}\left(2, \frac{b}{ax} + 1\right)\log\left(\frac{c(ax+b)}{x}\right) + 6b\text{PolyLog}\left(3, \frac{b}{ax} + 1\right) + \left((ax+b)\log\left(\frac{c(ax+b)}{x}\right) - 3b\log\left(-\frac{b}{ax}\right)\right)\log^2\left(\frac{c(ax+b)}{x}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Log[(c\*(b + a\*x))/x]^3,x]

[Out] (Log[(c\*(b + a\*x))/x]^2\*(-3\*b\*Log[-(b/(a\*x))]) + (b + a\*x)\*Log[(c\*(b + a\*x))/x]) - 6\*b\*Log[(c\*(b + a\*x))/x]\*PolyLog[2, 1 + b/(a\*x)] + 6\*b\*PolyLog[3, 1 + b/(a\*x)]/a

**Maple [F]** time = 0.549, size = 0, normalized size = 0.

$$\int \left(\ln\left(\frac{c(ax+b)}{x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(c*(a*x+b)/x)^3,x)`

[Out] `int(ln(c*(a*x+b)/x)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(ax + b) \log(ax + b)^3 + 3(ax \log(c) - ax \log(x)) \log(ax + b)^2}{a} + \int \frac{ax \log(c)^3 + b \log(c)^3 - (ax + b) \log(x)^3 + 3(ax \log(c) - ax \log(x)) \log(x)^2}{a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*(a*x+b)/x)^3,x, algorithm="maxima")`

[Out] `((a*x + b)*log(a*x + b)^3 + 3*(a*x*log(c) - a*x*log(x))*log(a*x + b)^2)/a + integrate((a*x*log(c)^3 + b*log(c)^3 - (a*x + b)*log(x)^3 + 3*(a*x*log(c) + b*log(c))*log(x)^2 + 3*((log(c)^2 - 2*log(c))*a*x + b*log(c)^2 + (a*x + b)*log(x)^2 - 2*(a*x*(log(c) - 1) + b*log(c))*log(x))*log(a*x + b) - 3*(a*x*log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\log\left(\frac{acx + bc}{x}\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*(a*x+b)/x)^3,x, algorithm="fricas")`

[Out] `integral(log((a*c*x + b*c)/x)^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$3b \int \frac{\log\left(ac + \frac{bc}{x}\right)^2}{ax + b} dx + x \log\left(\frac{c(ax + b)}{x}\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*(a\*x+b)/x)\*\*3,x)

[Out] 3\*b\*Integral(log(a\*c + b\*c/x)\*\*2/(a\*x + b), x) + x\*log(c\*(a\*x + b)/x)\*\*3

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \log\left(\frac{(ax+b)c}{x}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*(a\*x+b)/x)^3,x, algorithm="giac")

[Out] integrate(log((a\*x + b)\*c/x)^3, x)



$$3.96 \quad \int \log \left( \frac{c(b+ax)^2}{x^2} \right) dx$$

**Optimal.** Leaf size=28

$$x \log \left( \frac{c(ax+b)^2}{x^2} \right) + \frac{2b \log(ax+b)}{a}$$

[Out] (2\*b\*Log[b + a\*x])/a + x\*Log[(c\*(b + a\*x)^2)/x^2]

**Rubi [A]** time = 0.0068001, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2486, 31}

$$x \log \left( \frac{c(ax+b)^2}{x^2} \right) + \frac{2b \log(ax+b)}{a}$$

Antiderivative was successfully verified.

[In] Int[Log[(c\*(b + a\*x)^2)/x^2], x]

[Out] (2\*b\*Log[b + a\*x])/a + x\*Log[(c\*(b + a\*x)^2)/x^2]

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

#### Rule 31

```
Int[((a_.) + (b_.)*(x_.))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rubi steps

$$\int \log\left(\frac{c(b+ax)^2}{x^2}\right) dx = x \log\left(\frac{c(b+ax)^2}{x^2}\right) + (2b) \int \frac{1}{b+ax} dx$$

$$= \frac{2b \log(b+ax)}{a} + x \log\left(\frac{c(b+ax)^2}{x^2}\right)$$

**Mathematica [A]** time = 0.0025771, size = 28, normalized size = 1.

$$x \log\left(\frac{c(ax+b)^2}{x^2}\right) + \frac{2b \log(ax+b)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Log[(c\*(b + a\*x)^2)/x^2],x]

[Out] (2\*b\*Log[b + a\*x])/a + x\*Log[(c\*(b + a\*x)^2)/x^2]

**Maple [A]** time = 0.125, size = 40, normalized size = 1.4

$$x \ln\left(c\left(a + \frac{b}{x}\right)^2\right) - 2 \frac{b \ln(x^{-1})}{a} + 2 \frac{b}{a} \ln\left(a + \frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*(a\*x+b)^2/x^2),x)

[Out] x\*ln(c\*(a+b/x)^2)-2\*b/a\*ln(1/x)+2\*b/a\*ln(a+b/x)

**Maxima [A]** time = 1.19178, size = 38, normalized size = 1.36

$$x \log\left(\frac{(ax+b)^2 c}{x^2}\right) + \frac{2b \log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*(a\*x+b)^2/x^2),x, algorithm="maxima")

[Out]  $x \log((a*x + b)^2*c/x^2) + 2*b*\log(a*x + b)/a$

---

**Fricas [A]** time = 1.87655, size = 93, normalized size = 3.32

$$\frac{ax \log\left(\frac{a^2cx^2+2abcx+b^2c}{x^2}\right) + 2b \log(ax + b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*(a*x+b)^2/x^2),x, algorithm="fricas")`

[Out]  $(a*x*\log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2) + 2*b*\log(a*x + b))/a$

---

**Sympy [A]** time = 0.311584, size = 26, normalized size = 0.93

$$x \log\left(\frac{c(ax + b)^2}{x^2}\right) + \frac{2b \log(ax + b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*(a*x+b)**2/x**2),x)`

[Out]  $x*\log(c*(a*x + b)**2/x**2) + 2*b*\log(a*x + b)/a$

---

**Giac [A]** time = 1.14719, size = 39, normalized size = 1.39

$$x \log\left(\frac{(ax + b)^2c}{x^2}\right) + \frac{2b \log(|ax + b|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*(a*x+b)^2/x^2),x, algorithm="giac")`

[Out]  $x*\log((a*x + b)^2*c/x^2) + 2*b*\log(\text{abs}(a*x + b))/a$

### 3.97 $\int \log^2 \left( \frac{c(b+ax)^2}{x^2} \right) dx$

**Optimal.** Leaf size=67

$$\frac{8b \operatorname{PolyLog}\left(2, 1 - \frac{b}{ax+b}\right)}{a} + x \log^2 \left( \frac{c(ax+b)^2}{x^2} \right) - \frac{4b \log\left(\frac{b}{ax+b}\right) \log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}$$

[Out]  $(-4*b*\operatorname{Log}[b/(b+a*x)]*\operatorname{Log}[(c*(b+a*x)^2)/x^2])/a + x*\operatorname{Log}[(c*(b+a*x)^2)/x^2]^2 + (8*b*\operatorname{PolyLog}[2, 1 - b/(b+a*x)])/a$

**Rubi [A]** time = 0.156801, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2486, 2488, 2411, 2343, 2333, 2315}

$$\frac{8b \operatorname{PolyLog}\left(2, 1 - \frac{b}{ax+b}\right)}{a} + x \log^2 \left( \frac{c(ax+b)^2}{x^2} \right) - \frac{4b \log\left(\frac{b}{ax+b}\right) \log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Log}[(c*(b+a*x)^2)/x^2]^2, x]$

[Out]  $(-4*b*\operatorname{Log}[b/(b+a*x)]*\operatorname{Log}[(c*(b+a*x)^2)/x^2])/a + x*\operatorname{Log}[(c*(b+a*x)^2)/x^2]^2 + (8*b*\operatorname{PolyLog}[2, 1 - b/(b+a*x)])/a$

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)]^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

#### Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)]^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/h, x] + Dist[(p*r*s*(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(a + b*x)*(c + d*x), x], x] /; FreeQ[{a, b, c,
```

d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

### Rule 2343

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_)])\*(b\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_)^(r\_.))), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*Log[c\*x])/(x\*(d + e\*x^(r/n))), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]

### Rule 2333

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*((d\_) + (e\_.)/(x\_))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[(e + d\*x)^q\*(a + b\*Log[c\*x^n])^p, x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rubi steps

$$\begin{aligned}
\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx &= x \log^2\left(\frac{c(b+ax)^2}{x^2}\right) + (4b) \int \frac{\log\left(\frac{c(b+ax)^2}{x^2}\right)}{b+ax} dx \\
&= -\frac{4b \log\left(\frac{b}{b+ax}\right) \log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x \log^2\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{(8b^2) \int \frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)} dx}{a} \\
&= -\frac{4b \log\left(\frac{b}{b+ax}\right) \log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x \log^2\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{(8b^2) \text{Subst}\left(\int \frac{\log\left(\frac{b}{x}\right)}{x\left(-\frac{b}{a}+\frac{x}{a}\right)} dx, x, b+ax\right)}{a^2} \\
&= -\frac{4b \log\left(\frac{b}{b+ax}\right) \log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x \log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{(8b^2) \text{Subst}\left(\int \frac{\log(bx)}{\left(-\frac{b}{a}+\frac{1}{ax}\right)x} dx, x, \frac{1}{b+ax}\right)}{a^2} \\
&= -\frac{4b \log\left(\frac{b}{b+ax}\right) \log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x \log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{(8b^2) \text{Subst}\left(\int \frac{\log(bx)}{\frac{1}{a}-\frac{bx}{a}} dx, x, \frac{1}{b+ax}\right)}{a^2} \\
&= -\frac{4b \log\left(\frac{b}{b+ax}\right) \log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x \log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{8b \text{Li}_2\left(\frac{ax}{b+ax}\right)}{a}
\end{aligned}$$

**Mathematica [A]** time = 0.0156204, size = 106, normalized size = 1.58

$$\frac{8b \text{PolyLog}\left(2, \frac{ax+b}{b}\right)}{a} + x \log^2\left(\frac{c(ax+b)^2}{x^2}\right) - \frac{4b \log\left(\frac{b}{ax+b}\right) \log\left(\frac{c(ax+b)^2}{x^2}\right)}{a} - \frac{4b \log^2\left(\frac{b}{ax+b}\right)}{a} - \frac{8b \log\left(-\frac{ax}{b}\right) \log\left(\frac{b}{ax+b}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Log[(c\*(b + a\*x)^2)/x^2]^2,x]

[Out] (-8\*b\*Log[-((a\*x)/b)]\*Log[b/(b + a\*x)])/a - (4\*b\*Log[b/(b + a\*x)]^2)/a - (4\*b\*Log[b/(b + a\*x)]\*Log[(c\*(b + a\*x)^2)/x^2])/a + x\*Log[(c\*(b + a\*x)^2)/x^2]^2 + (8\*b\*PolyLog[2, (b + a\*x)/b])/a

**Maple [F]** time = 0.526, size = 0, normalized size = 0.

$$\int \left( \ln\left(\frac{c(ax+b)^2}{x^2}\right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(c*(a*x+b)^2/x^2)^2,x)`

[Out] `int(ln(c*(a*x+b)^2/x^2)^2,x)`

**Maxima [A]** time = 1.22382, size = 159, normalized size = 2.37

$$x \log\left(\frac{(ax+b)^2 c}{x^2}\right)^2 + \frac{4b \log(ax+b) \log\left(\frac{(ax+b)^2 c}{x^2}\right)}{a} + \frac{4\left(\left(\frac{c \log(ax+b)^2}{a} - \frac{2(\log(\frac{ax}{b}+1)\log(x)+\text{Li}_2(-\frac{ax}{b}))c}{a}\right)b - \frac{2(c \log(ax+b)-c \log(x))}{a}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="maxima")`

[Out] `x*log((a*x + b)^2*c/x^2)^2 + 4*b*log(a*x + b)*log((a*x + b)^2*c/x^2)/a + 4*((c*log(a*x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2*(c*log(a*x + b) - c*log(x))*b*log(a*x + b)/a)/c`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\log\left(\frac{a^2cx^2 + 2abcx + b^2c}{x^2}\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="fricas")`

[Out] `integral(log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2)^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$4b \int \frac{\log\left(a^2c + \frac{2abc}{x} + \frac{b^2c}{x^2}\right)}{ax+b} dx + x \log\left(\frac{c(ax+b)^2}{x^2}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*(a\*x+b)\*\*2/x\*\*2)\*\*2,x)

[Out] 4\*b\*Integral(log(a\*\*2\*c + 2\*a\*b\*c/x + b\*\*2\*c/x\*\*2)/(a\*x + b), x) + x\*log(c\*(a\*x + b)\*\*2/x\*\*2)\*\*2

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \log\left(\frac{(ax + b)^2 c}{x^2}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*(a\*x+b)^2/x^2)^2,x, algorithm="giac")

[Out] integrate(log((a\*x + b)^2\*c/x^2)^2, x)



$$3.98 \quad \int \log^3 \left( \frac{c(b+ax)^2}{x^2} \right) dx$$

**Optimal.** Leaf size=102

$$\frac{24b \text{PolyLog} \left( 2, \frac{ax}{ax+b} \right) \log \left( \frac{c(ax+b)^2}{x^2} \right)}{a} + \frac{48b \text{PolyLog} \left( 3, \frac{ax}{ax+b} \right)}{a} + x \log^3 \left( \frac{c(ax+b)^2}{x^2} \right) - \frac{6b \log \left( 1 - \frac{ax}{ax+b} \right) \log^2 \left( \frac{c(ax+b)^2}{x^2} \right)}{a}$$

[Out] x\*Log[(c\*(b + a\*x)^2)/x^2]^3 - (6\*b\*Log[(c\*(b + a\*x)^2)/x^2]^2\*Log[1 - (a\*x)/(b + a\*x)])/a + (24\*b\*Log[(c\*(b + a\*x)^2)/x^2]\*PolyLog[2, (a\*x)/(b + a\*x)])/a + (48\*b\*PolyLog[3, (a\*x)/(b + a\*x)])/a

**Rubi [A]** time = 0.131848, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2486, 2488, 2506, 6610}

$$\frac{24b \text{PolyLog} \left( 2, 1 - \frac{b}{ax+b} \right) \log \left( \frac{c(ax+b)^2}{x^2} \right)}{a} + \frac{48b \text{PolyLog} \left( 3, 1 - \frac{b}{ax+b} \right)}{a} + x \log^3 \left( \frac{c(ax+b)^2}{x^2} \right) - \frac{6b \log \left( \frac{b}{ax+b} \right) \log^2 \left( \frac{c(ax+b)^2}{x^2} \right)}{a}$$

Antiderivative was successfully verified.

[In] Int[Log[(c\*(b + a\*x)^2)/x^2]^3,x]

[Out] (-6\*b\*Log[b/(b + a\*x)]\*Log[(c\*(b + a\*x)^2)/x^2]^2)/a + x\*Log[(c\*(b + a\*x)^2)/x^2]^3 + (24\*b\*Log[(c\*(b + a\*x)^2)/x^2]\*PolyLog[2, 1 - b/(b + a\*x)])/a + (48\*b\*PolyLog[3, 1 - b/(b + a\*x)])/a

#### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] :> Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

#### Rule 2488

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] :> -Simp[(Log[-(b\*c - a\*d)/(d\*(a + b\*x))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/h, x] + Dist[(p\*r\*s\*

$(b*c - a*d)/h$ , Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

### Rule 2506

Int[Log[v\_]\*Log[(e\_.\*((f\_.\*((a\_.) + (b\_.\*(x\_))^(p\_.\*((c\_.) + (d\_.\*(x\_))^(q\_))^(r\_))^(s\_.\*(u\_)), x\_Symbol] :> With[{g = Simplify[((v - 1)\*(c + d\*x))/(a + b\*x)], h = Simplify[u\*(a + b\*x)\*(c + d\*x)]}, -Simp[(h\*PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1))/(b\*c - a\*d), x] + Dist[h\*p\*r\*s, Int[(PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] :> With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rubi steps

$$\begin{aligned} \int \log^3\left(\frac{c(b+ax)^2}{x^2}\right) dx &= x \log^3\left(\frac{c(b+ax)^2}{x^2}\right) + (6b) \int \frac{\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{b+ax} dx \\ &= -\frac{6b \log\left(\frac{b}{b+ax}\right) \log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x \log^3\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{(24b^2) \int \frac{\log\left(\frac{b}{b+ax}\right) \log\left(\frac{c(b+ax)^2}{x^2}\right)}{x(b+ax)} dx}{a} \\ &= -\frac{6b \log\left(\frac{b}{b+ax}\right) \log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x \log^3\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{24b \log\left(\frac{c(b+ax)^2}{x^2}\right) \text{Li}_2\left(1 - \frac{b}{b+ax}\right)}{a} + \frac{(48b^2) \int \frac{\log\left(\frac{b}{b+ax}\right) \log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{x(b+ax)} dx}{a} \\ &= -\frac{6b \log\left(\frac{b}{b+ax}\right) \log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x \log^3\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{24b \log\left(\frac{c(b+ax)^2}{x^2}\right) \text{Li}_2\left(1 - \frac{b}{b+ax}\right)}{a} + \frac{48b^2 \int \frac{\log\left(\frac{b}{b+ax}\right) \log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{x(b+ax)} dx}{a} \end{aligned}$$

**Mathematica [A]** time = 0.0245463, size = 98, normalized size = 0.96

$$\frac{24b \text{PolyLog}\left(2, \frac{ax}{ax+b}\right) \log\left(\frac{c(ax+b)^2}{x^2}\right)}{a} + \frac{48b \text{PolyLog}\left(3, \frac{ax}{ax+b}\right)}{a} + x \log^3\left(\frac{c(ax+b)^2}{x^2}\right) - \frac{6b \log\left(\frac{b}{ax+b}\right) \log^2\left(\frac{c(ax+b)^2}{x^2}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Log[(c\*(b + a\*x)^2)/x^2]^3,x]

[Out] (-6\*b\*Log[b/(b + a\*x)]\*Log[(c\*(b + a\*x)^2)/x^2]^2)/a + x\*Log[(c\*(b + a\*x)^2)/x^2]^3 + (24\*b\*Log[(c\*(b + a\*x)^2)/x^2]\*PolyLog[2, (a\*x)/(b + a\*x)])/a + (48\*b\*PolyLog[3, (a\*x)/(b + a\*x)])/a

**Maple [F]** time = 0.504, size = 0, normalized size = 0.

$$\int \left( \ln \left( \frac{c(ax+b)^2}{x^2} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*(a\*x+b)^2/x^2)^3,x)

[Out] int(ln(c\*(a\*x+b)^2/x^2)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{4 \left( 2(ax+b) \log(ax+b)^3 + 3(ax \log(c) - 2ax \log(x)) \log(ax+b)^2 \right)}{a} + \int \frac{ax \log(c)^3 + b \log(c)^3 - 8(ax+b) \log(x)^3 - \dots}{\dots} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*(a\*x+b)^2/x^2)^3,x, algorithm="maxima")

[Out] 4\*(2\*(a\*x + b)\*log(a\*x + b)^3 + 3\*(a\*x\*log(c) - 2\*a\*x\*log(x))\*log(a\*x + b)^2)/a + integrate((a\*x\*log(c)^3 + b\*log(c)^3 - 8\*(a\*x + b)\*log(x)^3 + 12\*(a\*x\*log(c) + b\*log(c))\*log(x)^2 + 6\*((log(c)^2 - 4\*log(c))\*a\*x + b\*log(c)^2 + 4\*(a\*x + b)\*log(x)^2 - 4\*(a\*x\*(log(c) - 2) + b\*log(c))\*log(x))\*log(a\*x + b) - 6\*(a\*x\*log(c)^2 + b\*log(c)^2)\*log(x))/(a\*x + b), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \log \left( \frac{a^2cx^2 + 2abcx + b^2c}{x^2} \right)^3, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*(a\*x+b)^2/x^2)^3,x, algorithm="fricas")

[Out] integral(log((a^2\*c\*x^2 + 2\*a\*b\*c\*x + b^2\*c)/x^2)^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$6b \int \frac{\log\left(a^2c + \frac{2abc}{x} + \frac{b^2c}{x^2}\right)^2}{ax + b} dx + x \log\left(\frac{c(ax + b)^2}{x^2}\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*(a\*x+b)\*\*2/x\*\*2)\*\*3,x)

[Out] 6\*b\*Integral(log(a\*\*2\*c + 2\*a\*b\*c/x + b\*\*2\*c/x\*\*2)\*\*2/(a\*x + b), x) + x\*log(c\*(a\*x + b)\*\*2/x\*\*2)\*\*3

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \log\left(\frac{(ax + b)^2c}{x^2}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*(a\*x+b)^2/x^2)^3,x, algorithm="giac")

[Out] integrate(log((a\*x + b)^2\*c/x^2)^3, x)

$$3.99 \quad \int \log\left(\frac{cx^2}{(b+ax)^2}\right) dx$$

**Optimal.** Leaf size=28

$$x \log\left(\frac{cx^2}{(ax+b)^2}\right) - \frac{2b \log(ax+b)}{a}$$

[Out] x\*Log[(c\*x^2)/(b + a\*x)^2] - (2\*b\*Log[b + a\*x])/a

**Rubi [A]** time = 0.0066862, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2486, 31}

$$x \log\left(\frac{cx^2}{(ax+b)^2}\right) - \frac{2b \log(ax+b)}{a}$$

Antiderivative was successfully verified.

[In] Int[Log[(c\*x^2)/(b + a\*x)^2], x]

[Out] x\*Log[(c\*x^2)/(b + a\*x)^2] - (2\*b\*Log[b + a\*x])/a

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

#### Rule 31

```
Int[((a_.) + (b_.)*(x_.))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rubi steps

$$\begin{aligned}\int \log\left(\frac{cx^2}{(b+ax)^2}\right) dx &= x \log\left(\frac{cx^2}{(b+ax)^2}\right) - (2b) \int \frac{1}{b+ax} dx \\ &= x \log\left(\frac{cx^2}{(b+ax)^2}\right) - \frac{2b \log(b+ax)}{a}\end{aligned}$$

**Mathematica [A]** time = 0.0028988, size = 28, normalized size = 1.

$$x \log\left(\frac{cx^2}{(ax+b)^2}\right) - \frac{2b \log(ax+b)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Log[(c\*x^2)/(b + a\*x)^2],x]

[Out] x\*Log[(c\*x^2)/(b + a\*x)^2] - (2\*b\*Log[b + a\*x])/a

**Maple [B]** time = 0.224, size = 79, normalized size = 2.8

$$\ln\left(\frac{c}{a^2}\left(\frac{b}{ax+b}-1\right)^2\right)x + 2\frac{b \ln((ax+b)^{-1})}{a} - 2\frac{b}{a} \ln\left(\frac{b}{ax+b}-1\right) + \frac{b}{a} \ln\left(\frac{c}{a^2}\left(\frac{b}{ax+b}-1\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x^2/(a\*x+b)^2),x)

[Out] ln(c\*(b/(a\*x+b)-1)^2/a^2)\*x+2/a\*b\*ln(1/(a\*x+b))-2/a\*b\*ln(b/(a\*x+b)-1)+1/a\*ln(c\*(b/(a\*x+b)-1)^2/a^2)\*b

**Maxima [A]** time = 1.24105, size = 38, normalized size = 1.36

$$x \log\left(\frac{cx^2}{(ax+b)^2}\right) - \frac{2b \log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x^2/(a\*x+b)^2),x, algorithm="maxima")

[Out]  $x \log(c x^2 / (a x + b)^2) - 2 b \log(a x + b) / a$

---

**Fricas [A]** time = 1.96246, size = 88, normalized size = 3.14

$$\frac{ax \log\left(\frac{cx^2}{a^2x^2 + 2abx + b^2}\right) - 2b \log(ax + b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x^2/(a*x+b)^2),x, algorithm="fricas")`

[Out]  $(a x \log(c x^2 / (a^2 x^2 + 2 a b x + b^2)) - 2 b \log(a x + b)) / a$

---

**Sympy [A]** time = 0.363537, size = 26, normalized size = 0.93

$$x \log\left(\frac{cx^2}{(ax + b)^2}\right) - \frac{2b \log(ax + b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*x**2/(a*x+b)**2),x)`

[Out]  $x \log(c x^2 / (a x + b)^2) - 2 b \log(a x + b) / a$

---

**Giac [A]** time = 1.26297, size = 39, normalized size = 1.39

$$x \log\left(\frac{cx^2}{(ax + b)^2}\right) - \frac{2b \log(|ax + b|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x^2/(a*x+b)^2),x, algorithm="giac")`

[Out]  $x \log(c x^2 / (a x + b)^2) - 2 b \log(\text{abs}(a x + b)) / a$

$$3.100 \quad \int \log^2 \left( \frac{cx^2}{(b+ax)^2} \right) dx$$

**Optimal.** Leaf size=67

$$\frac{8b \operatorname{PolyLog} \left( 2, 1 - \frac{b}{ax+b} \right)}{a} + x \log^2 \left( \frac{cx^2}{(ax+b)^2} \right) + \frac{4b \log \left( \frac{b}{ax+b} \right) \log \left( \frac{cx^2}{(ax+b)^2} \right)}{a}$$

[Out] x\*Log[(c\*x^2)/(b + a\*x)^2]^2 + (4\*b\*Log[(c\*x^2)/(b + a\*x)^2]\*Log[b/(b + a\*x))]/a + (8\*b\*PolyLog[2, 1 - b/(b + a\*x)])/a

**Rubi [A]** time = 0.154865, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2486, 2488, 2411, 2343, 2333, 2315}

$$\frac{8b \operatorname{PolyLog} \left( 2, 1 - \frac{b}{ax+b} \right)}{a} + x \log^2 \left( \frac{cx^2}{(ax+b)^2} \right) + \frac{4b \log \left( \frac{b}{ax+b} \right) \log \left( \frac{cx^2}{(ax+b)^2} \right)}{a}$$

Antiderivative was successfully verified.

[In] Int[Log[(c\*x^2)/(b + a\*x)^2]^2,x]

[Out] x\*Log[(c\*x^2)/(b + a\*x)^2]^2 + (4\*b\*Log[(c\*x^2)/(b + a\*x)^2]\*Log[b/(b + a\*x))]/a + (8\*b\*PolyLog[2, 1 - b/(b + a\*x)])/a

### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]^(s\_.), x\_Symbol] :> Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

### Rule 2488

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] :> -Simp[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d))/h, Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c,



d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

### Rule 2343

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_)])\*(b\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_)^(r\_.))), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*Log[c\*x])/(x\*(d + e\*x^(r/n))), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]

### Rule 2333

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*((d\_) + (e\_.)/(x\_))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[(e + d\*x)^q\*(a + b\*Log[c\*x^n])^p, x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rubi steps

$$\begin{aligned}
\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right) dx &= x \log^2\left(\frac{cx^2}{(b+ax)^2}\right) - (4b) \int \frac{\log\left(\frac{cx^2}{(b+ax)^2}\right)}{b+ax} dx \\
&= x \log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b \log\left(\frac{cx^2}{(b+ax)^2}\right) \log\left(\frac{b}{b+ax}\right)}{a} - \frac{(8b^2) \int \frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)} dx}{a} \\
&= x \log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b \log\left(\frac{cx^2}{(b+ax)^2}\right) \log\left(\frac{b}{b+ax}\right)}{a} - \frac{(8b^2) \text{Subst}\left(\int \frac{\log\left(\frac{b}{x}\right)}{x\left(\frac{-b}{a} + \frac{x}{a}\right)} dx, x, b+ax\right)}{a^2} \\
&= x \log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b \log\left(\frac{cx^2}{(b+ax)^2}\right) \log\left(\frac{b}{b+ax}\right)}{a} + \frac{(8b^2) \text{Subst}\left(\int \frac{\log(bx)}{\left(\frac{-b}{a} + \frac{1}{ax}\right)^x} dx, x, \frac{1}{b+ax}\right)}{a^2} \\
&= x \log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b \log\left(\frac{cx^2}{(b+ax)^2}\right) \log\left(\frac{b}{b+ax}\right)}{a} + \frac{(8b^2) \text{Subst}\left(\int \frac{\log(bx)}{\frac{1}{a} - \frac{bx}{a}} dx, x, \frac{1}{b+ax}\right)}{a^2} \\
&= x \log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b \log\left(\frac{cx^2}{(b+ax)^2}\right) \log\left(\frac{b}{b+ax}\right)}{a} + \frac{8b \text{Li}_2\left(\frac{ax}{b+ax}\right)}{a}
\end{aligned}$$

**Mathematica [A]** time = 0.0141174, size = 106, normalized size = 1.58

$$\frac{8b \text{PolyLog}\left(2, \frac{ax+b}{b}\right)}{a} + x \log^2\left(\frac{cx^2}{(ax+b)^2}\right) + \frac{4b \log\left(\frac{b}{ax+b}\right) \log\left(\frac{cx^2}{(ax+b)^2}\right)}{a} - \frac{4b \log^2\left(\frac{b}{ax+b}\right)}{a} - \frac{8b \log\left(-\frac{ax}{b}\right) \log\left(\frac{b}{ax+b}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Log[(c\*x^2)/(b + a\*x)^2]^2,x]

[Out] x\*Log[(c\*x^2)/(b + a\*x)^2]^2 - (8\*b\*Log[-((a\*x)/b)]\*Log[b/(b + a\*x)])/a + (4\*b\*Log[(c\*x^2)/(b + a\*x)^2]\*Log[b/(b + a\*x)])/a - (4\*b\*Log[b/(b + a\*x)]^2)/a + (8\*b\*PolyLog[2, (b + a\*x)/b])/a

**Maple [F]** time = 0.94, size = 0, normalized size = 0.

$$\int \left( \ln\left(\frac{cx^2}{(ax+b)^2}\right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(c*x^2/(a*x+b)^2)^2,x)`

[Out] `int(ln(c*x^2/(a*x+b)^2)^2,x)`

**Maxima [A]** time = 1.1449, size = 159, normalized size = 2.37

$$x \log\left(\frac{cx^2}{(ax+b)^2}\right)^2 - \frac{4b \log(ax+b) \log\left(\frac{cx^2}{(ax+b)^2}\right)}{a} + \frac{4 \left( \left( \frac{c \log(ax+b)^2}{a} - \frac{2(\log(\frac{ax}{b}+1) \log(x) + \text{Li}_2(-\frac{ax}{b}))c}{a} \right) b - \frac{2(c \log(ax+b) - c \log(x))b}{a} \right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="maxima")`

[Out] `x*log(c*x^2/(a*x + b)^2)^2 - 4*b*log(a*x + b)*log(c*x^2/(a*x + b)^2)/a + 4*((c*log(a*x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2*(c*log(a*x + b) - c*log(x))*b*log(a*x + b)/a)/c`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\log\left(\frac{cx^2}{a^2x^2 + 2abx + b^2}\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="fricas")`

[Out] `integral(log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2))^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-4b \int \frac{\log\left(\frac{cx^2}{a^2x^2+2abx+b^2}\right)}{ax+b} dx + x \log\left(\frac{cx^2}{(ax+b)^2}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*x\*\*2/(a\*x+b)\*\*2)\*\*2,x)

[Out] -4\*b\*Integral(log(c\*x\*\*2/(a\*\*2\*x\*\*2 + 2\*a\*b\*x + b\*\*2)))/(a\*x + b), x) + x\*log(c\*x\*\*2/(a\*x + b)\*\*2)\*\*2

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \log\left(\frac{cx^2}{(ax+b)^2}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x^2/(a\*x+b)^2)^2,x, algorithm="giac")

[Out] integrate(log(c\*x^2/(a\*x + b)^2)^2, x)

### 3.101 $\int \log^3 \left( \frac{cx^2}{(b+ax)^2} \right) dx$

**Optimal.** Leaf size=98

$$\frac{24b \operatorname{PolyLog}\left(2, \frac{ax}{ax+b}\right) \log\left(\frac{cx^2}{(ax+b)^2}\right)}{a} - \frac{48b \operatorname{PolyLog}\left(3, \frac{ax}{ax+b}\right)}{a} + x \log^3\left(\frac{cx^2}{(ax+b)^2}\right) + \frac{6b \log\left(\frac{b}{ax+b}\right) \log^2\left(\frac{cx^2}{(ax+b)^2}\right)}{a}$$

[Out] x\*Log[(c\*x^2)/(b + a\*x)^2]^3 + (6\*b\*Log[(c\*x^2)/(b + a\*x)^2]^2\*Log[b/(b + a\*x)])/a + (24\*b\*Log[(c\*x^2)/(b + a\*x)^2]\*PolyLog[2, (a\*x)/(b + a\*x)])/a - (48\*b\*PolyLog[3, (a\*x)/(b + a\*x)])/a

**Rubi [A]** time = 0.127085, antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2486, 2488, 2506, 6610}

$$\frac{24b \operatorname{PolyLog}\left(2, 1 - \frac{b}{ax+b}\right) \log\left(\frac{cx^2}{(ax+b)^2}\right)}{a} - \frac{48b \operatorname{PolyLog}\left(3, 1 - \frac{b}{ax+b}\right)}{a} + x \log^3\left(\frac{cx^2}{(ax+b)^2}\right) + \frac{6b \log\left(\frac{b}{ax+b}\right) \log^2\left(\frac{cx^2}{(ax+b)^2}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[Log[(c\*x^2)/(b + a\*x)^2]^3,x]

[Out] x\*Log[(c\*x^2)/(b + a\*x)^2]^3 + (6\*b\*Log[(c\*x^2)/(b + a\*x)^2]^2\*Log[b/(b + a\*x)])/a + (24\*b\*Log[(c\*x^2)/(b + a\*x)^2]\*PolyLog[2, 1 - b/(b + a\*x)])/a - (48\*b\*PolyLog[3, 1 - b/(b + a\*x)])/a

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

#### Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-(b*c - a*d)/(d*(a + b*x))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/h, x] + Dist[(p*r*s*
```

$(b*c - a*d)/h$ , Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

### Rule 2506

Int[Log[v\_]\*Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]^(s\_.)\*(u\_), x\_Symbol] :> With[{g = Simplify[((v - 1)\*(c + d\*x))/(a + b\*x)], h = Simplify[u\*(a + b\*x)\*(c + d\*x)]}, -Simp[(h\*PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1))/(b\*c - a\*d), x] + Dist[h\*p\*r\*s, Int[(PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] :> With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rubi steps

$$\begin{aligned} \int \log^3\left(\frac{cx^2}{(b+ax)^2}\right) dx &= x \log^3\left(\frac{cx^2}{(b+ax)^2}\right) - (6b) \int \frac{\log^2\left(\frac{cx^2}{(b+ax)^2}\right)}{b+ax} dx \\ &= x \log^3\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{6b \log^2\left(\frac{cx^2}{(b+ax)^2}\right) \log\left(\frac{b}{b+ax}\right)}{a} - \frac{(24b^2) \int \frac{\log\left(\frac{cx^2}{(b+ax)^2}\right) \log\left(\frac{b}{b+ax}\right)}{x(b+ax)} dx}{a} \\ &= x \log^3\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{6b \log^2\left(\frac{cx^2}{(b+ax)^2}\right) \log\left(\frac{b}{b+ax}\right)}{a} + \frac{24b \log\left(\frac{cx^2}{(b+ax)^2}\right) \text{Li}_2\left(1 - \frac{b}{b+ax}\right)}{a} - \frac{(48b^2) \int \dots}{a} \\ &= x \log^3\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{6b \log^2\left(\frac{cx^2}{(b+ax)^2}\right) \log\left(\frac{b}{b+ax}\right)}{a} + \frac{24b \log\left(\frac{cx^2}{(b+ax)^2}\right) \text{Li}_2\left(1 - \frac{b}{b+ax}\right)}{a} - \frac{48b \text{Li}_3\left(1 - \frac{b}{b+ax}\right)}{a} \end{aligned}$$

**Mathematica [A]** time = 0.0246982, size = 98, normalized size = 1.

$$\frac{24b \text{PolyLog}\left(2, \frac{ax}{ax+b}\right) \log\left(\frac{cx^2}{(ax+b)^2}\right)}{a} - \frac{48b \text{PolyLog}\left(3, \frac{ax}{ax+b}\right)}{a} + x \log^3\left(\frac{cx^2}{(ax+b)^2}\right) + \frac{6b \log\left(\frac{b}{ax+b}\right) \log^2\left(\frac{cx^2}{(ax+b)^2}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Log[(c\*x^2)/(b + a\*x)^2]^3,x]

[Out] x\*Log[(c\*x^2)/(b + a\*x)^2]^3 + (6\*b\*Log[(c\*x^2)/(b + a\*x)^2]^2\*Log[b/(b + a\*x)])/a + (24\*b\*Log[(c\*x^2)/(b + a\*x)^2]\*PolyLog[2, (a\*x)/(b + a\*x)])/a - (48\*b\*PolyLog[3, (a\*x)/(b + a\*x)])/a

**Maple [F]** time = 0.786, size = 0, normalized size = 0.

$$\int \left( \ln \left( \frac{cx^2}{(ax+b)^2} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x^2/(a\*x+b)^2)^3,x)

[Out] int(ln(c\*x^2/(a\*x+b)^2)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{4 \left( 2(ax+b) \log(ax+b)^3 - 3(ax \log(c) + 2ax \log(x)) \log(ax+b)^2 \right)}{a} - \int -\frac{ax \log(c)^3 + b \log(c)^3 + 8(ax+b) \log(x)}{a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x^2/(a\*x+b)^2)^3,x, algorithm="maxima")

[Out] -4\*(2\*(a\*x + b)\*log(a\*x + b)^3 - 3\*(a\*x\*log(c) + 2\*a\*x\*log(x))\*log(a\*x + b)^2)/a - integrate(-(a\*x\*log(c)^3 + b\*log(c)^3 + 8\*(a\*x + b)\*log(x)^3 + 12\*(a\*x\*log(c) + b\*log(c))\*log(x)^2 - 6\*((log(c)^2 + 4\*log(c))\*a\*x + b\*log(c)^2 + 4\*(a\*x + b)\*log(x)^2 + 4\*(a\*x\*(log(c) + 2) + b\*log(c))\*log(x))\*log(a\*x + b) + 6\*(a\*x\*log(c)^2 + b\*log(c)^2)\*log(x))/(a\*x + b), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \log \left( \frac{cx^2}{a^2x^2 + 2abx + b^2} \right)^3, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x^2/(a\*x+b)^2)^3,x, algorithm="fricas")

[Out] integral(log(c\*x^2/(a^2\*x^2 + 2\*a\*b\*x + b^2))^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-6b \int \frac{\log\left(\frac{cx^2}{a^2x^2+2abx+b^2}\right)^2}{ax+b} dx + x \log\left(\frac{cx^2}{(ax+b)^2}\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*x\*\*2/(a\*x+b)\*\*2)\*\*3,x)

[Out] -6\*b\*Integral(log(c\*x\*\*2/(a\*\*2\*x\*\*2 + 2\*a\*b\*x + b\*\*2))\*\*2/(a\*x + b), x) + x\*log(c\*x\*\*2/(a\*x + b)\*\*2)\*\*3

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \log\left(\frac{cx^2}{(ax+b)^2}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x^2/(a\*x+b)^2)^3,x, algorithm="giac")

[Out] integrate(log(c\*x^2/(a\*x + b)^2)^3, x)



$$3.102 \quad \int \frac{\text{PolyLog}\left(2, 1 + \frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)} dx$$

Optimal. Leaf size=35

$$-\frac{\text{PolyLog}\left(3, \frac{bc-ad}{d(a+bx)} + 1\right)}{bc-ad}$$

[Out]  $-(\text{PolyLog}[3, 1 + (b*c - a*d)/(d*(a + b*x))]/(b*c - a*d))$

**Rubi [A]** time = 0.0638609, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {6610}

$$-\frac{\text{PolyLog}\left(3, \frac{bc-ad}{d(a+bx)} + 1\right)}{bc-ad}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{PolyLog}[2, 1 + (b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x]$

[Out]  $-(\text{PolyLog}[3, 1 + (b*c - a*d)/(d*(a + b*x))]/(b*c - a*d))$

### Rule 6610

$\text{Int}[(u_*)\text{PolyLog}[n_, v_], x\_Symbol] \text{ :> With}[\{w = \text{DerivativeDivides}[v, u*v, x]\}, \text{Simp}[w*\text{PolyLog}[n + 1, v], x] \text{ /; !FalseQ}[w] \text{ /; FreeQ}[n, x]$

### Rubi steps

$$\int \frac{\text{Li}_2\left(1 + \frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)} dx = -\frac{\text{Li}_3\left(1 + \frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

**Mathematica [A]** time = 0.0115641, size = 30, normalized size = 0.86

$$\frac{\text{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right)}{ad-bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x]
```

```
[Out] PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]/(-(b*c) + a*d)
```

**Maple [A]** time = 0.062, size = 36, normalized size = 1.

$$\frac{1}{ad - bc} \operatorname{polylog}\left(3, 1 - \frac{ad - bc}{d(bx + a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, 1 + (-a*d + b*c)/d/(b*x + a))/(b*x + a)/(d*x + c), x)
```

```
[Out] 1/(a*d - b*c) * polylog(3, 1 - (a*d - b*c)/d/(b*x + a))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2\left(\frac{bc - ad}{(bx + a)d} + 1\right)}{(bx + a)(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2, 1 + (-a*d + b*c)/d/(b*x + a))/(b*x + a)/(d*x + c), x, algorithm="maxima")
```

```
[Out] integrate(dilog((b*c - a*d)/((b*x + a)*d) + 1)/((b*x + a)*(d*x + c)), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{Li}_2\left(\frac{bc - ad}{bdx + ad} + 1\right)}{bdx^2 + ac + (bc + ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="f
ricas")
```

```
[Out] integral(dilog((b*c - a*d)/(b*d*x + a*d) + 1)/(b*d*x^2 + a*c + (b*c + a*d)*
x), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2\left(\frac{bc-ad}{(bx+a)d} + 1\right)}{(bx+a)(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="g
iac")
```

```
[Out] integrate(dilog((b*c - a*d)/((b*x + a)*d) + 1)/((b*x + a)*(d*x + c)), x)
```

$$3.103 \quad \int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right) \log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)} dx$$

**Optimal.** Leaf size=85

$$\frac{\text{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right) \log\left(\frac{e(c+dx)}{a+bx}\right)}{bc-ad} - \frac{\text{PolyLog}\left(3, \frac{bc-ad}{d(a+bx)} + 1\right)}{bc-ad}$$

[Out] (Log[(e\*(c + d\*x))/(a + b\*x)]\*PolyLog[2, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/(b\*c - a\*d) - PolyLog[3, 1 + (b\*c - a\*d)/(d\*(a + b\*x))]/(b\*c - a\*d)

**Rubi [A]** time = 0.134357, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 50,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {2506, 6610}

$$\frac{\text{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right) \log\left(\frac{e(c+dx)}{a+bx}\right)}{bc-ad} - \frac{\text{PolyLog}\left(3, \frac{bc-ad}{d(a+bx)} + 1\right)}{bc-ad}$$

Antiderivative was successfully verified.

[In] Int[(Log[(-(b\*c) + a\*d)/(d\*(a + b\*x))]\*Log[(e\*(c + d\*x))/(a + b\*x)])/(a + b\*x)\*(c + d\*x)), x]

[Out] (Log[(e\*(c + d\*x))/(a + b\*x)]\*PolyLog[2, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/(b\*c - a\*d) - PolyLog[3, 1 + (b\*c - a\*d)/(d\*(a + b\*x))]/(b\*c - a\*d)

### Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] + Dist[h*p*r*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)]/(a + b*x)*(c + d*x), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right) \log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)} dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right) \operatorname{Li}_2\left(1 + \frac{bc-ad}{d(a+bx)}\right)}{bc-ad} + \int \frac{\operatorname{Li}_2\left(1 - \frac{-bc+ad}{d(a+bx)}\right)}{(a+bx)(c+dx)} dx$$

$$= \frac{\log\left(\frac{e(c+dx)}{a+bx}\right) \operatorname{Li}_2\left(1 + \frac{bc-ad}{d(a+bx)}\right)}{bc-ad} - \frac{\operatorname{Li}_3\left(1 + \frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

**Mathematica [A]** time = 0.0237251, size = 68, normalized size = 0.8

$$\frac{\operatorname{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right) \log\left(\frac{e(c+dx)}{a+bx}\right) - \operatorname{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad}$$

Antiderivative was successfully verified.

[In] Integrate[(Log[(-(b\*c) + a\*d)/(d\*(a + b\*x))]\*Log[(e\*(c + d\*x))/(a + b\*x)])/(  
(a + b\*x)\*(c + d\*x)),x]

[Out] (Log[(e\*(c + d\*x))/(a + b\*x)]\*PolyLog[2, (b\*(c + d\*x))/(d\*(a + b\*x))] - PolyLog[3, (b\*(c + d\*x))/(d\*(a + b\*x))])/(b\*c - a\*d)

**Maple [F]** time = 1.318, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)} \ln\left(\frac{ad-bc}{d(bx+a)}\right) \ln\left(\frac{e(dx+c)}{bx+a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln((a\*d-b\*c)/d/(b\*x+a))\*ln(e\*(d\*x+c)/(b\*x+a))/(b\*x+a)/(d\*x+c),x)

[Out] int(ln((a\*d-b\*c)/d/(b\*x+a))\*ln(e\*(d\*x+c)/(b\*x+a))/(b\*x+a)/(d\*x+c),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right) \log\left(-\frac{bc-ad}{(bx+a)d}\right)}{(bx+a)(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c), x
, algorithm="maxima")
```

```
[Out] integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)/((b*x + a)*d))/((b*x
+ a)*(d*x + c)), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( -\frac{bc-ad}{bdx+ad} \right) \log \left( \frac{dex+ce}{bx+a} \right)}{bdx^2 + ac + (bc + ad)x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c), x
, algorithm="fricas")
```

```
[Out] integral(log(-(b*c - a*d)/(b*d*x + a*d))*log((d*e*x + c*e)/(b*x + a))/(b*d*
x^2 + a*c + (b*c + a*d)*x), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a*d-b*c)/d/(b*x+a))*ln(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c), x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( \frac{(dx+c)e}{bx+a} \right) \log \left( -\frac{bc-ad}{(bx+a)d} \right)}{(bx+a)(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c), x  
, algorithm="giac")
```

```
[Out] integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)/((b*x + a)*d))/((b*x  
+ a)*(d*x + c)), x)
```

$$3.104 \quad \int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx} dx$$

**Optimal.** Leaf size=140

$$-\frac{2\text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right) \log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b} + \frac{2\text{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right)}{b} - \frac{\log\left(\frac{ad-bc}{d(a+bx)}\right) \log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}$$

[Out] -((Log[(-(b\*c) + a\*d)/(d\*(a + b\*x))]\*Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]^2)/b) - (2\*Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]\*PolyLog[2, (b\*(c + d\*x))/(d\*(a + b\*x))])/b + (2\*PolyLog[3, (b\*(c + d\*x))/(d\*(a + b\*x))])/b

**Rubi [A]** time = 0.180268, antiderivative size = 149, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 42,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2488, 2506, 6610}

$$-\frac{2\text{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right) \log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b} + \frac{2\text{PolyLog}\left(3, \frac{bc-ad}{d(a+bx)} + 1\right)}{b} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right) \log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]^2/(a + b\*x), x]

[Out] -((Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]^2)/b) - (2\*Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]\*PolyLog[2, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/b + (2\*PolyLog[3, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/b

### Rule 2488

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] :> -Simp[(Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d))/h, Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

### Rule 2506



```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] := With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx} dx &= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right) \log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b} - \frac{(2(bc-ad)) \int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right) \log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)} dx}{b} \\ &= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right) \log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b} - \frac{2 \log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \text{Li}_2\left(1 + \frac{bc-ad}{d(a+bx)}\right)}{b} - \frac{(2(bc-ad)) \int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right) \log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)} dx}{b} \\ &= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right) \log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b} - \frac{2 \log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \text{Li}_2\left(1 + \frac{bc-ad}{d(a+bx)}\right)}{b} + \frac{2 \text{Li}_3\left(1 + \frac{bc-ad}{d(a+bx)}\right)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0421648, size = 135, normalized size = 0.96

$$\frac{-2 \text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right) \log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right) + 2 \text{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right) - \log\left(\frac{ad-bc}{d(a+bx)}\right) \log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(a + b*x),
x]
```

```
[Out] (-(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2) - 2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*
PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*PolyLog[3, (b*(c + d*x))/(d*(a
+ b*x))])/b
```

**Maple [B]** time = 0.065, size = 879, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a), x)$

[Out] 
$$-1/(b*c*f-b*d*e)*\ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*\ln(1-(b*c*f-b*d*e)/(a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*c*f+1/(b*c*f-b*d*e)*\ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*\ln(1-(b*c*f-b*d*e)/(a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*d*e-2/(b*c*f-b*d*e)*\ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*\text{polylog}(2, (b*c*f-b*d*e)/(a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*c*f+2/(b*c*f-b*d*e)*\ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*\text{polylog}(2, (b*c*f-b*d*e)/(a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*d*e+2/(b*c*f-b*d*e)*\text{polylog}(3, (b*c*f-b*d*e)/(a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*c*f-2/(b*c*f-b*d*e)*\text{polylog}(3, (b*c*f-b*d*e)/(a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*d*e$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\log(dx+c)^3}{a} - \int \frac{\left(\log(-be+af)^2 - 2\log(-be+af)\log(-de+cf) + \log(-de+cf)^2\right) b dx + \left(\log(-be+af)^2 - 2\log(-be+af)\log(-de+cf) + \log(-de+cf)^2\right) c}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a), x, \text{algorithm} = \text{"maxima"})$

[Out] 
$$\log(d*x + c)^3/a - \text{integrate}(-((\log(-b*e + a*f))^2 - 2*\log(-b*e + a*f)*\log(-d*e + c*f) + \log(-d*e + c*f)^2)*b*d*x + (\log(-b*e + a*f))^2 - 2*\log(-b*e + a*f)*\log(-d*e + c*f) + \log(-d*e + c*f)^2)*b*c + (b*d*x + b*c)*\log(b*x + a)^2 - 2*(b*d*x*(\log(-b*e + a*f) - \log(-d*e + c*f)) + b*c*(\log(-b*e + a*f) - \log(-d*e + c*f)))*\log(b*x + a) + 2*(b*d*x*(\log(-b*e + a*f) - \log(-d*e + c*f)) + b*c*(\log(-b*e + a*f) - \log(-d*e + c*f)) - (2*b*d*x + b*c + a*d)*\log(b*x + a))*\log(d*x + c))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x)$$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \frac{bce-acf+(bde-adf)x}{ade-acf+(bde-bcf)x} \right)^2}{bx+a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((-a\*f+b\*e)\*(d\*x+c)/(-c\*f+d\*e)/(b\*x+a))^2/(b\*x+a),x, algorithm="fricas")

[Out] integral(log((b\*c\*e - a\*c\*f + (b\*d\*e - a\*d\*f)\*x)/(a\*d\*e - a\*c\*f + (b\*d\*e - b\*c\*f)\*x))^2/(b\*x + a), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln((-a\*f+b\*e)\*(d\*x+c)/(-c\*f+d\*e)/(b\*x+a))\*\*2/(b\*x+a),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( \frac{(be-af)(dx+c)}{(de-cf)(bx+a)} \right)^2}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((-a\*f+b\*e)\*(d\*x+c)/(-c\*f+d\*e)/(b\*x+a))^2/(b\*x+a),x, algorithm="giac")

[Out] integrate(log((b\*e - a\*f)\*(d\*x + c)/((d\*e - c\*f)\*(b\*x + a)))^2/(b\*x + a), x)

$$3.105 \quad \int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right) \log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)} dx$$

**Optimal.** Leaf size=109

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right) \text{PolyLog}\left(2, \frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)} + 1\right)}{bc-ad} - \frac{\text{PolyLog}\left(3, \frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)} + 1\right)}{bc-ad}$$

[Out] (Log[(e\*(c + d\*x))/(a + b\*x)]\*PolyLog[2, 1 + ((b\*c - a\*d)\*(e + f\*x))/((d\*e - c\*f)\*(a + b\*x))])/(b\*c - a\*d) - PolyLog[3, 1 + ((b\*c - a\*d)\*(e + f\*x))/((d\*e - c\*f)\*(a + b\*x))]/(b\*c - a\*d)

**Rubi [A]** time = 0.163821, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 62,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$ , Rules used = {2506, 6610}

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right) \text{PolyLog}\left(2, \frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)} + 1\right)}{bc-ad} - \frac{\text{PolyLog}\left(3, \frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)} + 1\right)}{bc-ad}$$

Antiderivative was successfully verified.

[In] Int[(Log[(e\*(c + d\*x))/(a + b\*x)]\*Log[(-(b\*c) + a\*d)\*(e + f\*x)/((d\*e - c\*f)\*(a + b\*x))])]/((a + b\*x)\*(c + d\*x)), x]

[Out] (Log[(e\*(c + d\*x))/(a + b\*x)]\*PolyLog[2, 1 + ((b\*c - a\*d)\*(e + f\*x))/((d\*e - c\*f)\*(a + b\*x))])/(b\*c - a\*d) - PolyLog[3, 1 + ((b\*c - a\*d)\*(e + f\*x))/((d\*e - c\*f)\*(a + b\*x))]/(b\*c - a\*d)

### Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] + Dist[h*p*r*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)]/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rubi steps

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right) \log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)} dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right) \operatorname{Li}_2\left(1 + \frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad} + \int \frac{\operatorname{Li}_2\left(1 - \frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)} dx$$

$$= \frac{\log\left(\frac{e(c+dx)}{a+bx}\right) \operatorname{Li}_2\left(1 + \frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad} - \frac{\operatorname{Li}_3\left(1 + \frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad}$$

**Mathematica [A]** time = 0.0266982, size = 96, normalized size = 0.88

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right) \operatorname{PolyLog}\left(2, \frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right) - \operatorname{PolyLog}\left(3, \frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad}$$

Antiderivative was successfully verified.

[In] Integrate[(Log[(e\*(c + d\*x))/(a + b\*x)]\*Log[(-(b\*c) + a\*d)\*(e + f\*x)]/((d\*e - c\*f)\*(a + b\*x)))]/((a + b\*x)\*(c + d\*x)), x]

[Out] (Log[(e\*(c + d\*x))/(a + b\*x)]\*PolyLog[2, ((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))] - PolyLog[3, ((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))])/((b\*c - a\*d))

**Maple [F]** time = 2.268, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)} \ln\left(\frac{e(dx+c)}{bx+a}\right) \ln\left(\frac{(ad-bc)(fx+e)}{(-cf+de)(bx+a)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e\*(d\*x+c)/(b\*x+a))\*ln((a\*d-b\*c)\*(f\*x+e)/(-c\*f+d\*e)/(b\*x+a))/(b\*x+a)/(d\*x+c), x)

[Out] int(ln(e\*(d\*x+c)/(b\*x+a))\*ln((a\*d-b\*c)\*(f\*x+e)/(-c\*f+d\*e)/(b\*x+a))/(b\*x+a)/(d\*x+c), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/
(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(log(b*x + a)^2 - 2*(log(b*x + a) - log(e))*log(d*x + c) + log(d*x + c)
)^2 - 2*log(b*x + a)*log(e)*log(f*x + e)/(b*c - a*d) + integrate(1/2*(2*(e
*log(-b*c + a*d)*log(e) - e*log(d*e - c*f)*log(e))*b*c + (b*d*f*x^2 + 2*b*c
*e - (2*d*e - c*f)*a + (3*b*c*f - a*d*f)*x)*log(b*x + a)^2 - 2*(d*e*log(-b*
c + a*d)*log(e) - d*e*log(d*e - c*f)*log(e))*a + 2*((f*log(-b*c + a*d)*log(
e) - f*log(d*e - c*f)*log(e))*b*c - (d*f*log(-b*c + a*d)*log(e) - d*f*log(d
*e - c*f)*log(e))*a)*x - 2*(b*d*f*x^2*log(e) - (e*(log(d*e - c*f) - log(e))
- e*log(-b*c + a*d))*b*c + (d*e*(log(d*e - c*f) - log(e)) - d*e*log(-b*c +
a*d) + c*f*log(e))*a + ((f*log(-b*c + a*d) - f*log(d*e - c*f) + 2*f*log(e)
)*b*c - (d*f*log(-b*c + a*d) - d*f*log(d*e - c*f))*a)*x)*log(b*x + a) + 2*(
b*d*f*x^2*log(e) + (e*log(-b*c + a*d) - e*log(d*e - c*f))*b*c - (d*e*log(-b
*c + a*d) - d*e*log(d*e - c*f) - c*f*log(e))*a + ((f*log(-b*c + a*d) - f*lo
g(d*e - c*f) + f*log(e))*b*c - (d*f*log(-b*c + a*d) - (f*log(d*e - c*f) + f
*log(e))*d)*a)*x - (b*d*f*x^2 + 2*b*c*f*x + b*c*e - (d*e - c*f)*a)*log(b*x
+ a))*log(d*x + c))/(a*b*c^2*e - a^2*c*d*e + (b^2*c*d*f - a*b*d^2*f)*x^3 -
(a*b*d^2*e + a^2*d^2*f - (c*d*e + c^2*f)*b^2)*x^2 + (b^2*c^2*e + a*b*c^2*f
- (d^2*e + c*d*f)*a^2)*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( -\frac{(bc-ad)fx+(bc-ad)e}{ade-acf+(bde-bcf)x} \right) \log \left( \frac{dex+ce}{bx+a} \right)}{bdx^2 + ac + (bc + ad)x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/
(b*x+a)/(d*x+c),x, algorithm="fricas")
```

[Out] `integral(log(-((b*c - a*d)*f*x + (b*c - a*d)*e)/(a*d*e - a*c*f + (b*d*e - b*c*f)*x))*log((d*e*x + c*e)/(b*x + a))/(b*d*x^2 + a*c + (b*c + a*d)*x), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(e*(d*x+c)/(b*x+a))*ln((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(b*x+a)/(d*x+c), x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right) \log\left(-\frac{(bc-ad)(fx+e)}{(de-cf)(bx+a)}\right)}{(bx+a)(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(b*x+a)/(d*x+c), x, algorithm="giac")`

[Out] `integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)*(f*x + e)/((d*e - c*f)*(b*x + a)))/((b*x + a)*(d*x + c)), x)`

$$3.106 \quad \int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)} dx$$

**Optimal.** Leaf size=204

$$\frac{2\text{PolyLog}\left(3, \frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af} - \frac{2\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\text{PolyLog}\left(2, \frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af} - \frac{\log\left(1 - \frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af}$$

[Out]  $-\left(\frac{\text{Log}\left[\frac{(b*e - a*f)*(c + d*x)}{(d*e - c*f)*(a + b*x)}\right]}{(b*e - a*f)}\right)^2 \frac{\text{Log}\left[1 - \frac{(b*e - a*f)*(c + d*x)}{(d*e - c*f)*(a + b*x)}\right]}{(b*e - a*f)} - \frac{2*\text{Log}\left[\frac{(b*e - a*f)*(c + d*x)}{(d*e - c*f)*(a + b*x)}\right]*\text{PolyLog}\left[2, \frac{(b*e - a*f)*(c + d*x)}{(d*e - c*f)*(a + b*x)}\right]}{(b*e - a*f)} + \frac{2*\text{PolyLog}\left[3, \frac{(b*e - a*f)*(c + d*x)}{(d*e - c*f)*(a + b*x)}\right]}{(b*e - a*f)}$

**Rubi [A]** time = 0.253539, antiderivative size = 206, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 49,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$ , Rules used = {2503, 2506, 6610}

$$\frac{2\text{PolyLog}\left(3, \frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)} + 1\right)}{be-af} - \frac{2\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\text{PolyLog}\left(2, \frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)} + 1\right)}{be-af} - \frac{\log\left(-\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af}$$

Antiderivative was successfully verified.

[In]  $\text{Int}\left[\frac{\text{Log}\left[\frac{(b*e - a*f)*(c + d*x)}{(d*e - c*f)*(a + b*x)}\right]^2}{(a + b*x)*(e + f*x)}, x\right]$

[Out]  $-\left(\frac{\text{Log}\left[\frac{(b*c - a*d)*(e + f*x)}{(d*e - c*f)*(a + b*x)}\right]}{(b*e - a*f)}\right)^2 \frac{\text{Log}\left[-\frac{(b*c - a*d)*(e + f*x)}{(d*e - c*f)*(a + b*x)}\right]}{(b*e - a*f)} - \frac{2*\text{Log}\left[\frac{(b*c - a*d)*(e + f*x)}{(d*e - c*f)*(a + b*x)}\right]*\text{PolyLog}\left[2, 1 + \frac{(b*c - a*d)*(e + f*x)}{(d*e - c*f)*(a + b*x)}\right]}{(b*e - a*f)} + \frac{2*\text{PolyLog}\left[3, 1 + \frac{(b*c - a*d)*(e + f*x)}{(d*e - c*f)*(a + b*x)}\right]}{(b*e - a*f)}$

**Rule 2503**

$\text{Int}\left[\text{Log}\left[\frac{(e_*)*((f_*)*((a_*) + (b_*)*(x_))^{(p_*)}*((c_*) + (d_*)*(x_))^{(q_*)})^{(r_*)}}{(u_*)}\right]^s, x_{\text{Symbol}}\right] \rightarrow \text{With}\left[\{g = \text{Coeff}\left[\text{Simplify}\left[1/(u*(a + b*x))\right], x, 0\right], h = \text{Coeff}\left[\text{Simplify}\left[1/(u*(a + b*x))\right], x, 1\right]\}, -\text{Simp}\left[\frac{\text{Log}\left[e*(f*(a + b*x))^p*(c + d*x)^q\right]^r}{(b*g - a*h)} \frac{\text{Log}\left[-\frac{(b*c - a*d)*(g + h*x)}{(d*g - c*h)*(a + b*x)}\right]}{(b*g - a*h)}\right], x\right] + \text{Dist}\left[\frac{(p*r*s*(b*c - a*d))}{(b*g - a*h)}, \text{Int}\left[\frac{\text{Log}\left[e*(f*(a + b*x))^p*(c + d*x)^q\right]^r}{(b*g - a*h)} \frac{\text{Log}\left[-\frac{(b*c - a*d)*(g + h*x)}{(d*g - c*h)*(a + b*x)}\right]}{(d*g - c*h)}\right], x\right]$



```
c*h)*(a + b*x))))/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))], x]
```

### Rule 2506

```
Int[Log[v_]*Log[(e_)*((f_)*((a_) + (b_)*(x_))^(p_)*((c_) + (d_)*(x_))^(q_))^(r_)]^(s_)*(u_), x_Symbol] := With[{g = Simplify[((v - 1)*(c + d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[{h*PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] + Dist[h*p*r*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rubi steps

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)} dx = -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{(2(bc-ad)) \int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)} dx}{be-af}$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{2 \log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \text{Li}_2\left(1 + \frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{(2(bc-ad)) \int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)} dx}{be-af}$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{2 \log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \text{Li}_2\left(1 + \frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af} + \frac{2 \text{Li}_3\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af}$$

**Mathematica [B]** time = 0.469379, size = 1636, normalized size = 8.02

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/((a + b*x)*(e + f*x)), x]
```

```

[Out] (-2*Log[a/b + x]^3 + 3*Log[a/b + x]^2*Log[a + b*x] - 6*Log[a/b + x]*Log[c/d
+ x]*Log[a + b*x] + 3*Log[c/d + x]^2*Log[a + b*x] + 6*Log[a/b + x]*Log[c/d
+ x]*Log[(d*(a + b*x))/(-b*c + a*d)] - 3*Log[c/d + x]^2*Log[(d*(a + b*x)
)/(-b*c + a*d)] + 3*Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 3*Log
[a/b + x]^2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] + 6*Log[a/
b + x]*Log[a + b*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] -
6*Log[c/d + x]*Log[a + b*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a +
b*x))] + 6*Log[c/d + x]*Log[(d*(a + b*x))/(-b*c + a*d)]*Log[((b*e - a*f)*(
c + d*x))/((d*e - c*f)*(a + b*x))] + 3*Log[(-b*c + a*d)/(d*(a + b*x))]*Lo
g[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 + 3*Log[a + b*x]*Log[(
b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 - 3*Log[a/b + x]^2*Log[e
+ f*x] + 6*Log[a/b + x]*Log[c/d + x]*Log[e + f*x] - 3*Log[c/d + x]^2*Log[e
+ f*x] - 6*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x)
)]*Log[e + f*x] + 6*Log[c/d + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a
+ b*x))]*Log[e + f*x] - 3*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b
x))]^2*Log[e + f*x] + 3*Log[a/b + x]^2*Log[(b*(e + f*x))/(b*e - a*f)] - 6*L
og[a/b + x]*Log[(f*(c + d*x))/(-d*e + c*f)]*Log[(b*(e + f*x))/(b*e - a*f)
] + 3*Log[(f*(c + d*x))/(-d*e + c*f)]^2*Log[(b*(e + f*x))/(b*e - a*f)] +
6*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*
(e + f*x))/(b*e - a*f)] - 6*Log[(f*(c + d*x))/(-d*e + c*f)]*Log[((b*e - a
*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] + 3*
Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[(b*(e + f*x))/(b
*e - a*f)] - 6*Log[a/b + x]*Log[c/d + x]*Log[(d*(e + f*x))/(d*e - c*f)] + 3
*Log[c/d + x]^2*Log[(d*(e + f*x))/(d*e - c*f)] + 6*Log[a/b + x]*Log[(f*(c +
d*x))/(-d*e + c*f)]*Log[(d*(e + f*x))/(d*e - c*f)] - 3*Log[(f*(c + d*x)
)/(-d*e + c*f)]^2*Log[(d*(e + f*x))/(d*e - c*f)] - 6*Log[c/d + x]*Log[((b*
e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)
] + 6*Log[(f*(c + d*x))/(-d*e + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e -
c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)] - 3*Log[((b*e - a*f)*(c +
d*x))/((d*e - c*f)*(a + b*x))]^2*Log[(-b*c + a*d)*(e + f*x))/((d*e - c*f)
*(a + b*x))] + 6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-b*c + a*d)] + 6*
(Log[a/b + x] + Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)] + 6*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - 6*Log[((b*e - a*f)
*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d
*e - c*f)*(a + b*x))] - 6*PolyLog[3, (d*(a + b*x))/(-b*c + a*d)] - 6*Poly
Log[3, (b*(c + d*x))/(b*c - a*d)] - 6*PolyLog[3, (b*(c + d*x))/(d*(a + b*x)
)] + 6*PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]/(3*b*e
- 3*a*f)

```

---

**Maple [A]** time = 0.063, size = 357, normalized size = 1.8

$$\frac{1}{af-be} \left( \ln \left( -\frac{(af-be)(ad-bc)}{b(cf-de)(bx+a)} + \frac{(af-be)d}{b(cf-de)} \right) \right)^2 \ln \left( 1 + \frac{(af-be)(ad-bc)}{b(cf-de)(bx+a)} - \frac{(af-be)d}{b(cf-de)} \right) + 2 \frac{1}{af-be} \ln \left( -\frac{(af-be)(ad-bc)}{b(cf-de)(bx+a)} + \frac{(af-be)d}{b(cf-de)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln((-a\*f+b\*e)\*(d\*x+c)/(-c\*f+d\*e)/(b\*x+a))^2/(b\*x+a)/(f\*x+e),x)

[Out] 1/(a\*f-b\*e)\*ln(-(a\*f-b\*e)\*(a\*d-b\*c)/b/(c\*f-d\*e)/(b\*x+a)+d/(c\*f-d\*e)\*(a\*f-b\*e)/b)^2\*ln(1+(a\*f-b\*e)\*(a\*d-b\*c)/b/(c\*f-d\*e)/(b\*x+a)-d/(c\*f-d\*e)\*(a\*f-b\*e)/b)+2/(a\*f-b\*e)\*ln(-(a\*f-b\*e)\*(a\*d-b\*c)/b/(c\*f-d\*e)/(b\*x+a)+d/(c\*f-d\*e)\*(a\*f-b\*e)/b)\*polylog(2,-(a\*f-b\*e)\*(a\*d-b\*c)/b/(c\*f-d\*e)/(b\*x+a)+d/(c\*f-d\*e)\*(a\*f-b\*e)/b)-2/(a\*f-b\*e)\*polylog(3,-(a\*f-b\*e)\*(a\*d-b\*c)/b/(c\*f-d\*e)/(b\*x+a)+d/(c\*f-d\*e)\*(a\*f-b\*e)/b)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((-a\*f+b\*e)\*(d\*x+c)/(-c\*f+d\*e)/(b\*x+a))^2/(b\*x+a)/(f\*x+e),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \frac{bce-acf+(bde-adf)x}{ade-acf+(bde-bcf)x} \right)^2}{bfx^2+ae+(be+af)x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((-a\*f+b\*e)\*(d\*x+c)/(-c\*f+d\*e)/(b\*x+a))^2/(b\*x+a)/(f\*x+e),x, algorithm="fricas")

[Out] integral(log((b\*c\*e - a\*c\*f + (b\*d\*e - a\*d\*f)\*x)/(a\*d\*e - a\*c\*f + (b\*d\*e - b\*c\*f)\*x))^2/(b\*f\*x^2 + a\*e + (b\*e + a\*f)\*x), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln((-a\*f+b\*e)\*(d\*x+c)/(-c\*f+d\*e)/(b\*x+a))^2/(b\*x+a)/(f\*x+e), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{(bx+a)(fx+e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((-a\*f+b\*e)\*(d\*x+c)/(-c\*f+d\*e)/(b\*x+a))^2/(b\*x+a)/(f\*x+e), x, algorithm="giac")

[Out] integrate(log((b\*e - a\*f)\*(d\*x + c)/((d\*e - c\*f)\*(b\*x + a)))^2/((b\*x + a)\*(f\*x + e)), x)

$$3.107 \quad \int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx} dx$$

**Optimal.** Leaf size=322

$$\frac{2\text{PolyLog}\left(3, \frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f} - \frac{2\text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right) \log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f} + \frac{2 \log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right) \text{PolyLog}\left(2, \frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f}$$

[Out] -((Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]^2)/f) + (Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]^2\*Log[1 - ((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))])/f - (2\*Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]\*PolyLog[2, (b\*(c + d\*x))/(d\*(a + b\*x))])/f + (2\*Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]\*PolyLog[2, ((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))])/f + (2\*PolyLog[3, (b\*(c + d\*x))/(d\*(a + b\*x))])/f - (2\*PolyLog[3, ((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))])/f

**Rubi [A]** time = 0.507152, antiderivative size = 334, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 5, integrand size = 42,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$ , Rules used = {2489, 2488, 2506, 6610, 2503}

$$\frac{2\text{PolyLog}\left(3, 1 - \frac{(e+fx)(bc-ad)}{(c+dx)(be-af)}\right)}{f} + \frac{2\text{PolyLog}\left(2, 1 - \frac{bc-ad}{b(c+dx)}\right) \log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f} - \frac{2 \log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right) \text{PolyLog}\left(2, 1 - \frac{(e+fx)(bc-ad)}{(c+dx)(be-af)}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]^2/(e + f\*x), x]

[Out] -((Log[(b\*c - a\*d)/(b\*(c + d\*x))]\*Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]^2)/f) + (Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]^2\*Log[((b\*c - a\*d)\*(e + f\*x))/((b\*e - a\*f)\*(c + d\*x))])/f + (2\*Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]\*PolyLog[2, 1 - (b\*c - a\*d)/(b\*(c + d\*x))])/f - (2\*Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]\*PolyLog[2, 1 - ((b\*c - a\*d)\*(e + f\*x))/((b\*e - a\*f)\*(c + d\*x))])/f + (2\*PolyLog[3, 1 - (b\*c - a\*d)/(b\*(c + d\*x))])/f - (2\*PolyLog[3, 1 - ((b\*c - a\*d)\*(e + f\*x))/((b\*e - a\*f)\*(c + d\*x))])/f

**Rule 2489**

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] :> Dist[d/h, Int[Log[e\*(f\*(a +

```

b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]

```

### Rule 2488

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] := -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]

```

### Rule 2506

```

Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_
))^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] := With[{g = Simplify[((v - 1)*(c +
d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]

```

### Rule 6610

```

Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

```

### Rule 2503

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] := With[{g = Coeff[Simplify[1/(u*(a + b*x))],
x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
))))]/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x))))]/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
x]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx} dx &= \frac{d \int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{c+dx} dx}{f} - \frac{(de-cf) \int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(c+dx)(e+fx)} dx}{f} \\
&= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right)}{f} - \frac{(2(bc-ad)) \int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(c+dx)(e+fx)} dx}{f} \\
&= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right)}{f} + \frac{2 \log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right)}{f} \\
&= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right)}{f} + \frac{2 \log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right)}{f}
\end{aligned}$$

**Mathematica [B]** time = 0.234267, size = 1080, normalized size = 3.35

$$\log(e+fx) \log^2\left(\frac{a}{b}+x\right) - \log\left(\frac{b(e+fx)}{be-af}\right) \log^2\left(\frac{a}{b}+x\right) - 2 \log\left(\frac{c}{d}+x\right) \log(e+fx) \log\left(\frac{a}{b}+x\right) + 2 \log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log(e+fx)$$

Antiderivative was successfully verified.

[In] Integrate[Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))]^2/(e + f\*x), x]

[Out]  $(-\text{Log}[-(b*c) + a*d]/(d*(a + b*x)))*\text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 + \text{Log}[a/b + x]^2*\text{Log}[e + f*x] - 2*\text{Log}[a/b + x]*\text{Log}[c/d + x]*\text{Log}[e + f*x] + \text{Log}[c/d + x]^2*\text{Log}[e + f*x] + 2*\text{Log}[a/b + x]*\text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*\text{Log}[e + f*x] - 2*\text{Log}[c/d + x]*\text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*\text{Log}[e + f*x] + \text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*\text{Log}[e + f*x] - \text{Log}[a/b + x]^2*\text{Log}[(b*(e + f*x))/(b*e - a*f)] + 2*\text{Log}[a/b + x]*\text{Log}[(f*(c + d*x))/(-(d*e) + c*f)]*\text{Log}[(b*(e + f*x))/(b*e - a*f)] - \text{Log}[(f*(c + d*x))/(-(d*e) + c*f)]^2*\text{Log}[(b*(e + f*x))/(b*e - a*f)] - 2*\text{Log}[a/b + x]*\text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*\text{Log}[(b*(e + f*x))/(b*e - a*f)] + 2*\text{Log}[(f*(c + d*x))/(-(d*e) + c*f)]*\text{Log}[(b*(e + f*x))/(b*e - a*f)] - \text{Log}[(b*(e + f*x))/(b*e - a*f)] - \text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*\text{Log}[(b*(e + f*x))/(b*e - a*f)] + 2*\text{Log}[a/b + x]*\text{Log}[c/d + x]*\text{Log}[(d*(e + f*x))/(d*e - c*f)] - \text{Log}[c/d + x]^2*\text{Log}[(d*(e + f*x))/(d*e - c*f)] - 2*\text{Log}[a/b + x]*\text{Log}[(f*(c + d*x))/(-(d*e) + c*f)]*\text{Log}[(d*(e + f*x))/(d*e - c*f)] + \text{Log}[(f*(c + d*x))/(-(d*e) + c*f)]^2*\text{Log}[(d*(e + f*x))/(d*e - c*f)] + 2*\text{Log}[c/d + x]*\text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*\text{Log}$

$$\begin{aligned} & [(d*(e + f*x))/(d*e - c*f)] - 2*\text{Log}[(f*(c + d*x))/(-(d*e) + c*f)]*\text{Log}[(b*e \\ & - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] * \text{Log}[(d*(e + f*x))/(d*e - c*f)] \\ & + \text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 * \text{Log}[(b*c + a*d \\ & )*(e + f*x))/((d*e - c*f)*(a + b*x))] - 2*\text{Log}[(b*e - a*f)*(c + d*x))/((d*e \\ & - c*f)*(a + b*x))] * \text{PolyLog}[2, (b*(c + d*x))/(d*(a + b*x))] + 2*\text{Log}[(b*e - \\ & a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] * \text{PolyLog}[2, ((b*e - a*f)*(c + d*x) \\ & )/((d*e - c*f)*(a + b*x))] + 2*\text{PolyLog}[3, (b*(c + d*x))/(d*(a + b*x))] - 2* \\ & \text{PolyLog}[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]/f \end{aligned}$$

**Maple [B]** time = 0.072, size = 4733, normalized size = 14.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e), x)`

[Out] 
$$\begin{aligned} & -2*b^2/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*\text{polylog}(2, -(b*c*f-b*d*e)/(-a*d*f \\ & +b*d*e))*(-a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b) \\ & * \ln(-a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*d^2*e \\ & ^2*a+2*b^3/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*\text{polylog}(2, -(b*c*f-b*d*e)/(-a \\ & *d*f+b*d*e))*(-a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e) \\ & /b))*\ln(-a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*d \\ & *e^2*c+b^3/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*\ln(1+(b*c*f-b*d*e)/(-a*d*f+b \\ & *d*e))*(-a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*\ln \\ & (-a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*d*e^2 \\ & *c-2*b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*\text{polylog}(2, -(b*c*f-b*d*e)/(-a*d*f+b \\ & *d*e))*(-a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*\ln \\ & (-a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*c*a^2*d \\ & *f-b^3/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*\ln(1+(b*c*f-b*d*e)/(-a*d*f+b*d*e))* \\ & (-a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*\ln(-a*f \\ & -b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*c^2*e-2*b^3 \\ & /b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*\text{polylog}(2, -(b*c*f-b*d*e)/(-a*d*f+b*d*e))* \\ & (-a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*\ln(-a*f \\ & -b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*c^2*e+2/(a*f- \\ & b*e)/f/(a*d-b*c)*\ln(-a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a \\ & *f-b*e)/b)*\text{polylog}(2, -(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*( \\ & a*f-b*e)/b)*b^2*c*e+2/(a*f-b*e)/f/(a*d-b*c)*\text{polylog}(3, -(a*f-b*e)*(a*d-b*c)/ \\ & b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*a*b*d*e+1/(a*f-b*e)/f/(a*d-b*c \\ & )*\ln(-a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*\ln \\ & (1+(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)-d/(c*f-d*e)*(a*f-b*e)/b)*b^2*c*e \\ & -2*b^2/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*\text{polylog}(3, -(b*c*f-b*d*e)/(-a*d*f+b \end{aligned}$$



$$\begin{aligned}
& *d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))^*c \\
& ^2*a*f-2*b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(3,-(b*c*f-b*d*e)/(-a*d \\
& *f+b*d*e))*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b \\
& ))*d^2*e*a^2-1/(a*f-b*e)/(a*d-b*c)*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x \\
& +a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*ln(1+(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a) \\
& -d/(c*f-d*e)*(a*f-b*e)/b)*a*b*c-2/(a*f-b*e)/(a*d-b*c)*ln(-(a*f-b*e)*(a*d-b* \\
& c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,-(a*f-b*e)*(a*d-b \\
& *c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*a*b*c-2/(a*f-b*e)/f/(a*d-b \\
& *c)*polylog(3,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e \\
& )/b)*b^2*c*e+2*b^3/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(3,-(b*c*f-b*d*e) \\
& )/(-a*d*f+b*d*e))*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f \\
& -b*e)/b))^*c^2*e+1/(a*f-b*e)/(a*d-b*c)*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/ \\
& (b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*ln(1+(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b* \\
& x+a)-d/(c*f-d*e)*(a*f-b*e)/b)*a^2*d+2/(a*f-b*e)/(a*d-b*c)*ln(-(a*f-b*e)*(a* \\
& d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,-(a*f-b*e)*(a \\
& *d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*a^2*d+2/(a*f-b*e)/(a*d \\
& -b*c)*polylog(3,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b \\
& *e)/b)*a*b*c-2/(a*f-b*e)/(a*d-b*c)*polylog(3,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d* \\
& e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*a^2*d+2*b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d- \\
& b*c)*polylog(2,-(b*c*f-b*d*e)/(-a*d*f+b*d*e))*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d \\
& *e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))^*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/( \\
& b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*d^2*e*a^2+b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b* \\
& c)*ln(1+(b*c*f-b*d*e)/(-a*d*f+b*d*e))*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x \\
& +a)+d/(c*f-d*e)*(a*f-b*e)/b))^*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d \\
& /(c*f-d*e)*(a*f-b*e)/b)^2*d^2*e*a^2-1/(a*f-b*e)/f/(a*d-b*c)*ln(-(a*f-b*e)*( \\
& a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*ln(1+(a*f-b*e)*(a*d \\
& -b*c)/b/(c*f-d*e)/(b*x+a)-d/(c*f-d*e)*(a*f-b*e)/b)*a*b*d*e-2/(a*f-b*e)/f/(a \\
& *d-b*c)*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b \\
& ))*polylog(2,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/ \\
& b)*a*b*d*e+b^2/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*ln(1+(b*c*f-b*d*e)/(-a*d*f \\
& +b*d*e))*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))^ \\
& *ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*c^2 \\
& *a*f+2*b^2/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(2,-(b*c*f-b*d*e)/(-a*d \\
& *f+b*d*e))*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b \\
& ))*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*c^2 \\
& *a*f-2*b^3/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*polylog(3,-(b*c*f-b*d*e)/(-a \\
& *d*f+b*d*e))*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e) \\
& /b))*d*e^2*c+2*b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(3,-(b*c*f-b*d*e) \\
& )/(-a*d*f+b*d*e))*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f- \\
& b*e)/b))^*c*a^2*d*f+2*b^2/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*polylog(3,-(b* \\
& c*f-b*d*e)/(-a*d*f+b*d*e))*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f- \\
& d*e)*(a*f-b*e)/b))*d^2*e^2*a-b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*ln(1+(b*c* \\
& f-b*d*e)/(-a*d*f+b*d*e))*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d* \\
& e)*(a*f-b*e)/b))^*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a \\
& *f-b*e)/b)^2*c*a^2*d*f-b^2/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*ln(1+(b*c*f-
\end{aligned}$$

```
b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)
*(a*f-b*e)/b))*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f
-b*e)/b)^2*d^2*e^2*a
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \frac{bce-acf+(bde-adf)x}{ade-acf+(bde-bcf)x} \right)^2}{fx + e}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorithm
="fricas")
```

```
[Out] integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
b*c*f)*x))^2/(f*x + e), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(f*x+e),x)
```

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{fx+e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((-a\*f+b\*e)\*(d\*x+c)/(-c\*f+d\*e)/(b\*x+a))^2/(f\*x+e),x, algorithm="giac")

[Out] integrate(log((b\*e - a\*f)\*(d\*x + c)/((d\*e - c\*f)\*(b\*x + a)))^2/(f\*x + e), x )

$$3.108 \quad \int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)} dx$$

**Optimal.** Leaf size=433

$$\frac{\text{PolyLog}\left(3, \frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad} - \frac{\text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)} \log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\right)}{bc-ad} + \frac{\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right) \text{PolyLog}\left(2, \frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad} + \dots$$

[Out]  $-(\text{Log}[-((b*c - a*d)/(d*(a + b*x))])*\text{Log}[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2)/(2*(b*c - a*d)) - (\text{Log}[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*\text{Log}[(b*(e + f*x))/(b*e - a*f)])/(2*(b*c - a*d)) + (\text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*\text{Log}[1 - ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/(2*(b*c - a*d)) - (\text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*\text{PolyLog}[2, (b*(c + d*x))/(d*(a + b*x))])/(b*c - a*d) + (\text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*\text{PolyLog}[2, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/(b*c - a*d) + \text{PolyLog}[3, (b*(c + d*x))/(d*(a + b*x))]/(b*c - a*d) - \text{PolyLog}[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]/(b*c - a*d)$

**Rubi [A]** time = 0.594495, antiderivative size = 445, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 6, integrand size = 65,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.092$ , Rules used = {2507, 2489, 2488, 2506, 6610, 2503}

$$\frac{\text{PolyLog}\left(3, 1 - \frac{(e+fx)(bc-ad)}{(c+dx)(be-af)}\right)}{bc-ad} + \frac{\text{PolyLog}\left(2, 1 - \frac{bc-ad}{b(c+dx)} \log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\right)}{bc-ad} - \frac{\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right) \text{PolyLog}\left(2, 1 - \frac{(e+fx)(bc-ad)}{(c+dx)(be-af)}\right)}{bc-ad} + \dots$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*\text{Log}[(b*(e + f*x))/(b*e - a*f)]/((a + b*x)*(c + d*x)), x]$

[Out]  $-(\text{Log}[(b*c - a*d)/(b*(c + d*x)])*\text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2)/(2*(b*c - a*d)) - (\text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*\text{Log}[(b*(e + f*x))/(b*e - a*f)]/(2*(b*c - a*d)) + (\text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*\text{Log}[(b*c - a*d)*(e + f*x))/((b*e - a*f)*(c + d*x))]/(2*(b*c - a*d)) + (\text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*\text{PolyLog}[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) - (\text{Log}[(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*\text{PolyLog}[2, 1 - ((b*c - a*d)*(e + f*x))/((b*e - a*f)*(c + d*x))])/(b*c - a*d) + \text{PolyLog}[3, 1 - (b*c - a*d)/(b*(c + d*x))]/(b*c - a*d) - \text{PolyLog}[3, 1 - ((b*c - a*d)*(e + f*x))/((b*e - a*f)*(c + d*x))]/(b*c - a*d)$

$x)/((b*e - a*f)*(c + d*x)]/(b*c - a*d)$

### Rule 2507

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*Log[(i\_.)\*((j\_.)\*((g\_.) + (h\_.)\*(x\_))^(t\_.))^(u\_.)]\*(v\_), x\_Symbol] := With[{k = Simplify[v\*(a + b\*x)\*(c + d\*x)]}, Simp[(k\*Log[i\*(j\*(g + h\*x)^t]^u)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s + 1))/(p\*r\*(s + 1)\*(b\*c - a\*d)), x] - Dist[(k\*h\*t\*u)/(p\*r\*(s + 1)\*(b\*c - a\*d)), Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s + 1)/(g + h\*x), x], x] /; FreeQ[k, x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && NeQ[s, -1]

### Rule 2489

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Dist[d/h, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s/(c + d\*x), x], x] - Dist[(d\*g - c\*h)/h, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s/((c + d\*x)\*(g + h\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && NeQ[b\*g - a\*h, 0] && NeQ[d\*g - c\*h, 0] && IGtQ[s, 1]

### Rule 2488

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := -Simp[(Log[-(b\*c - a\*d)/(d\*(a + b\*x))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d)/h, Int[(Log[-(b\*c - a\*d)/(d\*(a + b\*x))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

### Rule 2506

Int[Log[v\_]\*Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*(u\_), x\_Symbol] := With[{g = Simplify[((v - 1)\*(c + d\*x))/(a + b\*x)], h = Simplify[u\*(a + b\*x)\*(c + d\*x)]}, -Simp[(h\*PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(b\*c - a\*d), x] + Dist[h\*p\*r\*s, Int[(PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v,

x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rule 2503

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))  
^(r\_.)]^(s\_.)\*(u\_), x\_Symbol] := With[{g = Coeff[Simplify[1/(u\*(a + b\*x))],  
x, 0], h = Coeff[Simplify[1/(u\*(a + b\*x))], x, 1]}, -Simp[(Log[e\*(f\*(a + b  
\*x)^p\*(c + d\*x)^q]^r]^s\*Log[-(((b\*c - a\*d)\*(g + h\*x))/((d\*g - c\*h)\*(a + b\*x  
)))])]/(b\*g - a\*h), x] + Dist[(p\*r\*s\*(b\*c - a\*d))/(b\*g - a\*h), Int[(Log[e\*(f  
\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)\*Log[-(((b\*c - a\*d)\*(g + h\*x))/((d\*g -  
c\*h)\*(a + b\*x)))])]/((a + b\*x)\*(c + d\*x)), x], x] /; NeQ[b\*g - a\*h, 0] && Ne  
Q[d\*g - c\*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a  
\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u\*(a + b\*x))],  
x]

### Rubi steps

$$\begin{aligned} \int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)} dx &= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{f \int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx} dx}{2(bc-ad)} \\ &= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{d \int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{c+dx} dx}{2(bc-ad)} - \frac{(de-cf) \int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(c+dx)(e+fx)} dx}{2(bc-ad)} \\ &= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} \\ &= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} \\ &= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} \end{aligned}$$

**Mathematica [B]** time = 0.798781, size = 908, normalized size = 2.1

$$\log\left(\frac{b(e+fx)}{be-af}\right) \log^2(c+dx) - \log\left(\frac{d(e+fx)}{de-cf}\right) \log^2(c+dx) - 2 \log(a+bx) \log\left(\frac{b(e+fx)}{be-af}\right) \log(c+dx) - 2 \log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \log\left(\frac{b(e+fx)}{be-af}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Log[((b\*e - a\*f)\*(c + d\*x))/((d\*e - c\*f)\*(a + b\*x))])\*Log[(b\*(e + f\*x))/(b\*e - a\*f)])/((a + b\*x)\*(c + d\*x)),x]

```
[Out] (-Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 - 2*Log[a + b*x]*Log[c + d*x]*Log[(b*(e + f*x))/(b*e - a*f)] + Log[c + d*x]^2*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a + b*x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[(b*(e + f*x))/(b*e - a*f)] - Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(b*(e + f*x))/(b*e - a*f)] - 2*Log[c + d*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] - Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a + b*x]*Log[c + d*x]*Log[(d*(e + f*x))/(d*e - c*f)] - Log[c + d*x]^2*Log[(d*(e + f*x))/(d*e - c*f)] - 2*Log[a + b*x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[(d*(e + f*x))/(d*e - c*f)] + Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(d*(e + f*x))/(d*e - c*f)] + 2*Log[c + d*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)] - 2*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)] + Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] - 2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] + 2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))] - 2*PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])]/(2*b*c - 2*a*d)
```

**Maple [F]** time = 2.336, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)} \ln\left(\frac{(-af+be)(dx+c)}{(-cf+de)(bx+a)}\right) \ln\left(\frac{b(fx+e)}{-af+be}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e))/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e))/(b*x+a)/(d*x+c),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*
e))/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\log \left( \frac{bce-acf+(bde-adf)x}{ade-acf+(bde-bcf)x} \right) \log \left( \frac{bfx+be}{be-af} \right)}{bdx^2 + ac + (bc + ad)x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*
e))/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
b*c*f)*x))*log((b*f*x + b*e)/(b*e - a*f))/(b*d*x^2 + a*c + (b*c + a*d)*x),
x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e)
)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left( \frac{(fx+e)b}{be-af} \right) \log \left( \frac{(be-af)(dx+c)}{(de-cf)(bx+a)} \right)}{(bx+a)(dx+c)} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*  
e))/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log((f*x + e)*b/(b*e - a*f))*log((b*e - a*f)*(d*x + c)/((d*e - c*  
f)*(b*x + a)))/((b*x + a)*(d*x + c)), x)
```



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```

```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```

```

56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71     else # result do not contain complex
72         # this assumes optimal do not as well
73         if debug then
74             print("result do not contain complex, this assumes optimal do not
as well");
75         fi;
76         if leaf_count_result<=2*leaf_count_optimal then
77             if debug then
78                 print("leaf_count_result<=2*leaf_count_optimal");
79             fi;
80             return "A";
81         else
82             if debug then
83                 print("leaf_count_result>2*leaf_count_optimal");
84             fi;
85             return "B";
86         end if
87     end if
88     else #ExpnType(result) > ExpnType(optimal)
89         if debug then
90             print("ExpnType(result) > ExpnType(optimal)");
91         fi;
92         return "C";
93     end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417

```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'+'') or type(expn,'*') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```



```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157 ElementaryFunctionQ := proc(func)
158     member(func,[
159         exp,log,ln,
160         sin,cos,tan,cot,sec,csc,
161         arcsin,arccos,arctan,arccot,arcsec,arccsc,
162         sinh,cosh,tanh,coth,sech,csch,
163         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
164 end proc:
165
166 SpecialFunctionQ := proc(func)
167     member(func,[
168         erf,erfc,erfi,
169         FresnelS,FresnelC,
170         Ei,Ei,Li,Si,Ci,Shi,Chi,
171         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
172         EllipticF,EllipticE,EllipticPi])
173 end proc:
174
175 HypergeometricFunctionQ := proc(func)
176     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
177 end proc:
178
179 AppellFunctionQ := proc(func)
180     member(func,[AppellF1])
181 end proc:
182
183 # u is a sum or product. rest(u) returns all but the
184 # first term or factor of u.
185 rest := proc(u) local v;
186     if nops(u)=2 then
187         op(2,u)
188     else
189         apply(op(0,u),op(2..nops(u),u))
190     end if
191 end proc:
192
193 #leafcount(u) returns the number of nodes in u.
194 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```

```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

## 4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by

```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

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48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

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93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

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133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

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```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183             else:
184                 return "B"
185         else: #result contains complex but optimal is not
186             return "C"
187     else: # result do not contain complex, this assumes optimal do not as
188         well
189         if leaf_count_result <= 2*leaf_count_optimal:
190             return "A"
191         else:
192             return "B"
193     else:
194         return "C"
```